Energy Efficient Communication in Ad Hoc Networks from User's and Designer's Perspective*

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Abstract

Wireless ad hoc networks have gained a lot of attention in recent years. We consider a game that models the creation of such a network, where nodes are owned by selfish agents. We study a novel cost sharing model in which agents may pay for the transmission power of the other nodes. Each agent has to satisfy some connectivity requirement in the final network and the goal is to minimize its payment with no regard to the overall system performance. We analyze two fundamental connectivity games, namely broadcast and convergecast. We study pure Nash equilibria and quantify the degradation in the network performance called the price of anarchy resulting from selfish behavior. We derive tight

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bounds on the price of anarchy for these games.

We also study centralized network design. One of the most important problems in wireless ad hoc networks is the minimum-energy broadcast. Recently, there appeared many new applications such as real-time multimedia, battlefield communications and rescue operations that impose stringent end-to-end latency requirement on the broadcasting time. However, the existing algorithms that minimize the broadcasting energy tend to produce solutions with high latency. In this paper we consider the problem of bounded-hop broadcast. We present approximation and heuristic algorithms for this problem.

1 Introduction

The next generation communication networks are likely to be a combination of wireline and ad hoc networks, which are expected to fulfill a critical role where wired backbone networks are not available or not economical to build [10, 34]. A communication session in a wireless network is achieved either through a single-hop transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise. Depending on its power level and on the nature of environmental interference, a node can reach all nodes in a certain range. Typically, the signal power falls as $1/d^{\alpha}$, where d is the distance from the transmitter antenna and α is a constant between 2 and 4 depending on the environment [35]. All receivers have the same power threshold for signal detection, which is typically normalized to one. Under the above assumptions, the power required to establish a link between two nodes at distance d is d^{α}. In this paper we assume that nodes are located in a Euclidean space and consider the symmetric energy model. In ad hoc networks devices are usually equipped with battery that has a limited power. Thus, among the most crucial issues is that of developing energy-efficient topology control algorithms, which maximize the network lifetime [20].

Most of the existing works on wireless networks are based on the assumption that there is a centralized control, or the nodes run the same distributed algorithm. While this assumption may hold for some networks, such as military or government networks, it is not valid in general for Internet-like networks. In such a network, nodes are owned by different commercial entities, which are strongly driven by their economic interests. This naturally gives rise to many game-theoretic issues. The stable outcomes of the interaction between non-cooperative selfish agents correspond to Nash equilibria. A Nash equilibrium [32] can be viewed as a solution that selfish agents can agree upon, i.e., the agents have no incentive to deviate. Since Nash equilibria in network games can be much more expensive than the best centralized design, it is important to consider the implications of selfish behavior on the network performance. Recently, game-theoretical analysis of ad hoc networks has received a great deal of attention [14, 4, 30, 19, 38, 13].

Traditionally, in Computer Science research has been focused on finding a global optimum. With the emerging interest in computational issues related to game theory, Koutsoupias and Papadimitriou [28] introduced so called *price of anarchy*. The *pessimistic* price of anarchy is the ratio between the cost of the worst possible Nash equilibrium (the one with the maximum social cost) and the cost of the social optimum. Roughgarden and Tardos [36] derive the price of anarchy of selfish routing in networks. Ansheleich et al. [2] propose to consider the *optimistic* price of anarchy, which is the ratio between the cost of the best possible Nash equilibrium (the one with the minimum social cost) and that of the social optimum.

In this paper we consider the following game modeling the creation of a wireless ad hoc network. Each agent (node) has a specific connectivity requirement, i.e., each agent has to build a network in which this requirement is satisfied. We study a novel cost sharing model in which the transmission power of each node can be paid by different agents. In the model *without cost sharing*, each agent pays only for the transmission power of its own node. The goal of each agent is to pay as little as possible. Note that the price of anarchy measures the cost of *lack of coordination* between the agents. Thus, we assume that agents have *complete information*. We consider only pure strategies since mixed (probabilistic) Nash equilibria do not seem to be suitable for network design [17]. We study broadcast and convergecast problems, which are fundamental communication tasks in ad hoc networks. In the Broadcast Game, each agent has to establish a directed path between a designated root node and its own node while in the Convergecast Game, each agent has to establish a directed path between its own node and a designated root node.

The rapidly increasing capabilities and low costs of computing and communication devices have made it possible to use wireless networks in a wide range of applications such as real-time multimedia, battlefield communications, and rescue operations. The above applications impose stringent end-to-end latency requirement on the broadcasting time, which is the time taken by the message to reach all the nodes in the network. The latency of a broadcast scheduling algorithms is at least $\Omega(D)$, since the message needs to reach the furthest node from the source, where D is the network diameter. Gaber and Mansour [22] show that for any graph with $D = \Omega(\log^5 n)$, there is a centralized randomized schedule with latency $O(D)$, where n is the number of nodes.

Unfortunately, in the existing minimum-energy broadcast algorithms the latency of the broadcasting tends to be quite large since the minimum energy is attained when the number of the relay nodes is maximized, which results in a large network diameter. We consider the problem of *bounded-hop broadcast*, introduced in [12]. In this problem, we aim to construct a minimum-energy communication graph of bounded diameter rooted at the source node in which all nodes are covered. Note that there is a trade-off between reaching more nodes in a single hop using higher power and thus decreasing the network diameter and reaching fewer nodes using lower power and thus increasing the network diameter. The unrestricted version of this problem is NP-hard [6]. For a line topology, this problem can be solved optimally using the dynamic programming algorithm of Clementi et al. [11].

