

# Improved Multi-criteria Spanners for Ad-Hoc Networks Under Energy and Distance Metrics

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We study the problem of spanner construction in wireless ad-hoc networks through power assignments under two spanner models – distance and energy. In particular, we are interested in asymmetric power assignments so that the induced communication graph holds good distance and energy stretch factors simultaneously. In addition, we consider the following optimization objectives: low total energy consumption, low interference level, low hop-diameter, and high network lifetime.

Two node deployment scenarios are studied: random and deterministic. For  $n$  random nodes distributed uniformly and independently in a unit square we present several power assignments with varying construction time complexities. The results are based on various geometric properties of random points and shortest path tree constructions. Due to the probabilistic nature of this scenario, the probability of our results converges to one as the number of network nodes,  $n$ , increases. For the deterministic case we present two power assignments with non-trivial bounds. These are established on addition of shortcut edges that satisfy desired threshold stretch. To the best of our knowledge, these are the first results for spanner construction in wireless ad-hoc networks with provable bounds for both, energy and distance, metrics simultaneously. Our power assignments, in addition, try optimizing additional network properties such as network lifetime, interference, and hop-diameter.

Categories and Subject Descriptors: C.2.1 [Computer-Communication Networks]: Network Architecture and Design—*Network topology*; G.2.2 [Discrete Mathematics]: Graph Theory—*Graph algorithms; Network problems*

General Terms: Algorithms, Design, Theory

Additional Key Words and Phrases: Geometric and energy spanners, Approximation algorithms

## 1. INTRODUCTION

A wireless ad-hoc network consists of transceivers (nodes) which are located in the plane and communicate by radio. In contrast to wired networks, wireless ad-hoc networks have no fixed communication backbone. The temporary physical topology of the network is determined by the relative disposition of the wireless nodes, and the transmission range assignment of each of the nodes. The combination of these two factors produces a directed communication graph where the nodes correspond to the transceivers and the edges correspond to the communication links.

The transmission range of each node is determined by the assigned transmission power. It is common to assume that a transmission from node  $u$  can be received at node  $v$  if the transmission power of  $u$  is at least  $d(u, v)^\alpha$ , where  $d(u, v)$  is the Euclidean distance between

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$u$  and  $v$ , and  $\alpha$  is the *distance-power gradient*, usually taken to be in the interval  $[2, 4]$  (see [Pahlavan and Levesque 1995]).

Unlike nodes in wired networks, wireless devices are typically equipped with limited energy supplies, which are usually impossible or impractical to replenish. This makes *energy efficiency* one of the primary objectives in network design [Chandrakasan et al. 1999]. One of the most studied topology control problems in the context of energy preservation in wireless networks is the *minimum energy strong connectivity* problem:<sup>1</sup> given a set of nodes in the plane, find a power assignment so that the induced communication graph is strongly connected and the *total energy consumption* (also referred to as *cost*) is minimized.

Producing a *strongly connected* communication graph for wireless ad-hoc networks, through power assignments, was introduced by [Chen and Huang 1989] and has been studied since for the past 20 years. This is not surprising as many applications in civilian, industrial and military areas require a strongly connected underlying topology to carry out different networking tasks [Römer and Mattern 2004]. The problem appears to be NP-hard [Clementi et al. 1999] for the plane and polynomially solvable in the linear case (a special case when all the nodes are placed along a line segment). Thus, the majority of existing works produce approximation algorithms that induce a strongly connected graph with an upper guarantee on the total energy consumption (see [Chen and Huang 1989; Kirousis et al. 2000; Clementi et al. 2002; Ramanathan and Hain 2000]).

As stated above, energy efficiency is fundamental for successful network deployment. However, there are additional factors which need to be taken into account. A key component in the overall network performance is the *efficiency of routing algorithms* [Macker and Corson 1998]. There are many possible metrics to measure the efficiency of a routing algorithm, such as power, hop-count and residual energy [Giordano et al. 2001]. Ultimately, each node has a link to any other node in the system, so that the routing possibilities are unlimited and any routing graph is feasible. Unfortunately, this assumption is far from being realistic; it is impractical and usually impossible to allow each node to have a transmission range sufficient to reach all the other nodes. Instead, each node is assigned with enough power to reach only a relatively small subset of nodes. As a result, the topology of the induced communication graph has a strong effect on the routing algorithms efficiency. In this paper we focus on several key properties of the induced communication graph as outlined below.

- **Energy stretch factor:** Let  $\gamma_{u,v}$  be the minimum energy required to send a message from  $u$  to  $v$  (using other nodes if necessary). The *energy spanner* is aimed at minimizing the *energy stretch factor*  $t_E$  of the induced communication graph, i.e. for any  $u, v$ , the energy required to propagate a message from  $u$  to  $v$  is at most  $t_E \cdot \gamma_{u,v}$ . The energy spanner model reflects the power efficiency metric of routing protocols, which in the presence of constrained batteries is essential for extending the network lifetime. A very good survey of power-aware routing protocols in wireless networks can be found in [Lindsey et al. 2002].
- **Distance stretch factor:** The *distance spanner* minimizes the *distance stretch factor*  $t_D$  of the induced communication graph, that is for any pair of nodes, the minimum distance path from  $u$  to  $v$  is at most  $t_D \cdot d(u, v)$ . The distance stretch factor has a strong effect on the quality of *geographic routing protocols* [Gao et al. 2001]. These protocols use greedy forwarding decisions based on the geographic progress towards the destination, thus having a low distance stretch factor in the underlying topology graph is essential for efficient and successful geographic routing. For existing protocols using the geographic scheme see the survey in [Giordano et al. 2001].
- **Energy efficiency:** Since the energy consumption of a single node is proportional to the power it is assigned, a higher transmission range requires more energy, which grows non-

<sup>1</sup>A graph is strongly connected if for any pair of nodes,  $u, v$ , there is a path from  $u$  to  $v$  in the graph.

linearly with the distance. Nodes in a wireless network are typically battery-powered and have an initial battery charge which is sufficient for a limited amount of time. We evaluate energy efficiency through two measures: *total energy consumption* and *network lifetime*, which is defined as the time it takes the first node to run out of its initial battery charge.

- **Hop-diameter:** A low hop-diameter of the network allows faster data dissemination and easier routing.
- **Interference levels:** As nodes communicate through radio transmissions – interference is an inevitable consequence. Literally it means to what extent does a single transmission *interfere* with other transmissions in the network, as a signal transmitted by one node may disrupt other signals. The level of interference depends on the transmitting nodes proximity and the transmission ranges. A high level of interference decreases the number of transmissions that can happen simultaneously, while increasing the number of retransmissions. There are several common models for measuring the level of interference, all of which converge to evaluating the transmissions schedule length.

The majority of routing (and other) network protocols were traditionally developed for undirected graphs with symmetric (bidirectional) communication links. However, in wireless ad-hoc networks it is not uncommon to have asymmetric (unidirectional) links due to non-uniform background noise, non-uniform external interference and energy efficiency considerations. Some recent research addressed this phenomenon by providing several approaches for various network tasks (e.g. [Gerla et al. 2005; Lin et al. 2008; Bao and Garcia-Luna-Aceves 2001; Ramasubramanian et al. 2002; Zhang et al. 2008]). Moreover, [Moscibroda et al. 2006], proved that network topologies preserving connectivity of the given communication network using unidirectional links have significantly lower interference values and can therefore be scheduled much faster than connectivity preserving topologies using exclusively symmetric links. This result sheds new light on the question of practicality of directed as opposed to symmetric links in wireless ad-hoc and sensor networks: It shows that demanding communication links to be symmetric theoretically incurs a high overhead when it comes to scheduling. In this paper, we choose not to enforce symmetry over communication links, thus allowing unidirectional links to exist, which addresses a more general and realistic model of wireless ad-hoc networks.

We study the problem of topology control through power assignments so that the induced communication graph is strongly connected under the optimization objectives of stretch factor (for both models), energy efficiency, hop-diameter, and interference. We allow the nodes to have arbitrary positions in  $\mathbb{R}^2$ , while considering two cases.

**Probabilistic case** – We assume a random positioning of the wireless nodes so that they are uniformly and independently placed inside the unit square. Our results for this case are *with high probability*, or in short *w.h.p.*, which means that the probability of the result converges to one as the number of network nodes,  $n$ , increases. This assumption allows us to obtain better results than in the deterministic case.

**Deterministic case** – We make no assumptions on the particular positions of the wireless nodes, which makes the analysis more difficult.

This paper is organized as follows. In the rest of this section we present the model, discuss previous work and describe our contribution. Then, in Section 2 we show our results for the probabilistic case, followed by the deterministic case in Section 3. Section 4 outlines a possible distributed implementation of our algorithms. We present our simulations results in Section 5. Finally, in Section 6 we discuss some possible future research.

### 1.1. Model

Let  $G_V = (V, E_V)$  be a complete directed graph of the wireless nodes  $V$ ,  $|V| = n$ , positioned in the plane. We define the weight function,  $w : E_V \rightarrow \mathbb{R}^+$ , on the edge set  $E_V$  as  $w(u, v) =$

$d(u, v)^\alpha$ , where  $d(u, v)$  is the Euclidean distance between  $u$  and  $v$ . Note that the weight of an edge  $(u, v)$  matches the amount of energy which is required to transmit from  $u$  to  $v$ .

We proceed by first presenting some general definitions from graph theory used in this paper, and then discuss the wireless ad-hoc network model in detail.

**1.1.1. General definitions.** The following definitions are used for both, directed and undirected graphs. Let  $G = (V, E)$  be a subgraph of  $G_V$ . Denote the total weight of  $G$  by  $w(G) = \sum_{e \in E} w(e)$ . For an edge  $e = (u, v) \in E_V$ , let  $|e| = d(u, v)$ . Also, let  $e_u^*(G)$  be the longest outgoing edge from  $u$  in  $G$ , and by  $e^*(G)$ ,  $|e^*(G)| = \max_{u \in V} |e_u^*(G)|$  the longest edge in  $G$ .

