Fault-Tolerant Power Assignment and Backbone in Wireless Networks

Paz Carmi *
Ben-Gurion University of the Negev
Computer Science
Beer-Sheva 84105, Israel
carmip@cs.bgu.ac.il

Matthew J. Katz†
Ben-Gurion University of the Negev
Computer Science
Beer-Sheva 84105, Israel
matya@cs.bgu.ac.il

Michael Segal‡
Ben-Gurion University of the Negev
Communication Systems Engineering
Beer-Sheva 84105, Israel
segal@cse.bgu.ac.il

Hanan Shpungin§
Ben-Gurion University of the Negev
Computer Science
Beer-Sheva 84105, Israel
shpungin@cs.bgu.ac.il

January 10, 2006

Abstract

Given a wireless network, we need to assign transmission power to each of the nodes, that will enable communication between any two nodes (via other nodes). Moreover, due to possible faults, we would like to have at least k vertex-disjoint paths from any node to any other node, where k is some fixed integer, depending on the reliability of the nodes. The goal is to achieve this *directed* k-connectivity with minimal overall power consumption. The problem is NP-Hard for any $k \geq 1$, already for planar networks. Here we develop an $O(k)$-approximation algorithm for the planar case. Next, we address the problem of constructing a k-connected backbone, for which we present an $O(k^3)$-approximation algorithm.

1 Introduction

A wireless ad-hoc network consists of a set of transceivers, communicating with each other by radio. Each transceiver t is assigned a transmission power $p(t)$, which gives it some transmission range, denoted by r_t. It is customary to assume that the minimal transmission power required in order to transmit to a distance d is d^α, where the *distance-power gradient* α is usually taken to be in the interval $[2, 4]$ (see [30]). Thus, a transceiver s receives transmissions from t if $p(t) \geq d(t, s)^\alpha$, where $d(t, s)$ is the Euclidean distance between t and s. The transmission possibilities resulting from a power assignment induce a communication graph. Research efforts have focused on finding power

*Partially supported by a Kreitman Foundation doctoral fellowship.
†Partially supported by grant No. 2000160 from the U.S.-Israel Binational Science Foundation.
‡Partially supported by the REMON (4G) consortium.
§Partially supported by the Lynn and William Frankel Center for Computer Sciences.

assignments, for which the induced communication graph satisfies a certain connectivity property, while minimizing the total cost.

In the rest of this section we present the model, discuss previous work, and briefly describe our contribution.

1.1 The model

We are given a set $\mathcal{T}$ of n transceivers $t_1, t_2, \ldots, t_n$, positioned in $\mathbb{R}^d$, $d \geq 1$. When each transceiver is assigned a transmission power $p(t) = r_t^\alpha$, an ad-hoc network is created. A power assignment for $\mathcal{T}$ is a vector of transmission powers $\{p(t) \mid t \in \mathcal{T}\}$ and is denoted by $A(\mathcal{T})$ (usually abbreviated to A). The resulting (directed) communication graph is denoted by $H_A = (\mathcal{T}, E_A)$, where E_A is the set of directed edges resulting from the power assignment $A(\mathcal{T})$:

$$E_A = \{(t, s) \mid p(t) \geq d(t, s)^\alpha\}.$$

That is, there is a directed edge from t to s if t has sufficient transmission power to reach s. Throughout this paper we refer to transceivers as nodes. The cost C_A of the assignment is the sum of all transmission powers:

$$C_A = \sum_{t \in \mathcal{T}} p(t).$$

Recall that a graph $G = (V, E)$ is *k-vertex connected* if for any two nodes $u, v \in V$ there exist k vertex-disjoint paths connecting u to v. Equivalently, G is k-vertex connected if it remains connected after omitting any set of up to $k - 1$ vertices. The problems we consider in this paper are:

Problem 1.1 (k-vertex connected power assignment).
Input: *A set $\mathcal{T}$ of transceivers, and a parameter $k > 1$.*
Output: *A power assignment $A(\mathcal{T})$ with minimal possible cost C_A, where H_A is k-vertex connected.*

Problem 1.2 (k-connected backbone).
Input: *A set $\mathcal{T}$ of transceivers, and a parameter $k \geq 1$.*
Output: *A subset $\mathcal{D}$ of $\mathcal{T}$ and a power assignment $A(\mathcal{D})$ with minimal possible cost C_A, where H_A (restricted to $\mathcal{D}$) is k-vertex connected, and for each $t \in \mathcal{T} \setminus \mathcal{D}$, there exist $u_1, \ldots, u_k \in \mathcal{D}$, such that $d(u_i, t) \leq r_{u_i}$, where $d(u_i, t)$ is the Euclidean distance between u_i and t and r_{u_i} is the range assigned to u_i.*