Our results. First we consider the network creation games. For the Broadcast Game with cost sharing, we show that in contrast to the single-source connection game of [2], a pure Nash equilibrium may not exist and the optimistic price of anarchy is bounded away from one. We also establish that the pessimistic price of anarchy is $\Theta(n)$. We note that the Broadcast Game is impossible without cost sharing. For the Convergecast Game without cost sharing, we demonstrate that the optimistic price of anarchy is 1 while the pessimistic price of anarchy is $\Theta(n^{\alpha-1})$. Interestingly, the Convergecast Game has a lower pessimistic price of anarchy compared to that of the strong connectivity game in [15], which is $\Omega(n^{\alpha})$. We show that if we allow cost sharing, the pessimistic price of anarchy is improved significantly to $\Theta(n)$ while the optimistic price of anarchy remains the same.

Then we consider algorithms for centralized network design. For the bounded-hop broadcast problem, we present a simple algorithm that has an approximation factor of $\min(D^{\alpha-1}, 12(n/D)^{\log\beta})$, where D is the bound on the network diameter and β is a constant that depends on α. In particular, for $\alpha = 2$ we have $\beta \leq 2$, and the approximation factor is at most $O(\sqrt{n})$ for any D and at most $O(\log n)$ for $D = O(\log n)$ and $D = \Omega(n/\log n)$. Thereafter, we derive the bounded-diameter minimum spanning tree (BDMST) algorithm that achieves an approximation factor of $O(f(n) \cdot \log n)$, where $f(n)$ is the worst-case ratio between the energy cost of an optimal BDMST and that of an optimal solution for the bounded-hop broadcast problem. However, the running time of this algorithm is rather high because of the complexity of the BDMST problem. In addition, we propose the decremental distance heuristic, which is easy to implement from the practical point of view and has a polynomial running time (without large hidden coefficients). Finally, we show how to solve optimally in polynomial time the full-duplex (bi-directional) connection problem and the generalized multicast and the web-conferencing problems for a fixed number of nodes.

Paper organization. Our model is presented in Section 3. Section 2 describes the related work. We analyze the networks creation games in Section 4. Algorithms for the bounded-hop broadcast problem are presented in Section 5. Optimal algorithms for some connectivity problems are given in Section 6. We conclude with Section 7.

2 Related Work

Our paper is closely related to the work of Eidenbenz et al. [15], which considers topology control problems in ad hoc networks, where nodes choose their power levels in order to ensure the desired connectivity properties. Eidenbenz et al. give asymptotically tight bounds on the price of anarchy for the strong connectivity game and demonstrate that for the connectivity game it is NP-complete to decide whether a pure Nash equilibrium exists. In contrast to [15], we allow cost sharing in our games. Ansheleich et al. [2] consider a network design game for wired networks, where each agent has to connect a set of terminals. Ansheleich et al. show that determining whether a pure Nash equilibrium exists is NP-complete and present a polynomial-time algorithm that computes a 3-approximate Nash equilibrium. We define a cost sharing model similar to that of [2] for ad hoc networks. Fabricant et al. [16] study a wireline network creation game, where the cost of a node depends on the distances to the other nodes.

The problem of bounded-hop connectivity has been studied extensively. Optimal algorithms based on dynamic programming for the problem of bounded-hop strong connectivity and the problem of bounded-hop broadcast on a line are presented by Kirousis et al. [26] and Clementi at al. [11], respectively. Beier et al. [3] and Funke et al. [21] give efficient algorithms for finding a minimum-energy route between two given nodes that uses a bounded number of hops. The problem of bounded-hop broadcast is studied in a recent paper by Ambuhl et al. [1]. Ambuhl et al. give a polynomial-time algorithm based on dynamic programming for finding an optimal solution for 2-hop broadcast with running time $O(n^7)$ and derive a PTAS for the case in which the bound on the number of hops is fixed. Contrary to [1], in this paper we consider the whole range of possible values of D.

The problem of minimum-energy broadcasting in which the transmission power of each node has to be determined so that the total power is minimized has received recently a great deal of attention. Cagalj et al. [6] give a proof of NP-hardness of the minimum-energy broadcast problem in Euclidean space. Wieselthier et al. [40] propose three greedy heuristics, namely the minimum spanning tree (MST), the shortest path tree (SPT) and the broadcasting incremental power (BIP), and evaluate them through simulations. Wan et al. [39] present the first analytical results for this problem. In particular, they prove that the approximation ratio of MST is between 6 and 12 while the approximation ratio of SPT is at least $n/2$. Several approximation methods with analytical bounds for the asymmetric model are proposed by Liang [29]. Some of these results have been improved by Caragiannis at al. [8] for the symmetric model. Calinescu et al. [7] consider the problem of maximizing network lifetime as well as different energy efficient network connectivity problems. Cartigny et al. [9] develop localized algorithms for minimum-energy broadcasting.

