Bounded-Hop Energy-Efficient Liveness of Flocking Swarms

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Abstract—In this paper we consider a set of n mobile wireless nodes, which have no information about each other. The only information a single node holds is its current location and future mobility plan. We develop a two-phase distributed self-stabilizing scheme for producing a bounded hop-diameter communication graph.

The first phase is dedicated to the construction of an underlying topology for the dissemination of data needed for the second phase. In the second phase the required topology is constructed by means of an asymmetric power assignment under two modes — static and dynamic. The former aims to provide a steady topology for some time interval, while the latter uses the constant node locations changes to produce a constantly changing topology, which succeeds to preserve the required property of the bounded hop-diameter.

We provide an $O(\lambda,\lambda^2)$-bicriteria approximation (in terms of total energy consumption and network lifetime, respectively) algorithm in the static mode: for an input parameter λ we construct a static h-bounded hop communication graph, where $h = n/\lambda + \log\lambda$. In the dynamic mode, given a parameter h we construct an optimal (in terms of network lifetime) h-bounded hop communication graph when every node moves with constant speed in a single direction along a straight line during each time interval. Our results are validated through extensive simulations.

1 Introduction and Related Work

Consider the following scenario, a swarm of mobile scientific sensors are deployed in a backcountry area to perform a series of geological tests. The sensors sample the soil and change their position based on the readings. As the sensors work collectively, they require a wireless backbone to exchange their readings. The backbone has to be able to provide the required connectivity over some period of time despite the movements; desirably it will also have a low hop-diameter so that the routing complexity and delay are reduced. We propose schemes for the construction of such dynamic communication backbones in the presence of moving participants.

Typically, the transmission range r_v of a node v is determined by the power $p(v)$ which is assigned to v. It is customary to assume that the minimal transmission power required to transmit to distance d is d^α, where the *distance-power gradient* α is usually between 2 and 4, [40]. Thus, node v receives transmissions from node u iff $p(u) \geq d(u,v)^\alpha$, where $d(u,v)$ is the Euclidean distance between u and v (e.g. in Fig. 1 node v can receive transmissions from u, while w cannot). The majority of routing (and other) network protocols were traditionally developed for undirected graphs with symmetric (bidirectional) communication links. However, in wireless ad hoc networks it is not uncommon to have asymmetric (unidirectional) links due to non-uniform background noise, non-uniform external interference and energy efficiency considerations. Some recent research addressed this phenomenon by providing several approaches for various network tasks (e.g. [4], [27], [37], [43], [48]). We choose not to enforce symmetry over communication links, thus allowing unidirectional links to exist, which addresses a more general and realistic model of wireless ad hoc networks.

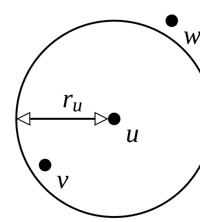

Fig. 1. The power assignment and transmission range model

By varying the power assignment of the nodes one can obtain different network topologies as suggested by Chen and Huang [11]. As the signal strength attenuates with the distance, two major factors determine the topology of a wireless ad hoc network at any given moment – the current locations of the wireless nodes and their transmission ranges. If the wireless nodes have fixed positions, the topology will remain unchanged as long as the transmission ranges remain the same. Mobility, however, makes topology control much more challenging.

As nodes are allowed to change their positions, the topology will change with respect to the node movement. It makes sense then to indicate the time interval, for which the induced topology is valid. In this paper we describe range assignments that induce a topology which is valid for some time interval $[t_s,t_f]$, where t_s and t_f are the start and finish times, respectively; we consider two possible modes for the topology construction – *static* and *dynamic*.

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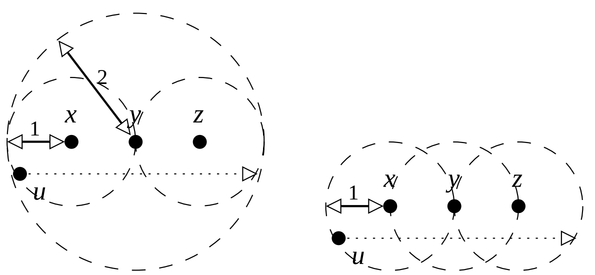
The **static mode**, preserves all the relevant communication links (those that are used for inducing the required topology) for the whole time interval $[t_s, t_f]$. Note that some other links might appear and disappear during the time interval, however the important links, which define the required topology remain unchanged. In other words, the communication graph, which is variant in time, always includes a subgraph which is unchanged for the whole time interval. The **dynamic mode** is different in that there is no constant subgraph which holds the topology property. However, as communication links are added and removed, depending on the movement of the nodes, the topology property requirement (e.g. connected dominating set) is satisfied during the entire period $[t_s, t_f]$. For example, in Fig. 2, there are 4 nodes; x, y, z are stationary, while u moves along the dotted arrow. The topology requirement is to induce a connected dominating set. In Figure 2(a) we show the static mode. The power assignment $p(u) = 0$, $p(x) = p(z) = 1$, and $p(y) = 4$ ensures that the following edges are always present: $(x,y),(y,x),(y,z),(z,y),(y,u)$. The dynamic mode (Figure 2(b)) is different. The power assignment $p(u) = 0$, $p(x) = p(y) = p(z) = 1$ ensures the existence of the following edges: $(x,y),(y,x),(y,z),(z,y)$. However, there is always an edge to u from one of the nodes, x, y or z, therefore the connected dominating set property is always maintained.