1.2 Previous work

Kirousis et al. [26] introduced the `MinRange(SC)` problem, which is Problem 1.1 with $k = 1$, and proved that it is NP-hard in three-dimensional Euclidean space for any value of $\alpha \geq 2$. The same paper provided a 2-approximation algorithm for the planar case, and an exact $O(n^4)$-time algorithm for the one-dimensional case. The planar case was shown to be NP-hard for any $\alpha \geq 2$ by Clementi et al. [17]. A simple 1.5-approximation algorithm for the case $\alpha = 1$ was presented in [4]. Some researchers add an additional constraint parameter to the problem, a bound h on the diameter of the induced communication graph, see results in [15, 18, 19]. Ambuhl et al. [3] presented some algorithms for weighted power assignment, solving optimally the broadcast, multi-source broadcast and strong connectivity problems in the linear case (they achieved the same running time for the connectivity problem as in [26]). They also presented some approximation algorithms for the multi-dimensional case. An excellent survey covering many variations of the problem is given in [16].

A natural generalization of the strong connectivity requirement is k-vertex connectivity. These networks also provide multi-path redundancy for load balancing or transmission fault tolerance. As power-optimal strong connectivity is NP-hard, so is power-optimal k-connectivity. Two versions of the problem arise: symmetric and asymmetric. In the symmetric version for any two nodes $t, s \in \mathcal{T}$, $p(t) \geq d(t,s)^\alpha \Leftrightarrow p(s) \geq d(s,t)^\alpha$, that is a node t can reach node s if and only if s can reach node t, we can also refer to this version as the undirected model. The asymmetric version allows directed links between two nodes. Krumke et al. [27] argued that the asymmetric version is harder than the symmetric version. A first non trivial result for planar asymmetric k-connectivity was presented by Shpungin and Segal in [31]. They derived an approximation factor of $O(k^2)$ for the planar case and some results for the linear case. Another possible connectivity property is k-edge connectivity, which implies that the removal of any k edges results in a disconnected graph. In [8], Calinescu and Wan presented various aspects of symmetric/asymmetric k-vertex connectivity and k-edge connectivity. They first proved NP-hardness of the symmetric two-edge and two-node connectivity problems and then provided a 4-approximation algorithm for both symmetric and asymmetric biconnectivity ($k = 2$), and a $2k$-approximation for both symmetric and asymmetric k-edge connectivity. Hajiaghayi et al. [23] give two algorithms for symmetric k-connectivity, with $O(k \log k)$ and $O(k)$-approximation factors and also some distributed approximation algorithms for $k = 2$ and $k = 3$ in geometric graphs. Jia et al. [22] present various approximation factors (depending on k) for symmetric k-connectivity, such as a $3k$-approximation algorithm for any $k \geq 3$ and a 6-approximation for $k = 3$. Additional results can be found in [2, 6, 7, 12, 18, 24, 28]. It is worth mentioning that unless otherwise specified, all the algorithms are centralized.

Dai and Wu [20] considered a construction of a k-connected k-dominating set as a backbone to balance efficiency and fault tolerance. They presented a number of localized construction protocols which lack the analysis of guaranteed performance bounds.

When the transmission ranges are equal and known in advance, the network is modeled as a unit-disk graph, and the problem of finding a minimum connected dominating set (CDS) in a unit disk graph has been shown to be NP-hard [13]. The work in [29] proposes a 10-approximation centralized algorithm for this problem. The work in [11] presents a polynomial-time approximation scheme that guarantees an approximation factor of $(1 + 1/s)$ with running time $n^{O((s \log s)^2)}$. Recently, the distributed construction of a small CDS has attracted a great deal of attention. The currently best known distributed algorithms are due to [33], with approximation factor of 8, and to [21] with approximation factor 6.91.

Other relevant work in the area of energy-efficient power assignment includes energy-efficient broadcasting and multicasting in wireless networks. The problem, given a source node s, is to find a minimum power assignment such that the induced communication graph contains a spanning tree rooted at s. This problem was shown to be NP-hard. In [14, 25, 34, 35], authors presented some heuristic solutions and gave some theoretical analysis. Srinivas and Modiano [32] provided a polynomial algorithm that optimally finds k node-disjoint paths for a given pair of nodes while minimizing the total node power needed on these k node-disjoint paths. They also provide a polynomial algorithm for solving the 2 edge-disjoint paths problem.