3 Model and Notation

We assume that there are n nodes, which are located on a Euclidean space. We denote by $\mathcal{P}$ the placement of nodes. A wireless network is represented by a directed *communication graph* $G = (V, E)$, where V is the set of nodes and E is the set of edges. The neighbors of a node u are determined by its *transmission power* P_u. Namely, node u can reach all nodes within its *transmission range* $R_u = P_u^{1/\alpha}$, where $2 \leq \alpha \leq 4$ is the the distance-power gradient.* Thus, edge (u, v) belongs to E if the distance between u and v, $d(u, v)$, is at most R_u. We consider the symmetric model in which the energy cost of an edge (u, v) is the same as that of (v, u).

In a minimum-energy network design problem, the goal is to assign transmission powers to

*For simplicity, we assume that the maximum transmission range of a node is unbounded.

the nodes in order to establish the desired connectivity requirement while minimizing the total energy of the nodes. Note that it is always sufficient to consider at most n different transmission ranges for each node, which are the distances to the other nodes and zero transmission range. We denote by OPT an optimal solution. We say that an algorithm A has the approximation factor of c, if the total power of the solution produced by A, $P(A)$, is at most c times that of OPT, $P(OPT)$, for any instance of the problem.

3.1 Broadcast and Convergecast Games

We assume that each node is an agent. A strategy of node u is a payment function P_v^u, that is how much energy u is willing to contribute to the transmission power of node v. The actual transmission power of node u is defined as $P_u = \sum_{v \in V} P_u^v$. The goal of node u is to minimize its total payment, that is $\sum_{v \in V} P_v^u$. We assume that nodes have complete information. The game is said to be in a *Nash equilibrium* if the connectivity requirement of each agent is satisfied and no agent can find a better (with lower cost) alternative strategy with respect to the current strategies of the other agents (i.e., assuming that the strategies of the other agents are fixed).

The social cost of the outcome of a game is the total power of the nodes. For a placement of nodes $\mathcal{P}$, we denote the cost of a Nash equilibrium by $C_{\mathcal{P}} = \sum_{u \in V} P_u$ and the cost of OPT by $C^*_{\mathcal{P}} = \sum_{u \in V} P^*_u$, where P^*_u is the transmission power of u under OPT. The *pessimistic price of anarchy* is $\max_{\mathcal{P}} C_{\mathcal{P}}/C^*_{\mathcal{P}}$ while the *optimistic price of anarchy* is $\min_{\mathcal{P}} C_{\mathcal{P}}/C^*_{\mathcal{P}}$.

We study the *Broadcast Game*: there is a designated root node r that has a message that must be delivered to all nodes, i.e., each node u has to establish a directed path $r \to u$. In this game, the transmission power of each node can be paid by different nodes. This model is inspired by the model of [2] for wired networks. We assume that each node, once it is connected, forwards data packets of the other nodes. Observe that the Broadcast Game requires cost sharing, since a node cannot establish a path from the root by controlling only its own power.

We also consider the *Convergecast Game*: each node has a message that must be delivered to a designated root node r, i.e., each node u has to establish a directed path $u \to r$. In this game, each agent u contributes only to the transmission power of its own node, i.e., $P_v^u = 0$ for $v \neq u$. This model has been introduced in [15]. We assume that multiple messages can be aggregated into a single packet and thus we count the energy cost of each edge only once. We will also extend the analysis of the Convergecast Game to the cost sharing model.

3.2 Bounded-Hop Broadcast

For each node u, we consider a shortest path in G between a designated root node r and u. The *diameter* of the communication graph is the maximal length of such a path. The *bounded-hop broadcast* problem is to find the energy assignment for broadcast that minimizes the total energy, that is $\sum_u P_u$, subject to the constraint that the diameter of the resulting communication graph is bounded by D.

Note that if $D = n$, the problem is just the classical minimum-energy broadcast problem, which is NP-hard [6]. On the other hand, if $D = 1$, there exists only a trivial solution in which r has the transmission range equal to the distance to the furthest node. We denote this distance by L, i.e., $L = \max_u d(r, u)$.

4 Network Creation Games

In this section we consider the network creation games.

4.1 Broadcast Game

We show that in contrast to the single-source connection game of [2], a pure Nash equilibrium for the Broadcast Game may not exist and the optimistic price of anarchy is bounded away from one. We also demonstrate that the pessimistic price of anarchy is $\Theta(n)$.

Definition 4.1 *We say that node u covers node v if there exists a directed simple path in the underlying communication graph G from r to v such that u precedes v on this path.*

The following observation characterizes the cost sharing in a Nash equilibrium.

Observation 1 *Consider a Nash equilibrium. We have that for each node v covered by u such that $d(u, v) < R_u$, $P_u^v = 0$, i.e., v does not contribute anything to the transmission power of u.*

If it is not the case, v would always have incentive to decrease P_u^v because it would still remain covered by u.

First we give an example of the Broadcast Game for which a pure Nash equilibrium does not exist.

Theorem 4.1 *The Broadcast Game may possess no pure Nash equilibrium.*

Proof: Consider the following scenario (see Figure 1). Node u is at distance d from the root in one direction and two other nodes v and w are at distance $d/2$ and $d+\epsilon$ in the opposite direction, respectively. Obviously, in a Nash equilibrium the transmission range R_r of r is either $d/2$, d, or $d + \epsilon$. We will show that all these situations are unstable.