The *hop-distance* from  $u$  to  $v$  in  $G$ ,  $h_{u,v}(G)$ , is defined as the minimum number of edges in any path from  $u$  to  $v$ . The *hop-diameter* of graph  $G$ , denoted  $h(G)$ , is the maximum hop-distance between any pair of nodes  $u, v \in V$  in  $G$ . The *weight* of a path  $P$  in  $G$  is  $w(P) = \sum_{e \in P} w(e)$ ; the *distance* of  $P$  is  $d(P) = \sum_{e \in P} |e|$ . For any pair of nodes  $u, v \in V$ , let  $P_{u,v}^{weight}$  and  $P_{u,v}^{dist}$  be the minimum weight and distance paths from  $u$  to  $v$  in  $G$ , respectively. We define  $\gamma_{u,v}(G) = w(P_{u,v}^{weight})$  and  $\delta_{u,v}(G) = d(P_{u,v}^{dist})$ .

Let  $MST_V$  be a minimum weight spanning tree of the undirected version of  $G_V$  (which is obtained easily by omitting the edge directions). For any  $s \in V$ , we denote by  $SPT(s)$  a shortest path tree of  $G_V$ , rooted at  $s \in V$ , i.e. for any  $u \in V$ ,  $\gamma_{s,u}(SPT(s)) = \gamma_{s,u}(G_V)$ .

In this paper we address the following spanner models.

**DEFINITION 1.1.** [Shpungin and Segal 2009]  $G$  is an energy  $t$ -spanner of  $G_V$  if for all  $u, v \in V$ ,

$$\gamma_{u,v}(G) \leq t \cdot \gamma_{u,v}(G_V).$$

**DEFINITION 1.2.** [Chew 1986]  $G$  is a distance  $t$ -spanner of  $G_V$  if for all  $u, v \in V$ ,

$$\delta_{u,v}(G) \leq t \cdot \delta_{u,v}(G_V) = t \cdot d(u, v).$$

If  $G$  is an energy (resp. distance)  $t$ -spanner of  $G_V$ , then we say that  $G$  has an energy (resp. distance) stretch factor of  $t$ . We denote by  $\gamma(G)$  and  $\delta(G)$  the energy and distance, respectively, stretch factors of  $G$ .

**1.1.2. Wireless ad-hoc network model.** A power assignment is a function  $p : V \rightarrow \mathbb{R}^+$ , which assigns each node  $v \in V$  a transmission range  $r_v = \sqrt[p(v)]{p(v)}$ . The transmission possibilities resulting from a power assignment induce a directed communication graph  $H_p = (V, E_p)$ , where

$$E_p = \{(u, v) : r_u \geq d(u, v)\}.$$

is a set of directed edges. The graph  $H_p$  is strongly connected if for every pair of nodes  $u, v \in V$ , there exists a directed path from  $u$  to  $v$  in  $H_p$ . The total energy consumption, also referred to as the cost, of the power assignment is given by

$$c(p) = \sum_{v \in V} p(v).$$

Let  $c^*$  be the cost of the minimum cost power assignment  $p$  so that  $H_p$  is strongly connected. We assume  $\alpha = 2$  for simplicity, although our results can be easily extended to any constant  $\alpha \geq 2$ .

Each node  $v$  has some initial battery charge  $b(v)$ , which is sufficient for a limited amount of time, proportional to the power assignment  $p(v)$ . It is common to take the lifetime of a wireless node  $v$  to be  $l(v) = b(v)/p(v)$ . The network lifetime is defined as the time it takes the first node to run out of its battery charge. For a power assignment  $p$  and initial battery charges  $b$ , the network lifetime is defined as

$$l(p) = \min_{v \in V} l(v).$$

Let  $l^*$  be the lifetime of the maximum lifetime power assignment  $p$  so that  $H_p$  is strongly connected. In this paper we assume unit initial battery charges  $b \equiv 1$ , i.e.  $b(v) = 1, \forall v \in V$ .

Interference is a direct consequence of any power assignment  $p$ . The level of interference determines the length of the transmission schedule. All common models of interference define the notion of *coverage*, which is the number of nodes or edges that are affected (interfered) by a transmission over a specific link in the induced communication graph  $H_p$ . [Burkhart et al. 2004] defined the coverage of an undirected edge  $(u, v)$  as the number of nodes covered by two transmission disks centered at  $u$  and  $v$  with radius  $d(u, v)$  (implying that  $u$  and  $v$  communicate with each other),

$$COV_I(H_p, (u, v)) = |\{w \in V, \min\{d(u, w), d(v, w)\} \leq d(u, v)\}|.$$

Another coverage model proposed by [Moaveni-Nejad and Li 2005] puts emphasis on the transmission range of a single node. The measure is the number of nodes covered by a transmission from node  $u$  alone,

$$COV_{II}(H_p, u) = |\{w \in V, d(u, w) \leq r_u\}|.$$

A third coverage model was introduced by [Khan et al. 2009]. They define the coverage as the number of edges affected by an edge  $(u, v) \in E_p$ ,

$$COV_{III}(H_p, (u, v)) = |\{(u', v') \in E_p, d(u, v) \leq d(u', v'), \\ \min\{d(u, u'), d(u, v'), d(v, u'), d(v, v')\} \leq d(u', v')\}|.$$

Note that even though  $COV_I(H_p, (u, v))$  is defined for undirected edges, we find it useful also in the context of asymmetric power assignments, as it describes the level of coverage for bi-directional links in the induced communication graph. Let,

$$\begin{aligned} COV_I^*(H_p) &= \max_{(u,v) \in E_p} COV_I(H_p, (u, v)), \\ COV_{II}^*(H_p) &= \max_{u \in V} COV_{II}(H_p, u), \\ COV_{III}^*(H_p) &= \max_{(u,v) \in E_p} COV_{III}(H_p, (u, v)). \end{aligned}$$

The interference of  $H_p$  is defined as

$$I(H_p) = \max\{COV_I^*(H_p), COV_{II}^*(H_p), COV_{III}^*(H_p)\}.$$

We assume the use of *frame-based* MAC protocols which divide the time into frames, containing a fixed number of slots. The main difference from the classic TDMA is that instead of having one access point which controls transmission slot assignments, there is a localized distributed protocol mimicking the behavior of TDMA. The advantage of a frame-based (TDMA-like) approach compared to the traditional IEEE 802.11 (CSMA/CA) protocol for a Wireless LAN is that collisions do not occur, and that idle listening and overhearing can be drastically reduced. When scheduling communication links, that is, specifying the sender-receiver pair per slot, nodes only need to listen to those slots in which they are the intended receiver – eliminating all overhearing. When scheduling senders only, nodes must listen in to all occupied slots, but can still avoid most overhearing by shutting down the radio after the MAC (slot) header has been received. In both variants (link and sender-based scheduling) idle listening can be reduced to a simple check if the slot is used or not. Several MAC protocols have been developed that take classical TDMA solutions using an access point to ad-hoc settings without any infrastructure by employing a distributed slot-selection mechanism that self-organizes a multi-hop network into a conflict-free schedule (see [Rajendran et al. 2003; van Hoesel and Havinga 2004]).

**Remark:** We would like to draw the reader's attention to the fact that our energy spanner definition differs from a related definition of power spanners (e.g. see [Schindelbauer et al.

Table I. Notation summary

Notation	Description
$G_V = (V, E_V)$	A complete graph on the set of wireless nodes $V$ , $ V  = n$
$d(u, v)$	Euclidean distance between $u$ and $v$
$\alpha$	Distance-power gradient (path loss coefficient)
$w : E_V \rightarrow \mathbb{R}^+$	The weight function on $G_V$ , $w(u, v) = d(u, v)^\alpha$
$ e $	Length of edge $e$ (i.e. for $e = (u, v)$ , $ e  = d(u, v)$ )
$w(P)$	The total weight of path $P$
$d(P)$	The total distance of path $P$
$w(G)$	The total weight of all the edges in $G$
$e^*(G)$	The longest edge in $G$
$e_u^*(G)$	The longest outgoing edge from $u$ in $G$
$h_{u,v}(G)$	The hop-distance from $u$ to $v$ in $G$
$MST_V$	The minimum weight spanning tree of $G_V$
$SPT_V(s)$	Shortest (minimum weight) path tree of $G_V$ rooted at $s$
$p : E_V \rightarrow \mathbb{R}^+$	The transmission power assignment
$p(u)$	The transmission power assigned to $u$
$H_p$	The communication graph as a result of power assignment $p$
$c^*$	The cost of a power assignment needed for strong connectivity
$l^*$	The lifetime of a power assignment needed for strong connectivity
$\delta_{u,v}(G)$	The length of the minimum distance path from $u$ to $v$ in $G$
$\gamma_{u,v}(G)$	The weight of the minimum weight path from $u$ to $v$ in $G$
$h(G)$	The hop-diameter of $G$
$c(p)$	The cost of the power assignment $p$
$l(p)$	The lifetime of the power assignment $p$
$I(p)$	The interference measure of $H_p$
$p_1, p_2, p_3, p_4, p_5$	Power assignments constructed in this paper

2007]), which sets  $\gamma_{u,v}(G_V)$  to be  $w(u, v)$ . The energy stretch factor is a more accurate estimator of energy efficiency since the cost of the minimum energy cost path from  $u$  to  $v$  in  $G_V$  may be significantly lower than  $w(u, v)$ .

A summary of the main notations used in this paper can be found in Table I.

## 1.2. Previous work

The first to introduce the concept of spanners was [Chew 1986]. The trivial 1-spanner (under both models) is to take the complete graph. Unfortunately the optimal stretch factor comes at a cost of high inefficiency. Therefore, much of the research considers an additional optimization objective, in addition to low dilation, such as a low number of edges, low total edge weight or low maximum node degree. Some of the works try to satisfy two or more of these properties at the same time. The NP-hardness of the problem was addressed in [Gudmundsson and Smid 2006; Klein and Kutz 2006; Cheong et al. 2007; Lloyd 1977]. The results can be roughly divided into two categories, geometric graphs and general graphs.

*Geometric graphs* – For a Euclidean graph  $G$  in  $\mathbb{R}^2$ , [Keil and Gutwin 1992] showed that for any constant  $t > 1$ , it is possible to construct a  $t$ -spanner of  $G$  with  $O(n)$  edges in  $O(n \log n)$  time. The same result for any fixed dimension  $d$  can be found in [Salowe 1991; Vaidya 1991; Callahan and Kosaraju 1993]. [Chandra et al. 1992] presented a construction of a  $t$ -spanner with a total weight of  $O(\log n) \cdot w(MST_V)$ .