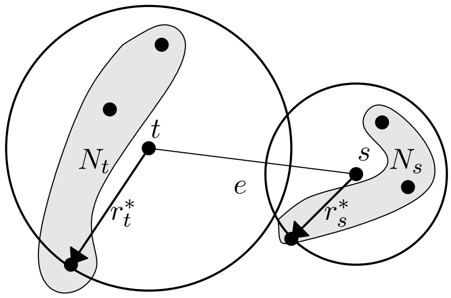


Figure 1: In A_k, each node in N_t reaches all nodes in N_s and vice versa.

1.3 Our contribution

We first present a polynomial-time $O(k)$-approximation algorithm for the two dimensional instance of the k-vertex connected power assignment problem. Next, we present an $O(k^3)$-approximation algorithm for the k-connected backbone problem. For $k = 1$, we obtain an efficient constant-factor approximation algorithm.

2 Improved fault-tolerant power assignment

Given a set $\mathcal{T}$ of n points in the plane (representing n transceivers) and an integer $k \geq 1$, we present an algorithm that assigns transmission ranges to the nodes in $\mathcal{T}$, such that the resulting (directed) communication graph is k-vertex connected, and the cost of the assignment of ranges is $O(k)$ times the cost of an optimal assignment of ranges.

For each node $t \in \mathcal{T}$, let $N_t \subseteq \mathcal{T}$ be a set of k closest nodes to t, and put $r_t^* = \max_{t' \in N_t} d(t, t')$. We now describe the range assignment algorithm. Compute a minimum spanning tree MST of the Euclidean graph induced by $\mathcal{T}$. Assign to each node $t \in \mathcal{T}$ the range r_t^*. Denote this initial range assignment by A'_k. For each edge $e = (t, s)$ of MST, increase the range of the nodes in $N_t \cup N_s$ (if necessary), such that each node $t' \in N_t$ can reach all nodes in N_s, and vice versa. Let A_k denote the resulting range assignment (see Figure 1).

It is easy to see that the resulting (directed) communication graph is k-vertex connected. That is, for any two nodes $t_1, t_2 \in \mathcal{T}$, there exist k vertex-disjoint paths from t_1 to t_2, that "follow" the path from t_1 to t_2 in MST.

Let E_t be the set of edges of MST that are adjacent to t. (It is well known that $|E_t| \leq 6$.) Let A_k^* be an optimal solution to our problem. Next we analyze the approximation factor of the proposed algorithm. We begin with the following simple observation, where C_{PA} denotes the cost of a power assignment PA, and $\sigma = \Sigma_{e \in \text{MST}} |e|^\alpha$.

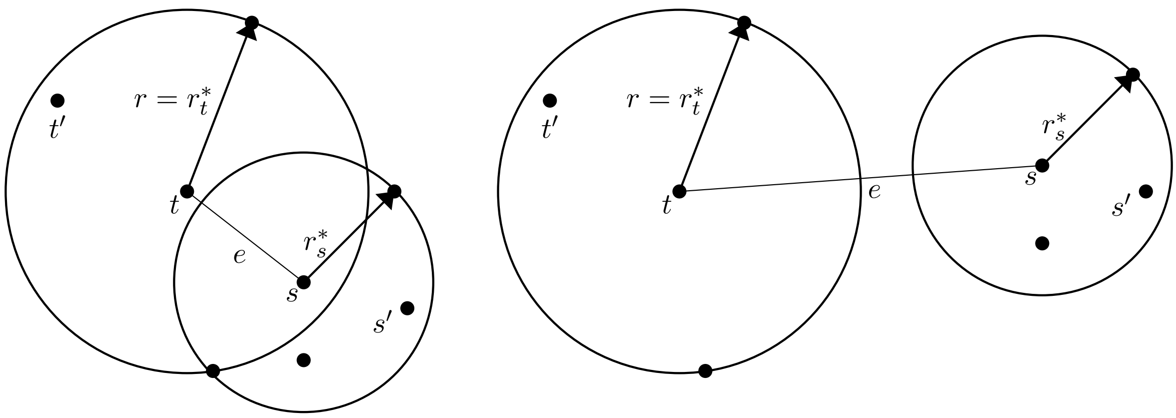
Observation 2.1. $C_{A_k^*} \geq C_{A'_k}$.