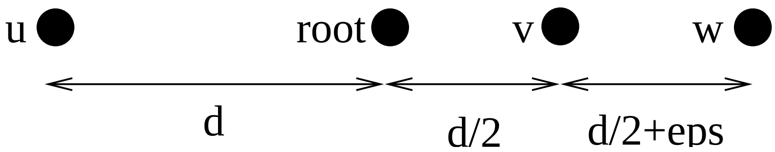

Figure 1: An example of the Broadcast Game without a pure Nash equilibrium.

If $R_r = d/2$, then u would always increase its payment to r to assure $R_r = d$ because it is the cheapest way for u to get covered.

If $R_r = d$, then by Observation 1, v does not pay anything to r. We argue that the payment of w to r is also zero because it would only to pay to v for a transmission range of $d/2 + \epsilon$. Therefore, u fully pays to the root for a transmission range of d. In this case w has incentive to increase its payment to the root from 0 to $(d + \epsilon)^\alpha - (d)^\alpha$ to make its transmission range be $d + \epsilon$ and decrease its payment to v to zero.

If $R_r = d+\epsilon$, Observation 1 implies that w fully pays for the energy of the root $P_r = (d+\epsilon)^\alpha$. However, w can benefit from decreasing its payment to the root to $(d/2)^\alpha$ and increasing its payment to v to $(d/2 + \epsilon)^\alpha$. ∎

The next theorem shows that the optimistic price of anarchy is greater than 1.

Theorem 4.2 *The optimistic price of anarchy for the Broadcast Game is bounded away from* 1.

Proof: Consider the following scenario (see Figure 2). There are three rays emanating from the root and the the angle between two adjacent rays is 120 degrees. On the first two rays there are two nodes at distance $d-x$ and $d+x$ from the root, respectively. On the third ray there is a node at distance d from the root.

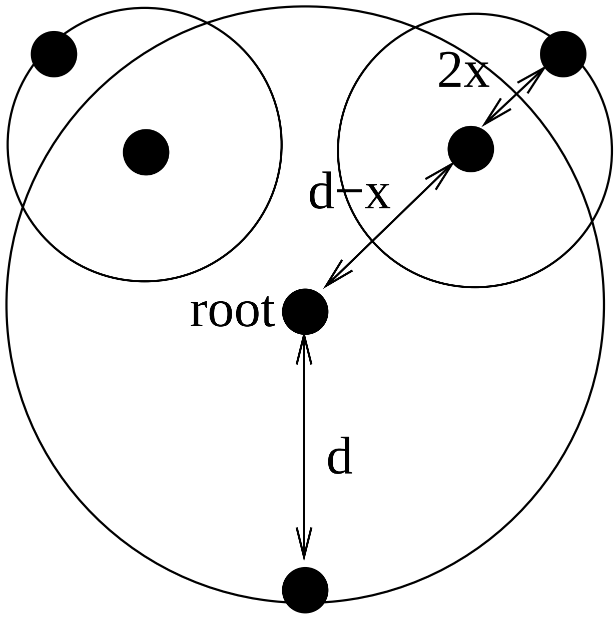

Figure 2: An example of the Broadcast Game in which the optimistic price of anarchy is bounded away from 1.

Consider a situation in which the node on the last ray pays to the root for a transmission range of d and the furthest node on each of the first two rays pays to the first node on its ray for a transmission range of $2x$. It is a unique Nash equilibrium provided that no furthest node can benefit from increasing the transmission range of the root, i.e.,

$$(d+x)^\alpha - d^\alpha > (2x)^\alpha. \tag{1}$$

We have that the price of anarchy is bounded away from 1 if

$$d^\alpha + 2(2x)^\alpha > (d+x)^\alpha. \tag{2}$$

Note that OPT just assigns a transmission range of $d + x$ to the root. In case $\alpha = 2$, the inequalities (1, 2) hold for $\frac{2}{7}d < x < \frac{2}{3}d$. ∎

Now we proceed to study the pessimistic price of anarchy. The following theorem establishes a simple upper bound of n on the price of anarchy, which turns out to be asymptotically tight.

Theorem 4.3 *The pessimistic price of anarchy for the Broadcast Game is at most n.*

The theorem holds due to the fact that no agent spends more energy than OPT does, otherwise it can connect to the root by paying at most C^*.

The following theorem derives a lower bound of $\Omega(n)$ on the pessimistic price of anarchy.

Theorem 4.4 *The pessimistic price of anarchy for the Broadcast Game is at least $\Omega(n)$.*

Proof: Consider the following scenario (see Figure 3). There is a circle c_1 with radius d whose center is the root and a circle c_2 with a smaller radius. A set S_1 of nodes is located on the external part of c_2 w.r.t. c_1 such that the first and the last nodes are located at the intersection of c_1 and c_2. Another set S_2 of nodes is located on the radius connecting the last point in S_1 to the root and the distance between the last point of S_1 and the first point of S_2 is x. We assume that $|S_1| = |S_2| = n/2$ and the distance between any two adjacent nodes in S_1 and in S_2 is $\delta = 2(d - x)/n$.