*General graphs* – [Chandra et al. 1992] also showed a construction of a  $t$ -spanner with weight  $O\left(n^{\frac{2+\epsilon}{t-1}}\right) \cdot w(MST_V)$ . [Althöfer et al. 1993] and [Chandra et al. 1995] construct a  $t$ -spanner in  $O(n^{3+4/(t-1)})$  time with  $O(1 + 2/(t-1))$  edges. [Peleg and Roditty 2010] provides an algorithm

with running time  $O(n^2 \log n)$  for constructing a  $(1 + \varepsilon)$ -spanner for a given disk graph of a set of points in the plane with  $O(n/\varepsilon \log M)$  edges, where  $M$  is the maximum transmission radius. However, they [Peleg and Roditty 2010] did not consider distance and energy stretch factors, simultaneously. The energy stretch is particularly important in this case, since the cost of the minimum energy cost path from  $u$  to  $v$  in the complete graph may be significantly lower than the energy required to directly send the message from  $u$  to  $v$ , i.e.,  $w(u, v)$ . In addition, [Peleg and Roditty 2010] ignores other optimization criteria such as cost, lifetime, interference and hop-diameter which influence the suggested solutions.

Additional references may be found in [Peleg 2000; Cohen 1998; Thorup and Zwick 2005; Eppstein 2000; Abu-Affash et al. 2010].

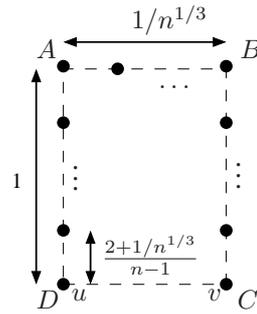


Fig. 1. An example of a bad tradeoff for a distance spanner ([Shpungin and Segal 2010])

One might think that applying traditional algorithms for the construction of spanners with low total edge weight in our model is possible. This is, however, not always possible for distance spanners, since the result of applying such an algorithm might result in a very large cost, due to our weight function.

Consider an example shown in Figure 1 adopted from [Shpungin and Segal 2010]. The nodes are placed in a continuous manner at a fixed distance of  $\frac{2+1/n^{1/3}}{n-1}$  from each other along the sides  $AD$ ,  $AB$ , and  $BC$  of a rectangle  $ABCD$ , with  $|AD| = |BC| = 1$ , and  $|AB| = |CD| = \frac{1}{n^{1/3}}$ . Let  $u$  and  $v$  be two nodes positioned at  $D$  and  $C$ , respectively. For any power assignment, if the communication graph has no edges between the nodes on the side  $AD$  and the nodes on the side  $BC$ , then the distance (in the communication graph) between  $u$  and  $v$  is  $\Theta(1)$ , which imposes a stretch factor of  $\Omega(n^{1/3})$ . Otherwise, if there is at least one edge which crosses from the  $AD$  side to  $BC$ , then the cost of that power assignment is at least  $1/n^{2/3}$ , which is  $\Omega(n^{1/3})$  times higher than  $w(MST_V) = \Theta(1/n)$  (in  $MST_V$  there are edges between any pair of adjacent nodes on the sides  $AD$ ,  $AB$ , and  $BC$ ). We can conclude that the product of the stretch factor and the cost is always  $\Omega(n^{1/3})w(MST_V)$  which implies that good approximation factors for both measures are impossible in the general case. Thus, there is no immediate deduction from geometric spanners to the proposed distance spanner problem due to the very high total energy cost.

For energy spanners, on the other hand, it is possible to use the algorithms developed for general graphs, but it seems that better results can be achieved due to the fact that nodes are positioned in the plane and the weight function holds the weak triangle inequality.

There was little research towards the development of energy efficient spanners in ad-hoc networks. In [Wang and Li 2006] the authors provided heuristics for the distance spanner construction without provable theoretical bounds. Then, in [Shpungin and Segal 2009] we showed several spanner constructions, for both models, while minimizing the total cost. In

[Schindelbauer et al. 2007] the authors show a relation between distance spanners and power spanners. Recall that the definition of power spanners significantly differs from energy spanners as it sets  $\gamma_{u,v}(G_V)$  to be  $w(u,v)$ . To the best of our knowledge this paper is the first to address the problem of spanner construction under the energy and distance metrics simultaneously.

The interference models addressed in this paper were studied before. In [Burkhart et al. 2004], the authors show that the common belief that graph sparseness induces low interference is not true. They give a clear definition of interference and develop connectivity-preserving and spanner constructions that are interference minimal. [Moaveni-Nejad and Li 2005] extends the interference definitions of [Burkhart et al. 2004] and show optimal and approximation algorithm for low interference network topology construction. [Khan et al. 2009] argued that the model proposed by [Burkhart et al. 2004] was not optimal and proposed power efficient, low interference topology constructions with node failures while preserving connectivity. Additional results can be found in [Johansson and Carr-Motyčková 2005; Chen et al. 2007].

An extensive research was also conducted on power efficiency in strongly connected ad-hoc networks. We refer the reader to the following works, [Kirousis et al. 2000; Ramanathan and Hain 2000; Carmi et al. 2006; Chang and Tassiulas 2000; Calinescu et al. 2003; Lloyd et al. 2005], for further information.

### 1.3. Our contribution

We study asymmetric power assignments in ad-hoc networks so that the induced communication graph is simultaneously an energy and distance spanner, while optimizing additional properties: cost, network lifetime, interference, and hop-diameter. We distinguish between two possible node deployments:

- **For random and uniform node distribution:** We construct three power assignments, with varying time complexity. The distance stretch factor of each of the power assignments is  $\sqrt{2}$ . We obtain energy stretch factors of 1 in  $O(n^3)$  time,  $\sqrt{n/k \log n}$  in  $O(kn^2)$  time, and  $\sqrt{n/\log n}$  in  $O(1)$  time (can also be implemented distributively). The cost for all of the power assignments is  $O(\log n) \cdot c^*$ , the network lifetime is  $\Omega(1) \cdot l^*$ , and the interference is  $O(\log n)$  (we note that it can be shown by using Chernoff bounds that w.h.p. the interference of any strongly connected topology is at least  $\Theta(\log n)$  in our model). The results are based on various geometric properties of random points and shortest path tree constructions. We choose to concentrate on shortest path trees since the length longest edge in such tree for uniformly distributed nodes is asymptotically equal to the length of the minimum spanning tree as we claim below. In particular, we analyze the behavior of shortest path trees that are rooted at the nodes relatively close to the remaining ones.
- **For arbitrary node positions:** We construct two power assignments. The first has a distance stretch of  $O(n^\varepsilon)$ , energy stretch of  $1 + O(\frac{1}{1-\varepsilon})$  and a cost of  $O(n^{2-2\varepsilon}) \cdot c^*$ , for any  $0 < \varepsilon < 1$ . The second has a distance stretch of  $O(h(T) \cdot n^\varepsilon)$ , energy stretch of  $1 + O(\frac{1}{1-\varepsilon})$ , a cost of  $O(n^{1-2\varepsilon}) \cdot w(T)$ , and hop-diameter of  $h(T)$ , where  $0 < \varepsilon < 1$  and  $T$  is any spanning subgraph of  $G_V$ . These are established on addition of shortcut edges that satisfy desired threshold stretch. Particularly, we start with a tree (MST or bounded-hop tree) and add shortcuts to produce a low weight distance spanner if the distance between nodes is less than some threshold value that depends on the weight of MST or length of the longest edge in bounded-hop tree. In addition, we explore the distance stretch factor of  $MST_V$ , which is of independent interest in the spanner construction research. We show that the distance stretch factor of  $MST_V$  is  $h(MST_V)$ .

Table II. Contribution summary (distance/energy stretch factors and cost/lifetime approximation bounds)

PA	Layout	Distance	Energy	Cost	Lifetime	Interference
$p_1$	Random	$\sqrt{2}$	1	$O(c^* \log n)$	$\Omega(l^*)$	$O(\log n)$
$p_2$	Random	$\sqrt{2}$	$\sqrt{n/k \log n}$	$O(c^* \log n)$	$\Omega(l^*)$	$O(\log n)$
$p_3$	Random	$\sqrt{2}$	$\sqrt{n/\log n}$	$O(c^* \log n)$	$\Omega(l^*)$	$O(\log n)$
$p_4$	Arbitrary	$O(n^\epsilon)$	$1 + O(\frac{1}{1-\epsilon})$	$O(c^* n^{2-2\epsilon})$	-	-
$p_5$	Arbitrary	$O(h(T) \cdot n^\epsilon)$	$1 + O(\frac{1}{1-\epsilon})$	$O(w(T)n^{1-2\epsilon})$	-	-

$[T \text{ is any spanning subgraph of } G_V]$

We also discuss a possible distributed implementation of our algorithms and provide thorough simulation results. A summary of our results can be found in Table II.

Note that all the upper bounds derived in this paper are compared with the best possible (optimal) lower bounds. Thus, the produced results serve as approximation guarantees for the considered problems.

## 2. THE PROBABILISTIC CASE

In this section we consider a wireless ad-hoc network with  $n$  random nodes distributed uniformly and independently in a unit square. First, we present several preliminary theoretical results for the uniform distribution used in later developments. Then, we construct three distinct power assignments with different stretch factors and running time complexity. We start with a basic simple homogeneous power assignment and analyze its performance criteria. This is followed by two power assignments which use the SPT (shortest path tree) structure to obtain better bounds.

### 2.1. Preliminaries

In [Berend et al. 2010] the authors showed the following lemma for the case that all the initial battery charges are equal.<sup>2</sup>

**LEMMA 2.1** ([BEREND ET AL. 2010]). *If for every  $v \in V$ ,  $b(v) = b$ , where  $b$  is a constant. Then,  $l^* \leq b/|e^*(MST_V)|^2$ .*

[Zhang and Hou 2008] derives a lower bound on the cost of a power assignment required to induce a  $k$ -fault resistant strongly connected communication graph ( $k = 1$  in our case) under the assumption that the nodes form a homogeneous Poisson point process with density  $\lambda$ . According to [Hall 1988], a random, uniform and independent  $n$ -point process in a unit square is essentially a Poisson process with  $\lambda = n$ , for large values of  $n$ . In the next theorem we bring the main result of [Zhang and Hou 2008] adapted to the case of  $k = 1$  and our point process.