The above observation is obvious, since in any solution to our problem, each node must be assigned sufficient power for it to be able to reach k other nodes. Otherwise the communication graph is not k-vertex connected.

Let A_{MST} be the power assignment where each node $t \in \mathcal{T}$ is assigned the transmission range $\max_{e \in E_t} |e|$. Kirousis et al. [26] proved that $\sigma \leq C_{A_1^*}$ and $C_{A_{\text{MST}}} \leq 2\sigma$, where A_1^* is an optimal solution to our problem with $k = 1$. This implies that $C_{A_{\text{MST}}} \le 2C_{A_1^*} \le 2C_{A_k^*}$. Now we are ready to prove the main theorem.

Theorem 2.2. $C_{A_k} = O(k) \cdot C_{A_k^*}$.

Proof. For each edge $e = (t, s)$ of MST, put $N_{t,s} = N_t \cup N_s$ and $r = \max\{r_t^*, r_s^*\}$. In A_k each of the (at most) $2k$ nodes in $N_{t,s}$ is assigned (due to the edge e) a transmission range of at most $|e| + 2r$. We distinguish between two cases (see Figure 2):
Case 1: $|e| \le r \Rightarrow |e| + 2r \le 3r$ (Figure 2(a)). Notice that at least one of the nodes t, s, say t, must have a transmission range of at least r in A_k^*. We thus charge the assignment (in A_k) of range at most $3r$ to the nodes in $N_{t,s}$ to t. Since the degree of t in MST is at most 6, t is charged in this way only $O(k)$ times.
Case 2: $|e| > r \Rightarrow |e| + 2r < 3|e|$ (Figure 2(b)). Notice that since e is an edge adjacent to t in MST, the range that is assigned to t in A_{MST} is at least $|e|$. We thus charge the assignment (in A_k) of range at most $3|e|$ to the nodes in $N_{t,s}$ to t. Again, since the degree of t in MST is at most 6, t is charged in this way only $O(k)$ times.
From the above it follows that the cost of A_k is bounded by $O(k)$ times the sum $C_{A_k^*} + C_{A_{\text{MST}}} \le 3C_{A_k^*}$, that is $C_{A_k} = O(k) \cdot C_{A_k^*}$. ■



(a) $\max_{t' \in N_t, s' \in N_s} d(t', s') \le 3r$ (b) $\max_{t' \in N_t, s' \in N_s} d(t', s') < 3|e|$

Figure 2: Proof of Theorem 2.2.

3 Power assignment for k-connected backbone

In this section we consider the *k-connected backbone* problem. Given a set $\mathcal{T}$ of n points in the plane (representing n transceivers), find a subset $\mathcal{D} \subseteq \mathcal{T}$ and assign ranges to the points in $\mathcal{D}$, such that

1. In the resulting (directed) communication graph, $\mathcal{D}$ is k-vertex connected. That is, for each $u, v \in \mathcal{D}$, there exist k vertex-disjoint paths from u to v.

2. For each $t \in \mathcal{T} \setminus \mathcal{D}$, there exist k transceivers $u_1, \ldots, u_k \in \mathcal{D}$ that can reach t. That is, for each $1 \le i \le k$, $d(u_i, t) \le r_{u_i}$, where $d(u_i, t)$ is the Euclidean distance between u_i and t and

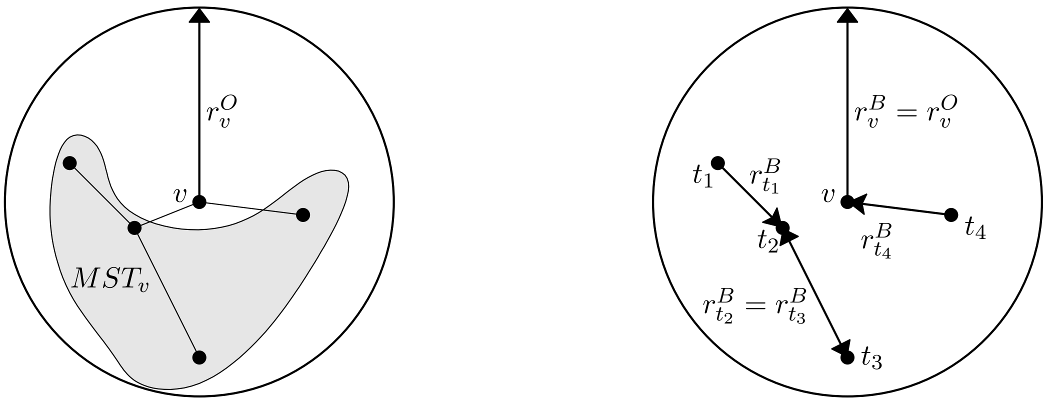
r_{u_i} is the range assigned to u_i, i.e., there is a directed edge from u_i to t in the communication graph.