Suppose that each node in S_1 pays to the root $d^\alpha/|S_1|$ and all but the first and the last nodes pay to the succeeding node in S_1 for the energy worth δ^α. Nodes in S_2 do not pay anything. The only alternative strategy for a node in S_1 is to decrease its payment to the root to zero and

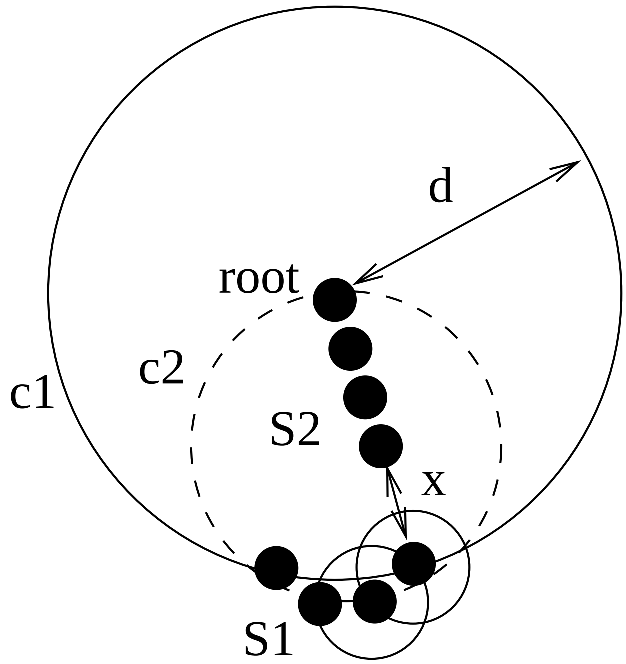

Figure 3: An example of the Broadcast Game in which the price of anarchy is $\Omega(n)$.

pay x^α to the first node in S_2 to connect the last node in S_1. Thus, the system is in a Nash equilibrium if $x^\alpha \geq d^\alpha/|S_1|$. We have that $C = d^\alpha + \frac{n-4}{2}\delta^\alpha$.

Note that OPT sets the transmission ranges of all but the first nodes in S_1 and S_2 to δ and assigns a transmission range of 0 and x to the first node in S_1 and to the first node in S_2, respectively. Thus, $C^* = x^\alpha + (n-2)\delta^\alpha$. Therefore, the pessimistic price of anarchy is at least

$$\frac{n}{2} \cdot \frac{d^\alpha + \frac{n-4}{2}\delta^\alpha}{d^\alpha + \frac{n(n-2)}{2}\delta^\alpha} = \Omega(n),$$

when n tends to infinity. ∎

4.2 Convergecast Game

We show that for the Convergecast Game, the optimistic price of anarchy is 1 while the pessimistic price of anarchy is $\Theta(n^{\alpha-1})$. Then we extend our results to the cost sharing model. We demonstrate that the pessimistic price of anarchy becomes $\Theta(n)$ while the optimistic price of anarchy remains 1. We note that the MST algorithm for strong connectivity by Kirousis et al. [26] implicitly finds an optimal solution for the minimum-energy convergecast in polynomial time.

The following theorem shows that the optimistic price of anarchy is one.

Theorem 4.5 *The optimistic price of anarchy for the Convergecast Game is* 1.

Clearly, OPT is a Nash equilibrium since no node can decrease its transmission range and still stay connected to the root.

Corollary 4.6 *There always exists a pure Nash equilibrium for the Convergecast Game.*

Next we derive an upper bound on the pessimistic price of anarchy.

Theorem 4.7 *The pessimistic price of anarchy for the Convergecast Game is at most* $O(n^{\alpha-1})$.

Proof: Consider a Nash equilibrium. Enumerate the nodes in order of non-increasing transmission range: $R_1 \geq R_2 \cdots \geq R_n$. Let Z be the sum of the transmission ranges of the nodes under OPT. We claim that $R_i \leq Z/i$. Consider a set of nodes S_i containing the first i nodes and the root. We claim that the distance between any two nodes in S_i at least R_i. If it is not the case, at least one node can decrease its transmission power without losing connectivity to the root, which contradicts the stability of a Nash equilibrium. We obtain that $Z \geq iR_i$ since OPT must connect all these nodes to the root. Hence, the cost of the Nash equilibrium is at most

$$C \leq \sum_{i=1}^{n} (Z/i)^\alpha = Z^\alpha \sum_{i=1}^{n} 1/i^\alpha \leq Z^\alpha \cdot const(\alpha),$$

where $const(\alpha)$ is a positive constant which depends on $\alpha > 1$. For example, for $\alpha = 2$ we have $const(\alpha) \leq \pi^2/6$. On the other hand, $C^* \geq n(Z/n)^\alpha$, since the transmission power is minimized when all nodes are evenly spaced. Dividing the bound for C by the bound for C^* we conclude the proof of the theorem. ∎

In the next theorem we establish a lower bound of $\Omega(n^{\alpha-1})$ on the pessimistic price of anarchy, which matches our upper bound.

Theorem 4.8 *The pessimistic price of anarchy for the Convergecast Game is at least* $\Omega(n^{\alpha-1})$.

Proof: Let ϵ be a small positive constant. Consider the following scenario (see Figure 4). All nodes are located on the line and the root is the first node. The distance between the root and the first regular (other than root) node is $1+\epsilon$ and the distance between any two adjacent regular nodes is 1. The transmission range of the regular nodes but the last one is 1 and the transmission range of the last node is $n-1+\epsilon$.