**THEOREM 2.2** ([ZHANG AND HOU 2008]). *If  $n$  wireless nodes are randomly, independently, and uniformly distributed in a unit square, then w.h.p.,  $c^* = \Omega(1)$ .*

In [Shpungin and Segal 2009] we derived an interesting result on the density of independent nodes, randomly placed in a unit square under the uniform distribution, as shown in the next lemma.

**LEMMA 2.3** ([SHPUNGIN AND SEGAL 2009]). *Let  $D^*$  be a maximum radius disk, which can be placed inside a unit square, so that there are no nodes inside  $D^*$ . Let  $\epsilon$  be the radius of  $D^*$ . Then w.h.p.,  $\epsilon < \sqrt{2 \log n/n}$ .*

<sup>2</sup>Note that Lemma 2.1 from [Berend et al. 2010] applies to an *arbitrary* node distribution, despite the fact that it appears in the context of random distribution, both in this paper and in [Berend et al. 2010].

One of the structures used later in the paper is the shortest path tree (SPT). The next theorem, which was derived in [Shpungin 2011] draws lower and upper bounds on  $w(e^*(MST_V))$  and  $w(e^*(SPT(s)))$ , for any  $s \in V$ .

**THEOREM 2.4** ([SHPUNGIN 2011]). *For  $n$  wireless nodes randomly, independently, and uniformly distributed in a unit square, and for any  $s \in V$  it holds*

$$w(e^*(SPT(s))) = \Theta(w(e^*(MST_V))) = \Theta(\log n/n).$$

The power assignments presented in Sections 2.2, 2.3, and 2.4 share a similar bound on the maximum power level. The following lemma derives bounds on energy efficiency and interference levels of these power assignments.

**LEMMA 2.5.** *Given a power assignment  $p$ , if  $p(u) = O(\log n/n)$  for every  $u \in V$ , then w.h.p.  $c(p) = O(\log n) \cdot c^*$ ,  $l(p) = \Omega(1) \cdot l^*$  and  $I(H_p) = O(\log n)$ .*

**PROOF.** The bound on the total cost is obtained by using Theorem 2.2,  $c(p) = O(\log n) = O(\log n) \cdot c^*$ . The bound on the network lifetime is due to a combination of Lemma 2.1 and Theorem 2.4. Hence, under the assumption that  $b \equiv 1$ ,

$$l(p) = \Omega\left(\frac{n}{\log n}\right) = \Omega\left(\frac{1}{|e^*(MST_V)|^2}\right) = \Omega(1) \cdot l^*.$$

It is easy to observe that  $I(H_p) \leq 2 \cdot COV_{II}^*(H_p)$ . The bound on  $COV_{II}^*(H_p)$  is derived by applying the ‘‘balls and bins’’ analysis. Divide the unit square into  $\frac{n}{\log n}$  grid cells<sup>3</sup>, each of size  $\sqrt{\frac{\log n}{n}} \times \sqrt{\frac{\log n}{n}}$ . We can look at the distribution of nodes in the grid as a random process where we independently and uniformly throw  $n$  balls into  $\frac{n}{\log n}$  bins. The authors in [Raab and Steger 1998] analyzed the maximum number of balls in each bin (in our case, nodes in a grid cell). They showed that w.h.p. each grid cell contains at most  $O(\log n)$  nodes. Therefore, as  $p(u) = O(\log n/n)$  for every  $u \in V$ ,  $O(\log n)$  nodes within the transmission range of  $u$  (a disk with radius  $\sqrt{p(u)}$ , centered at  $u$ , intersects at most  $O(1)$  grid cells). Thus,  $I(H_p) \leq 2 \cdot COV_{II}^*(H_p) = O(\log n)$ .  $\square$

The use of *w.h.p.* is omitted in the rest of the section. However, every claim which uses one of the probabilistic statements in Section 2.1 is *with high probability*.

## 2.2. Basic spanner construction

We begin by demonstrating a fast algorithm, which does not require any coordination between the nodes. Each node only has to be aware of the total number of nodes,  $n$ , which means it can be implemented in a distributed fashion. Our power assignment is based on Lemma 2.3; we try to exploit the density of the nodes in the unit square. More specifically, for every  $u \in V$  let  $p_1(u) = \frac{32 \cdot \log n}{n}$  be the power assignment. Surprisingly, this simple power assignment, which can be computed locally at each node, produces a power assignment with good performance in all three optimization categories. The next two lemmas show that  $H_{p_1}$  is strongly connected and analyze its stretch factors.

**LEMMA 2.6.** *W.h.p.,  $H_{p_1}$  is strongly connected.*

**PROOF.** For any pair of nodes  $u, v \in V$ , we show an existence of a directed path from  $u$  to  $v$  in  $H_{p_1}$ .

If  $d(u, v) \leq \sqrt{32 \log n/n}$  then it immediately follows that  $(u, v) \in H_{p_1}$ . Otherwise, denote  $x_0 = u$ , and let  $x_1, x_2, \dots, x_m$  be  $m$  points evenly placed on the line segment  $(u, v)$  so that

<sup>3</sup>For convenience we omit the use of floors and ceilings throughout the paper, which does not affect our analysis.

$d(x_i, x_{i+1}) = \sqrt{\frac{8 \log n}{n}}$ ,  $0 \leq i \leq m - 1$ , where  $m = \left\lceil d(u, v) \sqrt{\frac{n}{8 \log n}} \right\rceil$ . Let  $D_i$  be a disk with  $(x_{i-1}, x_i)$ ,  $1 \leq i \leq m$ , as a diameter. From Lemma 2.3, each such disk contains at least one node. Let  $z_i$  be an arbitrary node in  $D_i$ ,  $1 \leq i \leq m$ . We observe that  $d(u, z_1)$ ,  $d(z_m, v)$ , and  $d(z_i, z_{i+1})$ ,  $1 \leq i \leq m - 1$ , are each less than or equal to  $\sqrt{32 \log n/n}$ , and hence are in  $E_{p_1}$ . Consequently, the path  $\langle u, z_1, z_2, \dots, z_m, v \rangle$  is in  $H_{p_1}$ .  $\square$

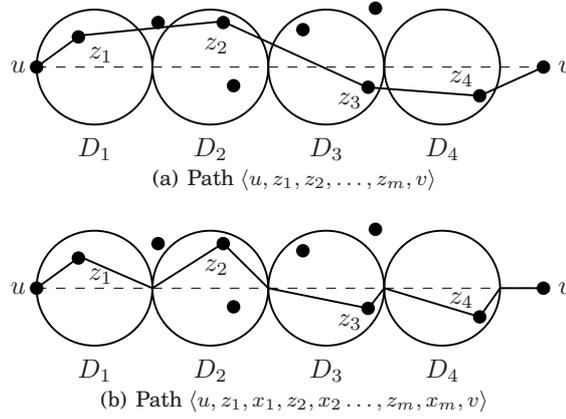


Fig. 2. The construction of a path from  $u$  to  $v$

LEMMA 2.7. *W.h.p.*,  $h(H_{p_1}) = \sqrt{\frac{n}{4 \log n}} + 1$ ,  $\gamma(H_{p_1}) \leq \sqrt{\frac{n}{4 \log n}} + 1$ ,  $\delta(H_{p_1}) \leq \sqrt{2}$ .

PROOF. For any arbitrary pair of nodes  $u, v \in V$ , recall the path constructed in the proof of Lemma 2.6,  $P = \langle u, z_1, z_2, \dots, z_m, v \rangle$ . As the nodes are inside of a unit square,  $d(u, v) \leq \sqrt{2}$ . Therefore,  $m \leq \sqrt{\frac{n}{4 \log n}}$ , and hence  $h(H_{p_1}) \leq \sqrt{\frac{n}{4 \log n}} + 1$ . Next, we show  $d(P) \leq \sqrt{2} \cdot d(u, v)$ , which guarantees  $\delta(H_{p_1}) \leq \sqrt{2}$ . If  $m = 0$  then  $d(P) = d(u, v)$ . Otherwise, let  $P'$  be a path obtained from  $P$  by enforcing it to visit the points  $x_i$ ,  $1 \leq i \leq m$ , as follows  $P' = \langle u = x_0, z_1, x_1, z_2, x_2, \dots, z_m, x_m, v \rangle$ . Clearly,  $d(P) \leq d(P')$  due to the triangle inequality of Euclidean distances. It is easy to verify that

$$d(x_i, z_{i+1}) + d(z_{i+1}, x_{i+1}) \leq \sqrt{2} \cdot d(x_i, x_{i+1}),$$

for  $0 \leq i \leq m - 1$ , and as a result

$$d(P') \leq \sqrt{2} \cdot d(u, x_m) + d(x_m, v) \leq \sqrt{2} \cdot d(u, v).$$

Finally, we bound the energy stretch factor. Let  $P^*$  be the minimum weight path from  $u$  to  $v$  in  $G_V$ . We consider two cases:

**Case 1:** If  $w(P^*) < 32 \log n/n$  then each edge  $e$  in  $P^*$  has a bounded weight,  $w(e) < 32 \log n/n$ . Therefore, path  $P^*$  is in  $H_{p_1}$  and  $\gamma_{u,v}(H_{p_1}) = w(P^*) = \gamma_{u,v}(G_V)$ .

**Case 2:** Otherwise,  $w(P^*) \geq 32 \log n/n$ . From the construction of  $p_1$  for every  $e \in E_{p_1}$ ,  $w(e) \leq 32 \log n/n$ . Thus,

$$\gamma_{u,v}(H_{p_1}) \leq \frac{32 \log n}{n} \cdot h(H_{p_1}) \leq w(P^*) \cdot h(H_{p_1}).$$

Note that a bound on  $\gamma_{u,v}(H_{p_1})$  is also a bound on  $\gamma(H_{p_1})$  as  $u$  and  $v$  are two arbitrary nodes.  $\square$

The following theorem summarizes the properties of  $p_1$  and is immediate from Lemmas 2.5, 2.6, and 2.7.