3. $\Sigma_{u \in \mathcal{D}} r_u^2$ is minimized.

Let A_k be the range assignment that was computed in Section 2. A_k assigns a range r_u to each $u \in \mathcal{T}$. Clearly A_k (i.e., the set of transceivers $\mathcal{T}$ together with the ranges assigned to them) is also a k-connected backbone; that is, it satisfies requirements 1 and 2 above. We prove that A_k is an $O(k^3)$-approximation; that is, $C_{A_k} = O(k^3) \cdot C_{\mathrm{OPT}_k}$, where $C_{A_k} = \sum_{u \in \mathcal{T}} r_u^2$ is the cost of A_k and C_{OPT_k} is the cost of an optimal solution OPT_k to the k-connected backbone problem.

We shall construct from OPT_k a connected backbone B, and show that (i) $C_B \leq c \cdot C_{\mathrm{OPT}_k}$, and (ii) $C_{A_k} \leq O(k^3) \cdot C_B$, implying that $C_{A_k} \leq O(k^3) \cdot C_{\mathrm{OPT}_k}$, where c is an appropriate constant.

OPT_k consists of a subset of transceivers $\mathcal{D}_{\mathrm{OPT}_k}$ together with the ranges r_v^O assigned to the transceivers in $\mathcal{D}_{\mathrm{OPT}_k}$. For each transceiver $t \in \mathcal{T} \setminus \mathcal{D}_{\mathrm{OPT}_k}$, we associate t with any one of the transceivers in $\mathcal{D}_{\mathrm{OPT}_k}$ that can reach t. Denote by $\mathcal{T}_v$ the set of transceivers that were associated with v, for $v \in \mathcal{D}_{\mathrm{OPT}_k}$. Now, for each $v \in \mathcal{D}_{\mathrm{OPT}_k}$, compute a minimum spanning tree MST_v of the (Euclidean graph induced by the) points in $\mathcal{T}_v \cup \{v\}$; see Figure 3(a). We are now ready to define B. B consists of the set of all transceivers $\mathcal{T}$. For each $t \in \mathcal{T}$ that is in $\mathcal{D}_{\mathrm{OPT}_k}$, we set r_t^B to be r_t^O, and for each $t \in \mathcal{T}$ that is not in $\mathcal{D}_{\mathrm{OPT}_k}$, we set r_t^B to be the length of the longest edge in MST_v that is adjacent to t, where v is the transceiver in $\mathcal{D}_{\mathrm{OPT}_k}$ with which t is associated; see Figure 3(b).



(a) MST_v (b) Defining B

Figure 3: The connected backbone B.

It is easy to verify that B is a connected backbone. Let t_i, t_j be two transceivers in $\mathcal{T}$. If t_i is not in $\mathcal{D}_{\mathrm{OPT}_k}$, then there exists a directed transmission path from t_i to the transceiver $v_i \in \mathcal{D}_{\mathrm{OPT}_k}$ with which t_i is associated. If t_j is not in $\mathcal{D}_{\mathrm{OPT}_k}$, then v_j, the transceiver in $\mathcal{D}_{\mathrm{OPT}_k}$ with which t_j is associated, can reach t_j. Finally, we can advance from v_i to v_j along transceivers of the optimal backbone. We first prove inequality (i).

Claim 3.1. *$C_B \leq c \cdot C_{\mathrm{OPT}_k}$, for some constant c.*

Proof. Let $D_p(r)$ denote the disk of radius r centered at p. Let $v \in \mathcal{D}_{\mathrm{OPT}_k}$ and notice that $\mathcal{T}_v \subseteq D_v(r_v^O)$. Carmi et al. [9] have shown that if one draws, for each edge $e \in \mathrm{MST}_v$, a disk D_e

whose diameter is e, then $\Sigma_{e \in \text{MST}_v} area(D_e) \leq 5 area(\cup_{e \in \text{MST}_v} D_e)$. (Ábrego et al. [1] have shown that the constant 5 can be replaced by 3, with a significantly more delicate argument. Their result appeared in an earlier (unpublished) draft of their manuscript.) This implies that

$$\Sigma_{u \in \mathcal{T}_v} area(D_u(r_u)) = O(area(\cup_{u \in \mathcal{T}_v} D_u(r_u))) \ .$$