(a) Static mode: $p(x) = p(z) = 1$, $p(y) = 4$; u is always reachable by y

(b) Dynamic mode: $p(x) = p(y) = p(z) = 1$; u is always reachable by some node

Fig. 2. Power assignment modes in mobile settings. Nodes x, y, z are stationary, while u moves along the dotted arrow and $p(u) = 0$. The topology requirement is *connected dominating set*.

We focus on the construction of strongly connected dynamic backbones (graphs) with a bounded-hop diameter. By definition, a graph $H = (V, E_H)$ is *h-bounded hop strongly connected* (in short, *h*-bounded), if for any pair of nodes $u, v \in V$, there exists a path from u to v in H with at most h edges. The hop-diameter of H, denoted $\Delta(H)$, is the minimum value of h for which H is h-bounded. Minimizing the hop-diameter of a communication backbone essentially minimizes the end-to-end delay according to many routing schemes which use the hop-count metric [29].

There are several challenges a network designer faces when developing a topology control algorithm in a mobile wireless network, such as: *How do the nodes discover each other? How do the nodes share the current layout information? How do the nodes share their mobility plans? How to discover new nodes? How to discover a node failure?* – All these questions must be answered before the construction of the communication backbone with the desirable property can begin. It seems reasonable to divide the problem of topology control into two main phases: *discovery* and *construction* [17]. In the first phase (described in Section 2), the nodes execute a very basic distributed algorithm, for the discovery of other nodes and disseminating the mobility and current layout information. The general idea is to execute this algorithm at constant time intervals. The second phase (described in Section 3) takes place between two consecutive executions of the first phase; let the time interval in between be $[t_s, t_f]$. Having acquired the mobility plans and current layout, each node now has all the required information to carry out the topology control algorithm and decide on its own power assignment. The topology constructed in the second phase is valid for the time interval $[t_s, t_f]$. Note that at the beginning of the first phase, each node is only aware of its current location and its own mobility plan. By adopting this scheme we are able to handle node failures and corruptions and thus to operate in an hostile environment. We also react to changes in the initial network settings, such as nodes arrival and departure. Some work was done on data dissemination and topology discovery for mobile networks [10], [44], and for stationary networks [18], [19], [23], [28], [30]. All these papers assume some underlying infrastructure for message passing, which is not the case we consider. Data dissemination algorithms might be used in the first phase after some basic underlying communication backbone is obtained.

Energy consumption is one of the most critical resources in wireless ad hoc networks as wireless devices have no constant power supply and have to rely on limited battery charges. Replacing the batteries is typically impractical or even impossible. Thus developing energy efficient communication backbone is of utmost importance [38]. In this work we use two metric to measure the energy efficiency of our power assignments: *network lifetime* (the time until the first node runs out of its battery charge) and *total energy consumption*. We note that a prolonged network lifetime, in addition to the obvious benefits, has an implicit effect of reducing the interference. As nodes communicate through radio signals, interference is inevitable; simultaneous transmissions can be heard by multiple nodes, which may lead to incorrect signal receptions. A common model used by multiple researchers is the *protocol interference model* ([8], [34], [39]), which measures the number of affected nodes or edges[1] in the communication graph by activating a specific link. Under the protocol model, a successful transmission occurs when a node falls inside the transmission range of its intended transmitter and falls outside the interference ranges of other non-intended transmitters. That is, if a node falls in the interference range of a non-intended transmitter, then this node is considered to be interfered and thus cannot receive correctly from its intended transmitter; otherwise, the interference is assumed to be negligible. As the network lifetime is reversely proportional to the maximum transmission power assigned ([21], [33], [45]) – higher network lifetime is coupled with lower transmission ranges and consequently reduced interference levels; this has a positive effect on the overall network performance in terms of schedule length, number of retransmissions, error rate, etc.