**THEOREM 2.8.** *For  $n$  wireless nodes randomly, independently, and uniformly distributed in a unit square, w.h.p.  $H_{p_1}$  is strongly connected and*

$$\begin{aligned} c(p_1) &= O(\log n) \cdot c^* & h(H_{p_1}) &= O(\sqrt{n/\log n}) \\ l(p_1) &= \Omega(1) \cdot l^* & \gamma(H_{p_1}) &= O(\sqrt{n/\log n}) \\ I(H_{p_1}) &= O(\log n) & \delta(H_{p_1}) &\leq \sqrt{2} \end{aligned}$$

### 2.3. Grid-based construction

The algorithm consists of three steps intuitively outlined as follows: (a) Select a subset of nodes  $S \subset V$ ,  $|S| = k$ , which are relatively close to the remaining ones; (b) Compute  $k$  shortest path trees, rooted at the nodes from  $S$ . Assign powers to all nodes, so that these trees are subgraphs of the induced communication graph; (c) Increase the power of each node, if needed, to ensure that  $H_{p_1}$  (from the previous section) is a subgraph of the induced communication graph. The idea is that by enforcing  $H_{p_1}$  to be a subgraph of the induced communication graph, the hop-diameter and distance stretch factor are at most  $h(H_{p_1})$  and  $\delta(H_{p_1})$ , respectively. We will show that the energy stretch factor is better than  $\gamma(H_{p_1})$  by using the constructed shortest path trees. We now describe the above steps in detail.

**Step 1** – Let  $k$  be an integer parameter,  $1 \leq k \leq \frac{n}{8 \cdot \log n}$ . Divide the unit square into  $k$  grid cells of size  $\sqrt{1/k} \times \sqrt{1/k}$ . Denote by  $N(i)$  the nodes in cell  $j$ ,  $1 \leq j \leq k$ . Note that a disk with radius  $\sqrt{2 \log n/n}$  fits entirely inside each of the grid cells, for any value of  $k$  in range  $\{1, \dots, \sqrt{\frac{n}{8 \cdot \log n}}\}$ . Therefore, from Lemma 2.3 we can conclude that there exists at least one node in each of the cells. Let  $S = \{s_1, s_2, \dots, s_k\}$  be an arbitrary subset of nodes so that  $s_j \in N(j)$  (as shown in Figure 3).

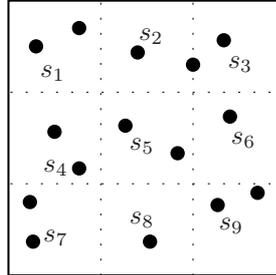


Fig. 3. The source nodes for the shortest path trees in the grid-based construction ( $k = 9$ )

**Step 2** – We define the initial power assignment for every  $u \in V$  as  $p'_2(u) = \max_{s_j \in S} |e_u^*(SPT(s_j))|^2$ .

**Step 3** – Finally, we ensure that  $H_{p_1}$  is a subgraph of the induced communication graph, as follows. For every  $u \in V$ ,  $p_2(u) = \max\{p'_2(u), p_1(u)\}$ .

The next observation is due to Theorem 2.4 and the definition of  $p_1$ .

**OBSERVATION 2.9.** *W.h.p., for every  $u \in V$ ,  $p_2(u) = O(\frac{\log n}{n})$ .*

Next, we analyze the energy stretch factor of  $H_{p_2}$ .

**LEMMA 2.10.** *W.h.p.,  $\gamma(H_{p_2}) = O\left(\sqrt{\frac{n}{k \cdot \log n}}\right)$ .*

PROOF. For any pair of nodes  $u, v \in V$  we show  $\gamma_{u,v}(H_{p_2}) = O\left(\sqrt{\frac{n}{k \cdot \log n}}\right) \cdot \gamma_{u,v}(G_V)$ . Similar to the proof of Lemma 2.7, let  $P^*$  be the minimum weight path from  $u$  to  $v$  in  $G_V$ . We consider two cases:

**Case 1:** If  $w(P^*) < \frac{32 \cdot \log n}{n}$  then  $P^*$  is in  $H_{p_2}$ .

**Case 2:** Otherwise,  $w(P^*) \geq \frac{32 \cdot \log n}{n}$ . Denote by  $i$  the grid cell that contains  $u$ , that is  $u \in N(i)$ . There exists a path  $P$  in  $H_{p_2}$  from  $u$  to  $v$  which consists of two parts. The first part,  $P_1$ , is a path from  $u$  to  $s_i$  and the second part,  $P_2$ , is from  $s_i$  to  $v$ . In what follows we show the existence of  $P$  and analyze its weight.

From the definition of  $p_2$ ,  $H_{p_1}$  is a subgraph of  $H_{p_2}$ ; let  $P_1 = \langle u, z_1, z_2, \dots, z_m, s_i \rangle$  be the path from  $u$  to  $s_i$  as constructed in the proof of Lemma 2.6. Since  $u \in N(i)$  then  $d(u, s_i) \leq \sqrt{\frac{2}{k}}$ , and as a result  $m \leq \sqrt{\frac{n}{4 \cdot k \cdot \log n}}$ . The weight of  $P_1$  can therefore be bounded,

$$w(P_1) \leq (m+1) \cdot \frac{32 \cdot \log n}{n} = O\left(\sqrt{\frac{\log n}{k \cdot n}}\right).$$

Let  $P_2$  be shortest path from  $s_i$  to  $v$  in  $G_V$ . According to the construction of  $H_{p_2}$ ,  $P_2$  is also a path in  $H_{p_2}$ . Let  $P'_1 = \langle s_i, z_m, \dots, z_1, u \rangle$  be a reverse of path  $P_1$ . Note that  $P'_1$  exists in  $G_V$  (a complete graph), but does not necessarily exist in  $H_{p_2}$ . Clearly,  $w(P'_1) = w(P_1)$ . Next, consider a path  $P'_2$ , which is a concatenation of  $P'_1$  with  $P^*$  (the concatenation is valid since both paths are in  $G_V$ ). We can therefore conclude.

$$w(P_2) \leq w(P'_2) \leq w(P'_1) + w(P^*) = w(P_1) + w(P^*).$$

By combining the above we can finally bound the weight of  $P$ :

$$\begin{aligned} w(P) &= w(P_1) + w(P_2) \leq 2 \cdot w(P_1) + w(P^*) \\ &= O\left(\sqrt{\frac{\log n}{k \cdot n}}\right) + w(P^*) = O\left(\sqrt{\frac{n}{k \cdot \log n}} \cdot \frac{\log n}{n}\right) + w(P^*) \\ &= O\left(\sqrt{\frac{n}{k \cdot \log n}}\right) w(P^*) + w(P^*). \end{aligned}$$

As a result,  $\gamma_{u,v}(H_{p_2}) = O\left(\sqrt{\frac{n}{k \cdot \log n}}\right) \cdot \gamma_{u,v}(G_V)$ .  $\square$

Note that  $H_{p_1}$  is a subgraph of  $H_{p_2}$ . The next theorem follows from Observation 2.9, Theorem 2.8, and Lemmas 2.5, 2.10.

**THEOREM 2.11.** *For  $n$  wireless nodes randomly, independently, and uniformly distributed in a unit square, w.h.p.  $H_{p_2}$  is strongly connected and*

$$\begin{aligned} c(p_2) &= O(\log n) \cdot c^* & h(H_{p_2}) &= O(\sqrt{n/\log n}) \\ l(p_2) &= \Omega(1) \cdot l^* & \gamma(H_{p_2}) &= O(\sqrt{n/k \log n}) \\ I(H_{p_2}) &= O(\log n) & \delta(H_{p_2}) &\leq \sqrt{2} \end{aligned}$$

#### 2.4. SPT-based construction

The final construction for the probabilistic case consists of  $n$  shortest path trees, rooted at every node, and  $H_{p_1}$  as subgraphs. As before, let  $SPT(v)$  be the shortest path tree rooted at  $v$ . We define the power assignment  $p_3$  for every  $u$  to be

$$p_3(u) = \max \left\{ \max_{v \in V} |e_u^*(SPT(v))|^2, p_1(u) \right\}.$$

The next theorem summarizes the properties of  $p_3$ .

**THEOREM 2.12.** *For  $n$  wireless nodes randomly, independently, and uniformly distributed in a unit square, w.h.p.  $H_{p_1}$  is strongly connected and*

$$\begin{aligned} c(p_3) &= O(\log n) \cdot c^* & h(H_{p_3}) &= O(\sqrt{n/\log n}) \\ l(p_3) &= \Omega(1) \cdot l^* & \gamma(H_{p_3}) &= 1 \\ I(H_{p_3}) &= O(\log n) & \delta(H_{p_3}) &\leq \sqrt{2} \end{aligned}$$

**PROOF.** From Theorem 2.4 we have  $p_3(u) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ . Using Lemma 2.5 we obtain the bounds for total cost, network lifetime and interference. The rest is similar to Theorem 2.11. Since  $H_{p_1}$  is a subgraph of  $H_{p_3}$ , it easily follows that  $\delta(H_{p_3}) \leq \delta(H_{p_1})$  and  $h(H_{p_3}) \leq h(H_{p_1})$ . The energy stretch factor,  $\gamma(H_{p_3}) = 1$ , is immediate from the construction.  $\square$

### 2.5. Construction time complexity

The first power assignment,  $p_1$  can be constructed in  $O(1)$  time, as each node only has to be aware of the total number of nodes,  $n$ . It also means that it can be implemented in a distributed fashion. The second power assignment  $p_2$  requires a construction of  $k$  shortest path trees. As  $G_V$  is a complete graph, a single tree is constructed in  $O(n^2)$  time. Thus, the total running time to  $O(k \cdot n^2)$ ,  $1 \leq k \leq \sqrt{\frac{n}{8 \cdot \log n}}$  (the other steps are negligible). Finally, the third power assignment  $p_3$  requires the computation of  $n$  shortest path trees, resulting in a  $O(n^3)$  running time. Note that the results in this section can be generalized to any convex fat area. An area is considered fat if for some bounded constant  $\alpha > 1$ , the ratio between the radius of the minimum enclosing circle and the radius of the maximum inscribed circle is at most  $\alpha$ .