Now, observe that the length of the longest edge in MST_v is smaller or equal to r_v^O. Thus,

$$area(\cup_{u \in \mathcal{T}_v} D_u(r_u)) \leq area(D_v(2r_v)) = O(area(D_v(r_v)) \ .$$

Putting together the last two equalities, we get that

$$\Sigma_{u \in \mathcal{T}_v} area(D_u(r_u)) = O(area(D_v(r_v))) \ .$$

Applying the above computation to each $v \in D_{\text{OPT}_k}$, we obtain

$$\Sigma_{v \in \mathcal{D}_{\text{OPT}_k}} \Sigma_{u \in \mathcal{T}_v} area(D_u(r_u)) = O(\Sigma_{v \in \mathcal{D}_{\text{OPT}_k}} area(D_v(r_v))) \ .$$

We conclude that $C_B = O(C_{\text{OPT}_k})$, since the right side of the equation above is equal to $O(C_{\text{OPT}_k})$, and the left side of this equation is equal to $O(C_B - C_{\text{OPT}_k})$. ■

We now prove inequality (ii) above. We already showed that B is a connected backbone. But, actually, B defines a feasible range assignment to the transceivers in $\mathcal{T}$ (i.e., an assignment of ranges to the transceivers in $\mathcal{T}$, such that the resulting communication graph is strongly connected). Kirousis et al. [26] showed that the range assignment, A_{MST}, induced by MST (where a transceiver t is assigned the length of the longest edge in MST adjacent to t) is feasible and is a 2-approximation of a minimum range assignment (where one wants in addition to minimize the sum of the ranges squared). Therefore,

$$C_{A_{\text{MST}}} \leq 2C_{A_1^*} \leq 2C_B \ ,$$

where, as in the previous section, A_1^* is a minimum range assignment.

Consider the complete graph induced by $\mathcal{T}$. We assign weights to the edges of the graph, such that the weight $w(e)$ of an edge e is $|e|^2$. Let G^2 denote this graph. Andreae and Bandelt [5] compute a TSP tour T^* in G^2, such that $w(T^*) \leq c' \cdot w(MST_{G^2})$, where MST_{G^2} is the minimum spanning tree of G^2 and c' is some constant. (Notice that, $w(MST_{G^2}) = O(C_{A_{\text{MST}}})$.)

We apply the algorithm of Shpungin and Segal [31] for assigning transmission ranges to the transceivers in $\mathcal{T}$ so that the resulting (directed) communication graph is k-vertex connected. For convenience we briefly describe their algorithm. For each transceiver $t \in \mathcal{T}$, assign t sufficient power so that it can reach the $k/2$ nodes preceding it in the tour T^* and the $k/2$ nodes succeeding it in T^*. Shpungin and Segal prove that the assignment obtained, A_{TSP}, is an $O(k^2)$-approximation of an optimal solution for the problem considered in Section 2. Moreover, they prove that the sum of the powers in A_{TSP} is $O(k^2)$ times the sum of the edge weights in T^*. Therefore, we get

$$C_{A_{\text{TSP}}} \leq O(k^2) \cdot w(T^*) \leq O(k^2) \cdot C_{A_{\text{MST}}} \ .$$

From Section 2 and the paragraph above we have

$$C_{A_k} \leq O(k) \cdot C_{A_k^*} \leq O(k) \cdot C_{A_{\text{TSP}}} \ ,$$

where A_k^* is an optimal solution for the problem of Section 2. Putting everything together we get

$$C_{A_k} \leq O(k) \cdot C_{A_k^*} \leq O(k) \cdot C_{A_{\mathrm{TSP}}} \leq O(k^3) \cdot w(T^*) \leq$$

$$O(k^3) \cdot C_{A_{\mathrm{MST}}} = O(k^3) \cdot C_{A_1^*} = O(k^3) \cdot C_B = O(k^3) \cdot C_{\mathrm{OPT}_k} .$$

The following theorem summarizes the result of this section.