We assume the use of *frame-based* MAC protocols which divide the time into frames, containing a fixed number of slots. The

1. These nodes and/or edges cannot be used simultaneously with the active link.

main difference from the classic TDMA is that instead of having one access point which controls transmission slot assignments, there is a localized distributed protocol mimicking the behavior of TDMA. The advantage of a frame-based (TDMA-like) approach compared to the traditional IEEE 802.11 (CSMA/CA) protocol for a Wireless LAN is that collisions do not occur, and that idle listening and overhearing can be drastically reduced. When scheduling communication links, that is, specifying the sender-receiver pair per slot, nodes only need to listen to those slots in which they are the intended receiver – eliminating all overhearing. When scheduling senders only, nodes must listen in to all occupied slots, but can still avoid most overhearing by shutting down the radio after the MAC (slot) header has been received. In both variants (link and sender-based scheduling) idle listening can be reduced to a simple check if the slot is used or not. Several MAC protocols were developed to adapt classical TDMA solutions which use access points to ad-hoc settings that have no infrastructure; these protocols employ a distributed slot-selection mechanism that self-organizes a multi-hop network into a conflict-free schedule (see [41], [47]).

This paper is organized as follows. In the rest of this section, we present our system settings, discuss previous work and state our results. The first phase (layout and mobility plan dissemination) is described in Section 2, followed by the second phase (topology control) in Section 3. Some numerical results are shown in Section 4. Finally, we conclude and discuss possible future directions in Section 5.

1.1 System settings

The topology control phase takes place after successful propagation of the mobility plans, and each node is aware of its own and other nodes mobility plans for some fixed time interval $[t_s, t_f]$, where t_s and t_f are the start and finish times, respectively. By $t \in [t_s, t_f]$ we indicate that $t_s \le t \le t_f$.

Let V be the set of n mobile wireless nodes. As the distance between any two nodes $u, v \in V$ may vary in time, we define $d_{u,v}(t)$ to be the Euclidean distance between u and v at time $t \in [t_s, t_f]$. A power assignment is a function $p : V \to \mathbb{R}^+$, which assigns each node $v \in V$ a transmission range $r_v = \sqrt[\alpha]{p(v)}$ (in this work we assume $\alpha = 2$ for simplicity, although our results can be easily extended to any constant α). The transmission possibilities resulting from a power assignment vary in time. Let $H_p(t) = (V, E_p(t))$, with $E_p(t) = \{(u,v) : r_u \ge d_{u,v}(t)\}$, be the induced directed communication graph at time $t \in [t_s, t_f]$. The cost of the power assignment is defined as $c(p) = \sum_{v \in V} p(v)$.

The lifetime of the network is defined as the time it takes the first node to run out of its battery charge. Each node $v \in V$ has some initial battery charge b_v, which is sufficient for some limited time, depending on the power assignment $p(v)$. It is common to take the lifetime of v to be $l_v = b_v / p(v)$, that is after a time interval of length $b_v / p(v)$ the battery is completely depleted. The lifetime of the whole network for a given power assignment p is $l(p) = \min_{v \in V} l_v$, see [33], [45], [21]. This paper assumes that all the nodes have the same initial battery charge b.

Definition 1.1. *A power assignment p induces a* static *h-bounded communication graph if there exists an h-bounded graph H_p, so that H_p is a subgraph of $H_p(t)$ for every $t \in [t_s, t_f]$.*

Definition 1.2. *A power assignment p induces a* dynamic *h-bounded communication graph if for every $t \in [t_s, t_f]$, $H_p(t)$ is h-bounded.*

In this paper we consider the problem of energy efficient power assignment which induces a communication graph with a bounded hop-diameter for the two modes, static and dynamic. In particular we solve the following two problems.

Problem 1.3 (Static Bounded Hop (SBH))**.** *Given the graph $G = (V, E_V)$, and a bound h for hop-diameter, find a power assignment p which induces a static h-bounded communication graph so that $c(p)$ is minimized and $l(p)$ is maximized.*

Problem 1.4 (Dynamic Bounded Hop (DBH))**.** *Given the graph $G = (V, E_V)$ and parameter $h > 0$, find a power assignment p which induces a dynamic h-bounded communication graph so that $l(p)$ is maximized.*

Note that the SBH problem aims to minimize the hop-diameter as one of its optimization objectives. This is due to the natural trade off between the cost of a power assignment and the hop-diameter of the induced communication graph. In Section 3.1 we propose an approximation algorithm which balances this trade off – the shorter the paths, the greater the ranges assigned. The definition of the DBH problem is somewhat simpler, as it is lacking the cost optimization constraint, and the desired hop-diameter is given as a parameter. As a result, in Section 3.2 we are able to optimally solve the problem.