## 3. THE DETERMINISTIC CASE

In this section we make no assumptions on node distribution; nodes are allowed to have arbitrary positions in  $\mathbb{R}^2$ . We start with some general statements which we use later on. Then, we present several power assignments that achieve non trivial energy and distance stretch factors. Our main challenge is the construction of low weight distance spanners; the energy stretch factor is then added by a simple alternation of the power assignments. The general idea of our constructions is to implement a simple greedy algorithm, which adds *shortcut* edges if some distance stretch factor threshold is violated. We examine two possible thresholds and provide theoretical bounds for the obtained power assignments.

Note that both power assignments presented in this section can be constructed in  $O(n^2)$  time.

### 3.1. Preliminaries

The following simple technical observation is very useful in all our developments.

**OBSERVATION 3.1.** *Given a graph  $G = (V, E)$  and a power assignment  $p$  so that for every  $u \in V$ ,  $p(u) = |e_u^*(G)|^2$ , it holds  $c(p) \leq 2 \cdot w(G)$  and  $G$  is a subgraph of  $H_p$ .*

**PROOF.** For every edge  $(u, v) \in E$ ,  $d(u, v) \leq |e_u^*(G)|$  and therefore  $(u, v) \in E_p$ , implying  $G$  is a subgraph of  $H_p$ . Also,

$$c(p) = \sum_{u \in V} p(u) = \sum_{u \in V} |e_u^*(G)|^2 \leq \sum_{u \in V} \sum_{(u, v) \in E} w(u, v) \leq 2 \cdot \sum_{e \in E} w(e).$$

Consequently,  $c(p) \leq 2 \cdot w(G)$ .  $\square$

[Chen and Huang 1989], and later [Kirousis et al. 2000] made the following statement, which has already become common folklore in the study of wireless networks. It derives a lower bound on the cost of any strongly connected power assignment, which in turn is a lower bound for any strongly connected spanner.

**THEOREM 3.2** ([CHEN AND HUANG 1989]).  $c^* \geq w(MST_V)$ .

[Chandra et al. 1992] studied the problem of spanner constructions for arbitrary positive weighted graphs (the energy spanner model according to our definitions). One of their results is given in the next theorem (adopted to our model).

**THEOREM 3.3** ([CHANDRA ET AL. 1992]). *Let  $G'$  be an arbitrary graph with a positive weight function  $w'$  and  $MST'$  be a minimum weight spanning tree of  $G'$  (with respect to  $w'$ ). For any  $t' > 1$  and any  $\varepsilon' > 0$ , it is possible to construct in polynomial time an energy  $t'$ -spanner of  $G'$  with a weight at most  $O(n^{\frac{2+\varepsilon'}{t'-1}})$  times the weight of  $MST'$ .*

Based on the theorem above we show how it is possible to add the energy stretch factor to the induced communication graph degrading the overall cost of a power assignment by using the construction proposed in Theorem 3.3.

**LEMMA 3.4.** *Let  $p$  be a power assignment so that  $c(p) = O(n^\varepsilon) \cdot w(MST_V)$ , with any  $0 < \varepsilon < 1$ . It is possible to obtain a power assignment  $p'$  so that  $\gamma(H_{p'}) = 1 + O(1/\varepsilon)$ ,  $c(p') = \Theta(c(p))$ , and  $H_p$  is a subgraph of  $H_{p'}$ .*

**PROOF.** From Theorem 3.3, by setting  $t' = 1 + \frac{2+\varepsilon'}{\varepsilon}$ , with any constant  $\varepsilon' > 0$ , it is possible to construct an energy  $t'$ -spanner of  $G_V$  with a total weight of  $O(n^\varepsilon) \cdot w(MST_V)$ . Denote this spanner as  $G'$ .

To ensure that  $G'$  is a subgraph of the induced communication graph, we increase power (if needed) as follows. For every  $u \in V$

$$p'(u) = \max\{p(u), |e_u^*(G')|^2\}.$$

We first concentrate on the cost of  $p'$  (the inequality is due to Observation 3.1).

$$\begin{aligned} c(p') &= \sum_{u \in V} \max\{p(u), |e_u^*(G')|^2\} \leq c(p) + 2 \cdot w(G') \\ &= O(n^\varepsilon) \cdot w(MST_V) + O(n^{\frac{2+\varepsilon'}{t'-1}}) \cdot w(MST_V) = O(n^\varepsilon) \cdot w(MST_V) = \Theta(c(p)). \end{aligned}$$

Clearly,  $G'$  and  $H_p$  are subgraphs of  $H_{p'}$ . The fact that  $G'$  is a subgraph of  $H_{p'}$  results in  $\gamma(H_{p'}) = O(t') = 1 + O(1/\varepsilon)$ .  $\square$

### 3.2. Augmentation by total edge length criterion

Our first algorithm is AUGMENT-TOTAL-EDGE-LENGTH. It is based on computing an  $MST_V = (V, E)$ , and then adding shortcuts to produce a low weight distance spanner. A shortcut is an edge  $(u, v)$ , which is added if  $d(u, v) < D/n^\varepsilon$ , where  $D = \sum_{e \in E} |e|$  is the total length of  $MST_V$ , and  $\varepsilon$  is a configurable parameter in the open interval  $(0, 1)$ .

The next lemma analyzes the total cost and distance stretch of the algorithm.

**LEMMA 3.5.** *For any  $0 < \varepsilon < 1$ ,  $c(p_4) = O(n^{2-2\varepsilon}) \cdot w(MST_V)$  and  $\delta(H_{p_4}) \leq n^\varepsilon$ .*

**PROOF.** From the last line of the algorithm AUGMENT-TOTAL-EDGE-LENGTH it follows that for every  $u \in V$ ,  $p_4(u) \leq \max\{|e_u^*(MST_V)|^2, D^2/n^{2\varepsilon}\}$ . Therefore due to Observation 3.1,  $c(p_4) \leq 2 \cdot w(MST_V) + \frac{n \cdot D^2}{n^{2\varepsilon}}$ . We bound the value of  $D^2$  by using the Cauchy-Schwartz inequality,  $D^2 = (\sum_{e \in E} |e|)^2 \leq n \cdot \sum_{e \in E} |e|^2 = n \cdot w(MST_V)$ .

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**AUGMENT-TOTAL-EDGE-LENGTH**


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```

1 compute  $MST_V = (V, E)$ 
2  $D \leftarrow \sum_{e \in E} |e|$ ;  $E' \leftarrow \emptyset$ 
3 foreach  $u, v \in V$  do
4   |   if  $d(u, v) < D/n^\varepsilon$  then
5   |   |   add  $(u, v)$  to  $E'$ 
6 let  $G = (V, E \cup E')$ 
7 foreach  $u \in V$  do
8   |    $p_4(u) = |e_u^*(G)|^2$ 

```

---

The distance stretch factor is almost straightforward from the definition of  $p_4$ . As  $MST_V$  is a subgraph in  $H_{p_4}$ , for any pair of nodes  $u, v \in V$ ,  $\delta_{u,v}(MST_V) \leq \delta_{u,v}(H_{p_4})$ . Two cases are considered:

**Case 1:**  $d(u, v) \geq D/n^\varepsilon$ . Since there exists a path from  $u$  to  $v$  in  $MST_V$ ,  $\delta_{u,v}(H_{p_4}) \leq \delta_{u,v}(MST_V) \leq D$ . Thus,  $\delta_{u,v}(H_{p_4}) \leq n^\varepsilon \cdot d(u, v)$ .

**Case 2:**  $d(u, v) < D/n^\varepsilon$ . In this case, the edge  $u, v$  is added and  $\delta_{u,v}(H_{p_4}) = d(u, v)$ .  $\square$

The following theorem summarizes the properties of  $p_4$  and adds the energy stretch factor to the induced communication graph.

**THEOREM 3.6.** *For any  $0 < \varepsilon < 1$ , there exists a power assignment  $p'_4$  so that  $c(p'_4) = O(n^{2-2\varepsilon}) \cdot c^*$ ,  $\delta(H_{p'_4}) = n^\varepsilon$ , and  $\gamma(H_{p'_4}) = 1 + O(\frac{1}{1-\varepsilon})$ .*

**PROOF.** From Lemma 3.5,  $c(p_4) = O(n^{2-2\varepsilon}) \cdot w(MST_V)$  and  $\delta(H_{p_4}) = n^\varepsilon$ ,  $0 < \varepsilon < 1$ . By using Lemma 3.4 we obtain a power assignment  $p'_4$  so that  $\gamma(H_{p'_4}) = 1 + O(\frac{1}{2-2\varepsilon})$ ,  $H_{p_4}$  is a subgraph of  $H_{p'_4}$ , which results in  $\delta(H_{p'_4}) = n^\varepsilon$ , and

$$c(p'_4) = \Theta(c(p_4)) = O(n^{2-2\varepsilon}) \cdot w(MST_V).$$

Combining with Theorem 3.2,  $c(p'_4) = O(n^{2-2\varepsilon}) \cdot c^*$ .  $\square$

### 3.3. Augmentation by maximum edge length criterion

The second algorithm, AUGMENT-MAX-EDGE utilizes the hop-diameter of the underlying structure. Let  $T$  be a bounded hop-diameter spanning tree of  $G_V$  (we discuss its computation in the remark below). For any pair of nodes  $u, v \in V$ , if  $d(u, v) < |e^*(T)|/n^\varepsilon$  we add the edge  $(u, v)$  to  $T$ .

---

**AUGMENT-MAX-EDGE-LENGTH**


---