Clearly, it is a Nash equilibrium since all nodes are connected to the root and no node can decrease its transmission power. The cost of this Nash equilibrium is

$$C = n - 2 + (n - 1 + \epsilon)^\alpha,$$

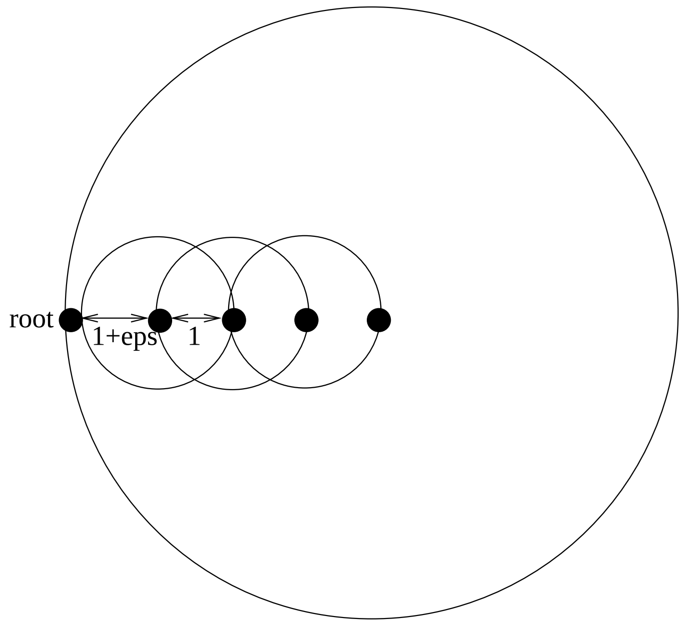

Figure 4: A bad example for the Convergecast Game.

while

$$C^* = n - 2 + (1+\epsilon)^\alpha,$$

since OPT sets the transmission range of all regular nodes but the first one to 1 and the transmission range of the first node to $1+\epsilon$. ∎

Note that if we allow cost sharing, the optimistic price of anarchy remains the same. That is due to the fact that OPT is still a Nash equilibrium if each node pays for its own transmission energy. However, the pessimistic price of anarchy drops to $\Theta(n)$, which shows the benefit of cost sharing. The proof of the upper bound is trivial. The proof of the lower bound is almost identical to that of Theorem 4.8 and is omitted from this abstract.

5 Bounded-hop Broadcast

In this section we present approximation algorithms for the bounded-hop broadcast problem.

5.1 Simple Algorithm

In this section we give a simple $\min(D^{\alpha-1}, 12(n/D)^{\log\beta})$-approximation algorithm, where β is a constant that depends on α. First we derive two obvious lower bounds on $P(OPT)$. The next lower bound will be useful for small values of D. Recall that $L = \max_u d(r, u)$.

Lemma 5.1 *For a given D, $P(OPT) \geq D\,(L/D)^{\alpha}$.*

Proof: The energy of OPT is at least $D\,(L/D)^{\alpha}$ since it must establish a path between r and the furthest node and the transmission power is minimized when nodes are evenly spaced and the number of relay nodes is maximized. ∎

The following lower bound will be useful for large values of D.

Theorem 5.2 ([39]) *For any value of D, $P(OPT) \geq P(MST)/12$.*

Now we present two basic algorithms. The first algorithm called root cover (RC) is a trivial approach in which the root covers all the nodes. Note that this algorithm has a good performance for small values of D. A similar claim has been made in [1] without a proof.

Theorem 5.3 *The approximation factor of the RC algorithm is at most $D^{\alpha-1}$.*

Proof: Note that the total energy of the root cover algorithm is L^{α}. On the other hand, Lemma 5.1 implies that the total energy of OPT is at least $D\,(L/D)^{\alpha}$. We obtain that $P(RC)/P(OPT) \leq D^{\alpha-1}$. ∎

In the second algorithm, we start from a shortest path tree T corresponding to the communication graph produced by the MST algorithm. The l-th layer of T contains all nodes at distance l from the root. We will contract the odd layers of T until the diameter drops below D. The layer contraction algorithm is presented in Figure 5. Observe that this algorithm would perform better for large values of D.

1. Apply the MST algorithm [39] and set the transmission range of each node accordingly.
2. If the diameter of the communication graph is smaller than D, return the current solution.
3. Construct a shortest path tree T of G rooted at r.
4. Contract all odd layers of T in the following way:
   (a) For each layer l s.t. l is even and each node u in layer l set the transmission range of u to $\max_{v,w:(u,v)\in T,(v,w)\in T}(d(u,v)+d(v,w))$. (Note that v is a child of u at layer $l+1$ and w is a child of v at layer $l+2$ and u now will be able to reach w directly.)
   (b) For each layer l s.t. l is odd and each node u in layer l set the transmission range of u to zero.
5. Goto Step 2.

Figure 5: The layer contraction (LC) algorithm.