1.2 Previous work

First we overview topology control in mobile networks in general and then discuss the bounded hop-diameter property.

1.2.1 Topology control in mobile networks

In theory it is impossible to devise a range assignment that will satisfy the topology requirement for a given period of time without being aware of the future location changes. Each node has its own *mobility plan*, which is composed of direction vectors, velocity, acceleration, and so on. Basch et al. [5], [6] proposed an elegant method to handle topology updates for mobile nodes. They proposed a framework to maintain an invariant of a set of moving objects in a discrete manner, called the *kinetic data structure* (KDS in short). They introduce the idea of keeping certificates as triggers for updates. When an object moves and a certificate fails, the consistency of the kinetic data structure is invalidated and an update is mandatory. Each failure of a certificate incurs a setup of up to a constant number of new certificates. Hence we are allowed to monitor the dynamics of a set of objects discretely and efficiently. The kinetic data structure requires that we know the mobility plan (a specification of the future motion) of all nodes, and that the trajectory of each disk can be described by some low-degree algebraic curve. These structures are extremely efficient for topology maintenance, but do not address the issue of energy efficiency or the construction of initial topology. The approach taken in this paper resembles the spirit of KDS. Additional results for topology control in mobile networks may be found in [25], [31], [32], [36]. However, none of these works addresses the bounded-hop strong connectivity property.

1.2.2 Bounded hop-diameter

The only results for h-bounded strong connectivity were obtained for stationary networks. For the linear case of node disposition, Kirousis et al. [35] develop an optimal power assignment algorithm in $O(n^4)$ time. In the Euclidean case, [15] obtains constant ratio approximation algorithms for the bounded-hop vertex connectivity for well spread instances. Beier et al. [7] discuss the problem of finding a bounded-hop path between pairs of nodes with minimized power consumption. They find an optimal path in $O(hn\log n)$ time. In [9] the authors obtain $(O(\log n), O(\log n))$ bicriteria approximation algorithms for the bounded-hop broadcast, bounded-hop connectivity and bounded-hop symmetric connectivity problems. In their output there are at most $h\log n$ hops and the cost is at most $\log n$ times the optimal. In [1] the authors present an exact algorithm for solving the 2-hop broadcast problem with a running time of $O(n^7)$ as well as a PTAS with a running time of $O(n^{\mu})$ where $\mu = O((h^2/\varepsilon)^{2^h})$. Funke and Laue [24] provide a PTAS for the h-broadcast algorithm in $O(n)$ time. Shpungin and Segal [46] give approximation algorithms for k-fault resistant bounded-hop broadcast for the linear and planar layout of nodes. They develop power assignments with a total cost of $O(k)$ and $O(k^2)$ times the optimum for the linear and planar cases, respectively. More on bounded range assignments can be found in [13], [16]. These results apply to stationary networks only and do not address the network lifetime optimization objective, with or without the total cost, like we do in this paper.

1.3 Our contribution

In this paper we consider a set of n mobile wireless nodes, which have no information in advance about each other. The only information a single node holds is its current location and future mobility plan. Each node is capable of adjusting its transmission power to cover any range. We choose to divide the problem of topology control in this type of network into two main phases: (1) Dissemination of current layout and mobility plans; (2) Topology construction. These phases are periodically executed, which allows us to quickly react to outdated and corrupted data. The above self-stabilizing scheme [20] allows us to operate even in a hostile environment.

We design a simple, distributed algorithm for the first phase; after which, each node holds the mobility plans and locations of every node in the network. Then, for the second phase we:

- Propose an $O(\lambda, \lambda^2)$-bicriteria approximation algorithm for the SBH problem, so that given a parameter $1 \le \lambda \le n-1$, we construct a power assignment which induces a static h-bounded communication graph, $h = n/\lambda + \log\lambda$, with a cost of at most λ times the optimum and network lifetime of at least $1/\lambda^2$ times the optimum.
- Show an optimal polynomial time algorithm for the DBH problem under the assumption that every node moves in a single direction along a straight line with constant speed. Given a parameter $1 \le h \le n-1$, we construct a power assignment which induces a dynamic h-bounded communication graph with an optimal network lifetime.
- Develop a polynomial time approximation scheme (PTAS) for the DBH problem with a substantially better running time that in the optimal solution.

In addition, our algorithms are validated through extensive simulations.