```

1 compute a bounded hop-diameter tree  $T = (V, E)$ 
2  $E' \leftarrow \emptyset$ 
3 foreach  $u, v \in V$  do
4   |   if  $d(u, v) < |e^*(T)|/n^\varepsilon$  then
5   |   |   add  $(u, v)$  to  $E'$ 
6 let  $G = (V, E \cup E')$ 
7 foreach  $u \in V$  do
8   |    $p_5(u) = |e_u^*(G)|^2$ 

```

---

The obtained distance stretch factor and the cost are given in the following lemma.

**LEMMA 3.7.** *For any  $0 < \varepsilon < 1$ ,  $c(p_5) = O(n^{1-2\varepsilon}) \cdot w(T)$  and  $\delta(H_{p_5}) \leq h(T) \cdot n^\varepsilon$ .*

PROOF. The proof resembles the one in Lemma 3.5.

$$c(p_5) \leq 2 \cdot w(T) + \frac{n \cdot |e^*(T)|^2}{n^{2\varepsilon}} = O(n^{1-2\varepsilon}) \cdot w(T).$$

Since  $T$  is a tree, for any two nodes  $u, v \in V$  there exists only one path  $P$  from  $u$  to  $v$  in  $T$ . The total distance of  $P$  is bounded by  $d(P) \leq h(T) \cdot |e^*(T)|$ . Following Observation 3.1,  $G$  is a subgraph of  $H_{p_5}$ ; therefore  $\delta_{u,v}(H_{p_5}) \leq \delta_{u,v}(T)$ . Again, two cases are considered:

**Case 1:** If  $d(u, v) < |e^*(T)|/n^\varepsilon$  then a shortcut is added and  $\delta_{u,v}(H_{p_5}) = d(u, v)$ .

**Case 2:** Otherwise,  $d(u, v) \geq |e^*(T)|/n^\varepsilon$ . Then,

$$\delta_{u,v}(T) = d(P) \leq h(T) \cdot |e^*(T)| \leq h(T) \cdot n^\varepsilon \cdot d(u, v).$$

Our proof is complete.  $\square$

The properties of  $p_5$  are summarized below.

**THEOREM 3.8.** *For any  $0 < \varepsilon < 1$  and spanning tree  $T$  of  $G_V$ , there exists a power assignment  $p'_5$  so that  $c(p'_5) = O(n^{1-2\varepsilon}) \cdot w(T)$ ,  $h(H_{p'_5}) = h(T)$ ,  $\delta(H_{p'_5}) = h(T) \cdot n^\varepsilon$ , and  $\gamma(H_{p'_5}) = 1 + O(\frac{1}{1-\varepsilon})$ .*

PROOF. The proof is very similar to the proof of Theorem 3.6. We can use Theorem 3.4 as  $T$  is a spanning subgraph of  $G_V$  and therefore,  $w(T) \geq w(MST_V)$ . The hop-diameter of  $H_{p'_5}$  is due to the fact that  $T$  is a subgraph of  $H_{p'_5}$ .  $\square$

**Remark:** Note that we can choose any low cost bounded hop-diameter tree as the underlying structure  $T$  in the algorithm AUGMENT-MAX-EDGE. For instance, it is possible to use a tree developed in [Kesselman et al. 2005]. The authors constructed a BDMST (bounded hop-diameter spanning tree of  $G_V$ ) with an arbitrary diameter  $h > 1$ , and a cost  $O(f(n) \cdot \log n)$  times the optimal cost of a minimum weight BDMST, where  $f(n)$  is the worst-case ratio between the cost of an optimal BDMST and that of an optimal solution for the bounded hop-diameter broadcast problem. It is also possible to use the bounded-hop construction in [Elkin et al. 2011].

### 3.4. Distance stretch factor of a minimum weight spanning tree

Finally, we show an observation which demonstrates a connection between the hop-diameter of  $MST_V$  and its distance stretch factor. The following lemma might be known as a folklore, but since we are not aware of any similar published statement, we present it here for completeness.

**LEMMA 3.9.**  *$MST_V$  is a distance  $h(MST_V)$ -spanner of  $G_V$ .*

PROOF. The proof is inductive on the number of nodes  $n$ . Clearly, for  $n = 2$ , the distance stretch factor of  $MST_V$  is  $h(MST_V) = 1$ . Assume that the claim holds for any number of nodes less than  $n$ . Let  $(V_1, V_2)$  be the cut induced by the longest edge in  $MST_V$ ,  $e^*(MST_V)$ . We define  $T_1$  and  $T_2$  to be the subtrees of  $MST_V$ , spanning  $V_1$  and  $V_2$ , respectively. It is easy to verify that  $T_1$  and  $T_2$  are minimum weight spanning trees, and  $|V_1|, |V_2| < n$ . Due to the induction hypothesis,  $T_1$  (resp.  $T_2$ ) has a distance stretch factor of  $h(T_1)$  (resp.  $h(T_2)$ ), which is at most  $h(MST_V)$ . This means that for any pair of nodes  $u, v$  in either of the trees, w.l.o.g. in  $T_1$ ,

$$\delta_{u,v}(MST_V) = \delta_{u,v}(T_1) \leq h(MST_V) \cdot d(u, v).$$

Next we bound the value of  $\delta_{u,v}(MST_V)$  for any  $u \in V_1$  and  $v \in V_2$  (nodes not in the same tree). Note that  $e^*(MST_V)$  is the minimum length edge in the cut  $(V_1, V_2)$  in  $G_V$ . Therefore,

$d(u, v) \geq e^*(MST_V)$ . Since in  $MST_V$  there is only one path between every pair of nodes we can conclude that

$$\delta_{u,v}(MST_V) \leq h(MST_V) \cdot |e^*(MST_V)| \leq h(MST_V) \cdot d(u, v).$$

This concludes our proof.  $\square$

To make use of this property, we define the following power assignment. For every  $u \in V$ ,  $p_6(u) = |e_u^*(MST_V)|^2$ . The next theorem summarizes the properties of  $p_6$ .

**THEOREM 3.10.**  $c(p_6) \leq 2 \cdot c^*$ ,  $l(p_6) = l^*$ , and  $\delta(H_{p_6}) = h(MST_V)$ .

**PROOF.** The cost and distance stretch factor follow immediately from Observation 3.1, Theorem 3.2, and Lemma 3.9. The lifetime can be easily derived from Lemma 2.1.  $\square$

#### 4. DISTRIBUTED IMPLEMENTATION

In order to make our solutions feasible, i.e. to allow them to work in real life node deployments, we outline how it is possible to implement them in a decentralized (distributed) manner (without the need for coordination by a central unit). In the proposed distributed implementations we make a use of the works [Awerbuch 1987] and [Bertsekas and Gallager 1987]. [Awerbuch 1987] shows how to find a leader in a distributed fashion in a network with  $n$  nodes in  $O(n)$  time using  $O(n \log n)$  messages. The distributed algorithm for building an SPT is shown in [Bertsekas and Gallager 1987]; its time complexity is  $O(h(G))$  and message complexity is  $O(n|E|)$ , where  $G = (V, E)$  is the initial graph. We assume that some initial undirected connected topology exists for the distributed schemes to take place and that nodes can compute the distance to any other node. In order to find locally its neighbors, each node transmits at maximum possible transmission range and discovers its neighbors. Moreover, we may also follow the different approach. The paper by Dolev et al. [Dolev et al. 2012] presents distributed algorithm, CONSTRUCT UNDERLYING TOPOLOGY, which provides the initial construction of the network. The algorithm is executed at every node and can be divided into two steps. First, the nodes discover their immediate neighbors by allowing them to transmit at maximum possible transmission range. Then this information is flooded so that each node could locally construct the entire network which can later be used for computing spanners.

Regarding the probabilistic case, for the *basic* spanner construction we can use the algorithm in [Awerbuch 1987] which also counts the number of nodes in the network with the same time and message complexity as the leader election problem. In order to implement the *grid-based* scheme, we first apply an algorithm for the leader election problem with a consequent choice (by elected leader) of  $k$  nodes running distributed SPT algorithm from [Bertsekas and Gallager 1987], in parallel. As in the previous case, we also need to count the number of nodes in the network in order to choose the final power assignment. The running time remains  $O(n)$ , while the message complexity becomes  $O(n \log n + kn|E|)$ . Finally, the *SPT-based* algorithm executes  $n$  shortest-path tree constructions in parallel. In this case as well we apply the algorithm from [Awerbuch 1987] for counting the total number of nodes. Thus, the running time remains  $O(n)$  while the message complexity deteriorates to  $O(n^2|E|)$ .