Theorem 3.2. *A_k is an $O(k^3)$-approximation for the k-connected backbone problem.*

Remark: If $k = 1$, then we get the standard connected backbone problem, and A_1 is a constant-factor approximation for it. In a preliminary version of this paper [10], we proved that the following somewhat simpler algorithm also computes a constant-factor approximation for the standard backbone problem. Compute a minimum spanning tree MST of the Euclidean graph induced by $\mathcal{T}$. Let D_I be the set of all internal nodes of MST, i.e., all nodes of MST of degree at least 2. For each $u \in D_I$, assign to u the range r_u, where r_u is the length of the longest edge in MST that is adjacent to u. Then the subset D_I together with the ranges r_u is a connected backbone.

References

[1] B. Ábrego, G. Araujo, E. Arkin, S. Fernández, F. Hurtado, M. Kano, J. S. B. Mitchell, E. O. na Pulido, E. Rivera-Campo, J. Urrutia and P. Valencia, "Matching points with geometric objects", manuscript, Universitat Politècnica de Catalunya, Nov. 2003.

[2] E. Althaus, G. Calinescu, I. Mandoiu, S. Prasad, N. Tchervenski and A. Zelikovsly, "Power Efficient Range Assignment in Ad-Hoc Wireless Networks", *IEEE Wireless Communications and Networking Conference*, 2003.

[3] C. Ambuhl, A. Clementi, M. Ianni, G. Rossi, A. Monti and R. Silvestri, "The Range Assignment Problem in Non-Homogeneous Static Ad-Hoc Networks", *2nd International Conference on AD-HOC Networks and Wireless (AdHoc-Now 03)*, 2004.

[4] C. Ambuhl, A. Clementi, P. Penna, G. Rossi and R. Silvestri, "Energy Consumption in Radio Networks: Selfish Agents and Rewarding Mechanisms", *Proceedings of the 10th International Colloquium on Structural Information & Communication Complexity (SIROCCO)*, pp. 1–16, 2003.

[5] T. Andreae and H.-J. Bandelt, "Performance guarantees for approximation algorithms depending on parameterized triangle inequalities", *SIAM Journal of Discrete Mathematics*, 8(1):1–16, 1995.

[6] D. Blough, M. Leoncini, G. Resta and P. Santi, "On the Symmetric Range Assignment Problem in Wireless Ad Hoc Networks", *Proc. 2nd IFIP International Conference on Theoretical Computer Science*, pp. 71–82, 2002.

[7] G. Calinescu, I. Mandoiu and A. Zelikovsly, "Symmetric Connectivity with Minimum Power Consumption in Radio Networks", *Proc. 2nd IFIP International Conference on Theoretical Computer Science*, pp. 119–130, 2002.

[8] G. Calinescu and P. J. Wan, "Range assignment for High Connectivitity in Wireless Ad Hoc Networks", *2nd International Conference on AD-HOC Networks and Wireless (AdHoc-Now 03)*, pp. 235–246, 2003.

[9] P. Carmi, M.J. Katz and J.S.B. Mitchell, "The minimum area spanning tree problem", *Proc. Workshop on Algorithms and Data Structures (WADS)*, LNCS 3608, 195–204, 2005.

[10] P. Carmi, M.J. Katz, M. Segal and H. Shpungin, "Fault-tolerant power assignment and backbone in wireless networks", *Proc. IEEE Internat. Workshop on Foundations and Algorithms for Wireless Networking*, 2006.

[11] X. Cheng, X. Huang, D. Li and D.-Z. Du, "Polynomial-time approximation scheme for minimum connected dominating set in ad hoc wireless networks," *Networks*, to appear.

[12] J. Cheriyan, S. Vempala and A. Vetta, "Approximation algorithms for minimum-cost k-vertex connected subgraphs", *Proceedings of the thirty-fourth annual ACM symposium on Theory of computing*, pp. 306-312, 2002.

[13] B. Clark, C. Colbourn and D. Johnson, "Unit Disk Graphs," *Discrete Mathematics*, Vol. 86, pp. 165-177, 1990.

[14] A. E. F. Clementi, P. Crescenzi, P. Penna, G. Rossi and P. Vocca, "On the Complexity of Computing Minimum Energy Consumption Broadcast Subgraphs", *18th Annual Symposium on Theoretical Aspects of Computer Science*, pp. 121–131, 2001.

[15] A. E. F. Clementi, A. Ferreira, P. Penna, S. Perennes and R. Silvestri, "The minimum range assignment problem on linear radio networks", *European Symposium on Algorithms*, pp. 143–154, 2000.