Theorem 5.4 *The approximation factor of the LC algorithm is at most $12(n/D)^{\log\beta}$, where β is a constant that depends on α.*

Proof: From Theorem 5.2 it follows that $P(OPT) \geq P(MST)/12$. If Step 4 is never executed, we are done. Otherwise, during each iteration we decrease the diameter (the number of layers) by half. We argue that Step 4 incurs at most a constant blowup in the energy. Let β be the smallest constant s.t. for any positive x, y the following holds

$$x^\alpha + y^\alpha \geq ((x+y)^\alpha - y^\alpha)/\beta. \tag{3}$$

Note that for $\alpha = 2$, we have $\beta \leq 2$. Consider a node u at an odd layer of T whose power is increased by Step 4 and let v and w be the nodes for which the maximum of $d(u,v) + d(v,w)$ is achieved. The transmission range of u is increased to $d(u,v) + d(v,w)$ while the transmission range of v is decreased to zero. Thus, the overall increase in energy is bounded by

$$\frac{d(u,w)^\alpha - d(v,w)^\alpha}{d(u,v)^\alpha + d(v,w)^\alpha} \leq \beta \text{ (by inequality (3))}.$$

Summing over all nodes, we get the total energy is increased by at most a factor of β. Observe that we count the decrease in the energy of even-layer nodes exactly once since each node has a unique parent in T.

Obviously, after at most $\log(n/D)$ iterations the diameter is at most D. Therefore, $P(LC) \leq \beta^{\log(n/D)} \cdot P(MST)$. The theorem follows. ∎

It follows from the proof that $\beta \leq 2$ for $\alpha = 2$. Finally, we consider the *combined algorithm* that selects the best solution among the root cover and the layer contraction algorithms.

Corollary 5.5 *The approximation factor of the combined algorithm is at most* $\min(D^{\alpha-1}, 12(n/D)^{\log\beta})$.

In particular, for $\alpha = 2$ we have $\beta \leq 2$, and the approximation factor is at most $O(\sqrt{n})$ for any D and at most $O(\log n)$ for $D = O(\log n)$ and $D = \Omega(n/\log n)$.

5.2 Bounded-Diameter MST Algorithm

In this section we describe an algorithm, which reduces our problem to the BDMST problem. In the BDMST problem, we are given a connected, undirected graph $G_r = (V_r, E_r)$ on $n_r = |V_r|$ vertices and an integer bound K. The goal is to find a spanning tree T of G_r whose diameter does not exceed K so as to minimize the weight of T. The BDMST problem is NP-hard for $4 \leq K < n_r$ [23]. Kortsarz and Peleg [27] show a lower bound of $\Omega(\log n_r)$ on the approximation ratio and describe a $O(K \log n_r)$ approximation algorithm that combines a greedy heuristic and exhaustive search. Naor and Schieber [31] give a polynomial time $O(\log n_r)$-approximation algorithm for directed graphs that uses linear programming, where the path lengths from the root to the rest of the vertices are at most twice the given bound.

We construct a complete weighted graph in which the weight of an edge is the energy required to establish it. Then we apply the algorithm of [31] to this graph and obtain an arborescence, which is used to specify the energy assignment. The BDMST algorithm is described in Figure 6.

1. Construct a complete weighted graph G' with the set of nodes V, where the weight of an edge (u, v) is $d(u, v)^\alpha$ (the energy required to establish this edge).
2. Compute a BDMST of G', T, using the algorithm of [31] with $K = D$.
3. Set the transmission power of each node to be equal to the weight of the heaviest incident edge in T directed outward the root.
4. If the diameter of G is greater than D, apply Steps $3, 4$ of the layer contraction algorithm. (We may need to decrease the diameter of G by half since the diameter of T is bounded by $2D$.)

Figure 6: The BDMST algorithm.

Note that Step 4 of the BDMST algorithm incurs only a constant blowup in the total energy (see the proof of Theorem 5.4). Suppose that the energy cost of an optimal BDMST with diameter D (we count only the heaviest incident edge to each node) is at most $f(n)$ times larger than the energy cost of an optimal solution for the bounded-hop broadcast problem. We obtain that the BDMST algorithm achieves an approximation factor of $O(f(n) \cdot \log n)$. We note that in [39] the *complete* cost (all edges are counted) of a minimum spanning tree with unbounded diameter is shown to be a constant factor larger than the energy cost of an optimal broadcast with unbounded latency. That is not true in our model since for a star topology and $D = 1$, the *complete* cost of an optimal BDMST is $n - 1$ times larger than the cost of an optimal solution for the bounded-hop broadcast problem. However, in this case the *energy* cost is exactly the same for both solutions and it may still turn out that $f(n)$ is a constant.

5.3 Decremental Distance Heuristic for Bounded-Hop Broadcast

In this section we describe the decremental distance heuristic. In a nutshell, we begin with the solution of the MST algorithm. Then we greedily find either the maximal absolute power decrease or the minimal average power increase that decreases the distance of some nodes that are more than D hops apart from the root. This process terminates when the diameter constraint is satisfied. The decremental distance heuristic is presented in Figure 7. Intuitively, this algorithm will tend to increase the power of the nodes that are closer to the root, which allows us to simultaneously decrease the distance from the root for many nodes.

The following theorem considers the running time of the decremental distance algorithm.

Theorem 5.6 *The decremental distance algorithm terminates after at most $n^2 - nD$ iterations*

1. Apply the MST algorithm [39] and set the transmission power of each node accordingly.
2. If the diameter of the communication graph is smaller than D, return the current solution.
3. For each node u and for each possible level of the transmission power $P > P_u$ Do:
   (a) Let m be the number of nodes at distance more than D from the root for which this distance is decreased if the transmission power of u is increased to P.
   (b) Assume that the transmission power of u becomes P. Go over all nodes $w \neq u$ and find the the minimal transmission power of w, $P'' < P_w$ (if any) so that (i) the distance from the root to any node does not increase beyond D and (ii) the coverage is maintained. Let P' be the total power decrease over all considered w. (When the transmission range of u is increased, the transmission ranges of the other nodes may be decreased without affecting feasibility of the current solution.)
4. Select a pair u, P with $m > 0$ with the maximum negative value of $(P - P_u - P')$ (the absolute energy decrease); otherwise select a pair u, P with the minimum positive value of $(P - P_u - P')/m$ (the average energy increase).
5. Set the transmission power of u to be P.
6. Decrease the transmission power of all nodes that contribute to P' on Step $3(b)$.
7. Goto Step 2.