In order to deal with the deterministic case for augmentation by total edge length criterion, we first find the  $MST_V$ , using the appropriate algorithm from [Awerbuch 1987], and compute its total length  $D$  in  $O(n)$  time using  $O(n \log n)$  messages through a convergecast process towards the leader. Using  $D$ , each node, in parallel, computes the edges required to be added to the constructed graph and chooses the largest outgoing edge. The total time and message complexities are dominated by the initial  $MST_V$  construction step. For augmentation by maximum edge length criterion we notice that the difference from the augmentation by total edge length criterion scheme is by replacing  $D$  by maximum edge length of the bounded hop-diameter tree. One of possible constructions was given by [Elkin et al. 2011] that show how to

build a tree of  $O(n/\rho + \log \rho)$  hop-diameter having weight  $O(\rho w(MST_V))$ , where  $\rho$  is a positive parameter. As was pointed in [Elkin et al. 2011] this tree can be found distributively in  $O(n)$  time using  $O(n \log n)$  messages. As before, using the convergecast process started from leaves where each node transmits towards the leader the maximum outgoing edge in its subtree, we find the maximal edge. All the rest remains the same.

## 5. SIMULATIONS

We tested the performance of our power assignment algorithms through simulations. The wireless nodes were randomly, uniformly and independently placed in a unit square, with the values of  $n$  ranging from 100 to 1000 with steps of 100. Each point in the plot is an average of 5 tries.

In our simulations we compared the behavior of 7 different power assignments. The power assignments developed in Section 2,  $p_1$ ,  $p_2$  with  $k = \frac{n}{8 \log n}$ , and  $p_3$  are denoted as **basic**, **grid** and **spt**, respectively. For the power assignments constructed in Section 3 we consider two possible values of  $\varepsilon$ , 0.1 and 0.9; we denote by **atel1** and **atel2** the output of the AUGMENT-TOTAL-EDGE-LENGTH algorithm with  $\varepsilon = 0.1$  and  $\varepsilon = 0.9$ , respectively; similarly, **amel1** and **amel2** denote the output of the AUGMENT-MAX-EDGE-LENGTH algorithm.

We measured different performance criteria of the power assignments: *energy efficiency* (total cost and lifetime), *general graph characteristics* (hop-diameter and interference), and *the stretch factors* under the two models (energy and distance).

### 5.1. Energy efficiency

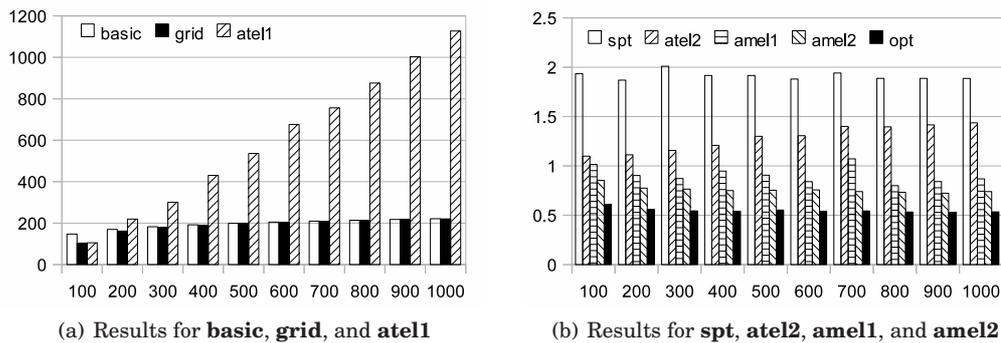


Fig. 4. Total energy consumption as a function of the number of nodes

We are able to compare the performance of our power assignments with lower and upper bounds of an optimal solution (denoted as **opt**) as derived in Lemma 2.1 and Theorem 3.2.

As expected, the total energy consumption of **basic**, **grid**, and **atel1** is higher than for the rest of the power assignments (Figure 4(a)). It is interesting to observe that **basic** and **grid** have an almost identical energy consumption. This is probably due to the fact that the requirement that  $H_{p_1}$  is a subgraph of  $H_{p_2}$  dominates the power assigned to nodes. It comes without much surprise that the network lifetime of these power assignments is quite low as well (Figure 5(a)).

The other power assignments (**spt**, **atel2**, **amel1**, and **amel2**), however, are very energy efficient. It can be seen, that both, the total energy consumption (Figure 4(b)) and network lifetime (Figure 5(b)) are within a small constant from the best possible.

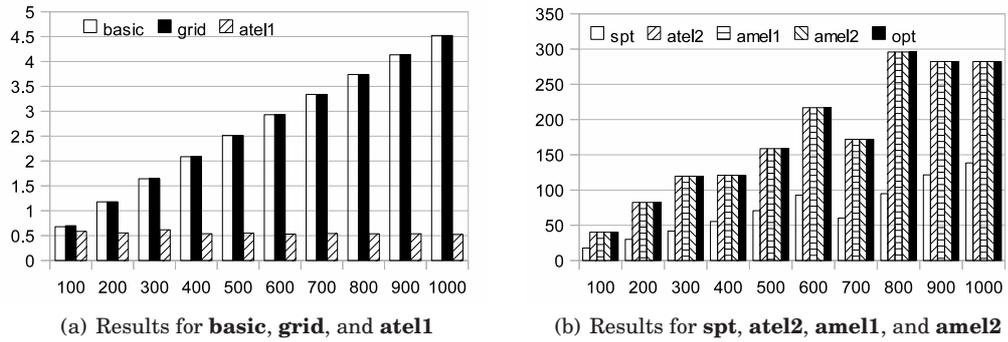


Fig. 5. Network lifetime as a function of the number of nodes

## 5.2. General graph characteristics

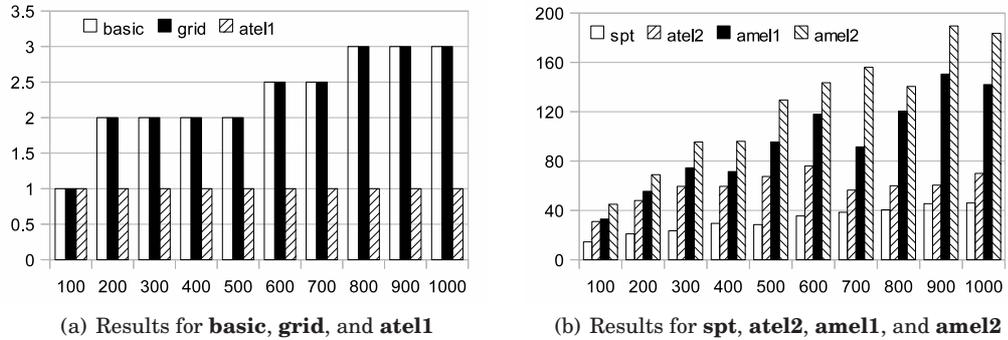


Fig. 6. Network hop-diameter as a function of the number of nodes

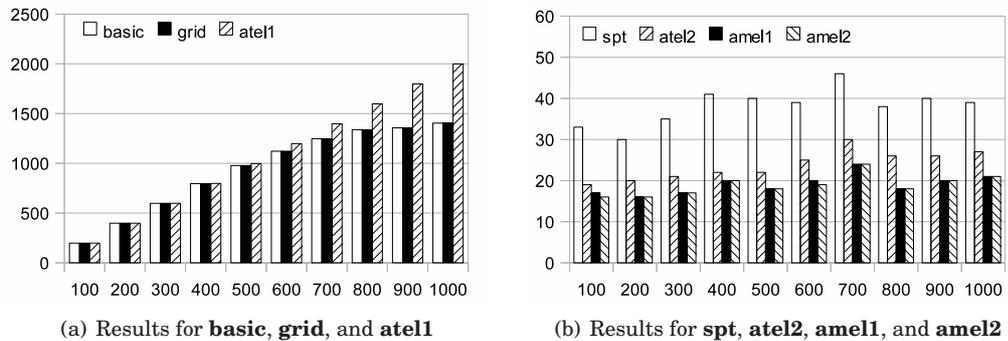


Fig. 7. Maximum interference as a function of the number of nodes

High energy consumption power assignments (**basic**, **grid**, and **atel1**) produce communication graphs which have a low hop-diameter (Figure 6(a)). In fact the hop-diameter is a very small constant, 1 – 3, which significantly simplifies the task of routing. On the other hand, the

interference is naturally increased, compared to lower level power assignments, as it can be seen in Figure 7.

The lower level power assignments (**spt**, **atel2**, **amel1**, and **amel2**) produce a more balanced communication graph, in terms of both hop-diameter and interference (Figure 6(b) and Figure 7(b)). It should be noted that **spt** has the best performance, with the lowest interference and hop-diameter.

### 5.3. Stretch factors

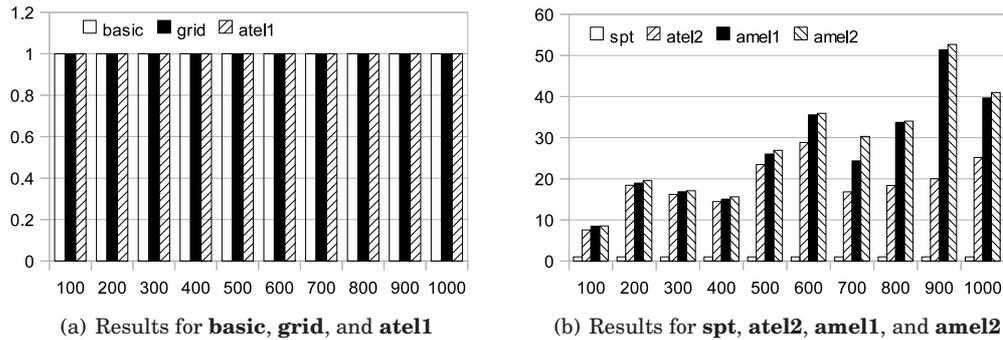


Fig. 8. Energy stretch factor as a function of the number of nodes

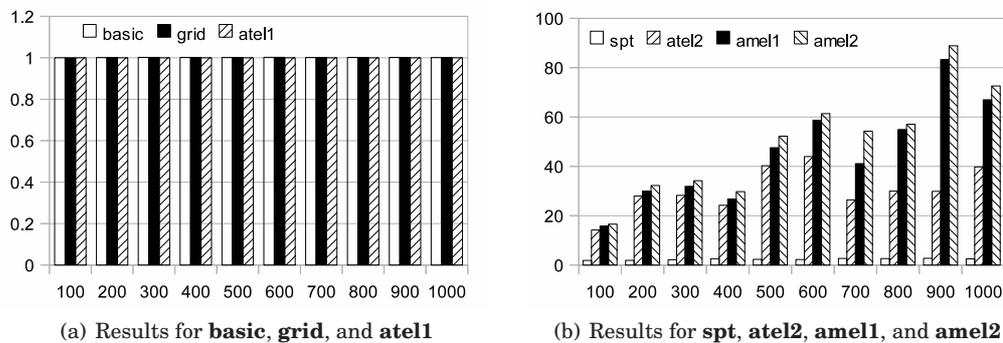


Fig. 9. Distance stretch factor as a function of the number of nodes

It is intuitive that energy consuming power assignments will have better stretch factors as depicted in Figure 8 and Figure 9. Clearly, the power assignments **basic**, **grid**, and **atel1** acquire (almost) all the minimum cost paths (in both models) and therefore the stretch factors are very close to 1 (Figure 8(a) and Figure 9(a)).

The power assignments **atel2**, **amel1**, and **amel2** result in a stable stretch factor for both measures, while **spt** provides a very good, small constant stretch factor for both, energy and distance, models.

## 6. CONCLUSIONS AND FUTURE WORK

In this paper we studied the problem of spanner construction in wireless ad hoc networks through power assignments. We presented power assignment algorithms which achieve provable theoretical bounds on the stretch factor in both, energy and distance, spanner models. In addition we addressed several other optimization objectives: low total energy consumption, low interference level, low hop-diameter, and high network lifetime. We studied two possible node deployments: random and arbitrary. To the best of our knowledge, the presented results are the first results for spanner construction in wireless ad-hoc networks with provable bounds for both, energy and distance, metrics simultaneously.

We outlined a possible distributed implementation of our power assignments and analyzed the complexity of such schemes. Finally, we tested the performance of our power assignments through simulation and measured each of the optimization criteria for 6 different power assignments.

One of the obvious conclusions of this paper is that the SPT-based power assignment has the best performance in every considered optimization criteria. However, it also has the worst construction time.

A natural future research direction would be to expand our understanding of the deterministic case and provide stronger bounds. The simulations results certainly show that the bounds can be refined. It would also be of great interest to scale down by a constant the basic and grid-based power assignments to make them more energy efficient, without compromising other optimization objectives. Finally, we would like to identify the best values for  $\varepsilon$  for  $p_4$  and  $p_5$  in the deterministic case.

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