[16] A. E. F. Clementi, G. Huiban, P. Penna, G. Rossi and Y. C. Verhoeven, "Some Recent Theoretical Advances and Open Questions on Energy Consumption in Ad-Hoc Wireless Networks", *3rd Workshop on Approximation and Randomization Algorithms in Communication Networks*, 2003.

[17] A. E. F. Clementi, P. Penna and R. Silvestri, "Hardness Results for the Power Range Assignment Problem in Packet Radio Networks", *Proceedings of the 2nd International Workshop on Approximation Algorithms for Combinatorial Optimization Problems (APPROX)*, pp. 197–208, 1999.

[18] A. E. F. Clementi, P. Penna and R. Silvestri, "On the Power Assignment Problem in Radio Networks", *Electronic Colloquium on Computational Complexity (ECCC)*, 2000.

[19] A. E. F. Clementi, P. Penna and R. Silvestri, "The Power Range Assignment Problem in Radio Networks on the Plane", *Proc. 17th Annual Symposium on Theoretical Aspects of Computer Science*, pp. 651-660, 2000.

[20] F. Dai and J. Wu, "On Constructing k-Connected k-Dominating Set in Wireless Networks", in *IPDPS*, 2005.

[21] S. Funke, A. Kesselman, U. Meyer and M. Segal, "A simple improved distributed algorithm for minimum CDS in unit disk graphs", *IEEE WiMOB*, 2005.

[22] X. Jia, D. Kim, P. Wan and C. Yi, "Power Assignment for k-Connectivity in Wireless Ad Hoc Networks", *Proceedings of the IEEE INFOCOM*, 2005.

[23] M. T. Hajiaghayi, N. Immorlica and V. S. Mirrokni, "Power Optimization in Fault-Tolerant Topology Control Algorithms for Wireless Multi-Hop Networks", *Proceedings of the 9th Annual International Conference on Mobile Computing and Networking*, pp. 300–312, 2003.

[24] M. T. Hajiaghayi, G. Kortsarz, V. S. Mirrokni and Z. Nutov, "Power Optimization for Connectivity Problems", *Proceedings of IPCO*, 2005.

[25] G. Huiban and Y. C. Verhoeven, "A Self-Stabilized Distributed Algorithm for the Range Assignment in Ad-Hoc Wireless Networks", *Soumis Parallel Processing Letters.*

[26] L. M. Kirousis, E. Kranakis, D. Krizanc and A. Pelc, "Power Consumption in Packet Radio Networks", *Theoretical Computer Science*, Vol. 243(1-2), pp. 289-305, 2000.

[27] S. O. Krumke, R. Liu, E. L. Lloyd, M. V. Marathe, R. Ramanathan and S. S. Ravi, "Topology Control Problems under Symmetric and Asymmetric Power Thresholds", *ADHOC-NOW 2003*, pp. 187–198, 2003

[28] E. L. Lloyd, R. Liu, M. V. Marathe, R. Ramanathan and S. S. Ravi, "Algorithmic Aspects of Topology Control Problems for Ad Hoc Networks", *MONET 10(1-2)*, pp. 19–34, 2005.

[29] M. V. Marathe, H. Breu, H. B. Hunt III, S. S. Ravi and D. J. Rosenkrantz, "Simple Heuristics for Unit Disk Graphs," *Networks*, Vol. 25, pp. 59-68, 1995.

[30] K. Pahlavan and A. Levesque, "Wireless Information Networks", *Wiley-Interscience*, 1995.

[31] H. Shpungin and M. Segal, "k-Fault Resistance in Wireless Ad-Hoc Networks", *DIALM-M-POMC 2005*, pp. 89-96, 2005.

[32] A. Srinivas and E. Modiano, "Minimum Energy Disjoint Path Routing in Wireless Ad-hoc, *Proceedings of the 9th annual international conference on Mobile computing and networking*, pp. 122-133, 2003.

[33] P. J. Wan, K. Alzoubi and O. Frieder, "Distributed Construction of Connected Dominating Set in Wireless ad hoc networks," *Proc. of INFOCOM 2002.*

[34] P. J. Wan, G. Calinescu, X. Y. Li and O. Frieder, "Minimum-Energy Broadcast Routing in Static Ad Hoc Wireless Networks", *ACM Wireless Networks*, 2002.

[35] J. Wieselthier, G. Nguyen and A. Ephremides, "On the Construction of Energy-Efficient Broadcast and Multicast Trees in Wireless Networks", *Proceedings of the IEEE INFOCOM*, pp. 586-594, 2000.