Figure 7: The decremental distance (DD) heuristic.

and has running time of at most $O(n^8)$.

Proof: Clearly, during each iteration we decrease the distance from the root to at least one node that is more than D hops apart and condition (i) guarantees that we do not increase the distance from the root to any node beyond D. Therefore, the algorithm terminates after at most $n^2 - nD$ iterations.

We argue that Step 3, which dominates the running time of each iteration, takes at most $O(n^6)$ time. Note that we have to consider at most n possible transmission for each node

u, w. Thus, there can be at most n^4 possible quadruples u, P, w, P''. Clearly, we can check the conditions (i) and (ii) in $O(n^2)$ time. The theorem follows. ∎

6 Communication Problems

In this section we consider different minimum-energy communication problems in ad hoc networks and reduce them to connectivity problems in a directed weighted graph. A similar reduction has been used in [29, 8].[†]

Given the placement of nodes $\mathcal{P}$, we construct a directed graph $G' = (V', E')$ as follows. For each node $v_i \in V$, we create $n-1$ new nodes $v_i^j : j = 1, \ldots, n, j \neq i$. Then for each pair of nodes (v_i, v_i^j), we add a directed edge between v_i and v_i^j of weight $d(v_i, v_j)^\alpha$. We also add a directed edge of weight zero between v_i^j and v_m if $d(v_i, v_j) \geq d(v_i, v_m)$. Note that G' contains n^2 vertices and $O(n^3)$ edges.

Full-Duplex Connection. We are a given two endpoints $s, d \in V$. We wish to establish a full-duplex connection between s and d. We use an algorithm proposed by Natu and Fang [33] in order to solve the following problem. Given a directed weighted graph G_r and a pair of nodes s, d, our goal is to find a minimum-weight subgraph H of G_r that contains paths from s to d and from d to s. Natu and Fang propose an algorithm with running time $O(m_r n_r + n_r^2 \log n_r)$, where n_r is the number of nodes and m_r is the number of edges in G_r. We apply the algorithm of [33] with G' as the input graph to obtain an optimal solution in $O(n^5)$ time.

Generalized Multicast. We are a given a subset $P = \{v_1, \ldots, v_q\} \subset V$ of senders and a

[†]We note that our reduction works also for the asymmetric energy model, where the energy cost of an edge (u, v) may differ from that of (v, u).

subset $P' = \{u_1, \ldots, u_{q'}\} \subset V$ of receivers that have fixed cardinality q, q'. We wish the transmission from each sender to reach all the receivers, i.e., to establish directed paths from every node of P to every node of P'. We assume that different transmissions can be aggregated if they pass through the same edge. We can use the algorithm provided by Feldman and Ruhl [18] for the *p-Directed Steiner Network problem*. In this problem, we are given a directed weighted graph G_r and p pairs of nodes $\{(s_1, d_1), \ldots, (s_p, d_p)\}$ and our goal is to find a minimum-weight subgraph H of G_r that contains all paths $s_i \rightarrow d_i$. Feldman and Ruhl propose an algorithm with running time $O(m_r n_r^{4p-2} + n_r^{4p-1} \log n_r)$, where n (resp. m) is the number of nodes (resp. edges) in G_r. We apply the algorithm of [18] to G' with qq' input pairs of nodes $\{(v_i, u_j)\}$. In this way, we get an optimal solution in $O(n^{8qq'-1})$ time.

Web-Conferencing. We are a given a subset $P = \{v_1, \ldots, v_q\} \subset V$ of fixed cardinality p that contains nodes participating in a conference. We wish that all nodes in P would be able to communicate with each other. We can use another algorithm provided by Feldman and Ruhl [18], where they solve the *p-Strongly Connected Steiner Subgraph problem*. In this problem, we are given a directed weighted graph G_r and p vertices $w_1, \ldots, w_p$ and we need to find a minimum-weight strongly connected subgraph H of G_r, that contains $w_1, \ldots, w_p$. Feldman and Ruhl give an algorithm with running time $O(m_r n_r^{2p-3} + n_r^{2p-2} \log n_r)$, where n_r (resp. m_r) is the number of nodes (resp. edges) in G_r. We can apply this algorithm to G' obtaining an optimal solution in $O(n^{4q-3})$ time.

7 Conclusion and Open Problems

We have presented tight analysis of the broadcast and the convergecast games in ad-hoc networks under a novel model of energy cost sharing. We show that cost sharing significantly improves the price of anarchy. Unfortunately, the price of anarchy still remains quite high, which suggests a need for some sort of coordination mechanism. An interesting future research direction is to study the case of mobile nodes.

We have also proposed approximation and heuristic algorithms for the bounded-hop broadcast problem. There is an open question of whether the BDMST algorithm achieves an approximation factor of $O(\log n)$.

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