Note that the SBH problem is NP-Hard since the problem of minimum power strong connectivity is NP-Hard even in stationary networks without the hop-diameter limitation [14].

2 Layout and Mobility Plans Dissemination

One of the main challenges of every ad-hoc wireless network is topology discovery. We propose a simple distributed algorithm to form a temporary underlying topology, which can be used for the dissemination of current location and mobility plans.

We assume that all participants have synchronized clocks (say by a common input from a GPS, or by using a self-stabilizing clock synchronization algorithm). Thus, they all repeatedly and simultaneously start the two phase algorithm, which in turn implies convergence and stabilization following the first restart.

Another assumption is that the first phase is happening very fast, so the location of nodes remains unchanged during its execution. To simplify things, we allow the nodes to transmit at the maximum possible transmission range $R_{\max}$ which can be adjusted. Now, suppose the algorithm is executed at some time t' - so the underlying topology to be used for the data dissemination is actually $H_{p_{\max}}(t')$, where $p_{\max}(u) = (R_{\max})^2$, for every $u \in V$. Without loss of generality, let $H_{p_{\max}}(t')$ be strongly connected (otherwise the algorithm is valid for every strongly connected component).

CONSTRUCT UNDERLYING TOPOLOGY

```
// Neighbors Discovery
N(u) ← ∅
transmit hello⟨u⟩ in range R_max
while not timeout do
    if received hello⟨v⟩ then
        add v to N(u)
// Construct network underlying topology
transmit neighbors⟨u,N(u)⟩ in range R_max
initialize G_u to be an empty graph
add edges {(u,v) : v ∈ N(u)} to G_u (create new nodes if required)
Forwarded ← {u}
Unknown ← N(u)
while Unknown ≠ ∅ do
    if received neighbors⟨v,N(v)⟩ then
        if v ∉ Forwarded then
            add edges {(v,w) : w ∈ N(v)} to G_u (create new nodes
            if required)
            foreach w ∈ N(v) do
                if w ∉ Forwarded and w ∉ Unknown then
                    add w to Unknown
            remove v from Unknown
            add v to Forwarded
            transmit neighbors⟨v,N(v)⟩ in range R_max
```

We present a distributed algorithm, CONSTRUCT UNDERLYING TOPOLOGY, which enables the nodes to acquire the knowledge about the topology of $H_{p_{\max}}(t')$. The algorithm is executed at every node at time t' and can be roughly divided into two steps. First (lines 1-5), the nodes discover their immediate neighbors in $H_{p_{\max}}(t')$. Then (lines 6-20) this information is flooded so that each node could locally construct $H_{p_{\max}}(t')$, which can later be used for layout and mobility plans dissemination.

The first step is carried out by each node transmitting its own unique id at range $R_{\max}$ (line 2). Since a node can receive a message only if it is within the transmission range from the sender, then node u can safely add every node v to its neighbor list $N(u)$ once it receives v's `hello` message.

In the second step, each node obtains the knowledge of the underlying topology $H_{p_{\max}}(t')$ through the flooding of the neighbor lists. Every node u transmits its own neighbor list (line 6) and then constructs the underlying graph G with the aid of two lists, *Forwarded* and *Unknown*. If $v \in \mathit{Forwarded}$, then u has received the `neighbors`$\langle v, N(v)\rangle$ message and graph G_u contains all the edges adjacent to v in $H_{p_{\max}}(t')$. If $v \in \mathit{Unknown}$, then u has not received the `neighbors`$\langle v, N(v)\rangle$ message.

For every new `neighbors`$\langle v, N(v)\rangle$ message received at u, the edges from v to every $w \in N(v)$ are added to G_u (line 13) (if some edge contains an endpoint which does not appear in G_u, it is added to the graph). Then, each node $w \in N(v)$, which is neither in *Forwarded* nor in *Unknown*, is added to *Unknown* (line 17). Finally, `neighbors`$\langle v, N(v)\rangle$ is forwarded (line 20).

Note that during the second step nodes need not transmit at $R_{\max}$ as they only need to reach the farthest node in their neighbor list. That is, node u can forward `neighbors` messages using $\max_{v \in N(u)} d_{u,v}(t')$ as the transmission range. The next theorem shows the correctness, time and message complexity of the CONSTRUCT UNDERLYING TOPOLOGY algorithm.

Theorem 2.1. *The message complexity of* CONSTRUCT UNDERLYING TOPOLOGY *is* $O(|E_{p_{\max}}(t')| \cdot n)$, *and after at most* $\Delta(H_{p_{\max}}(t'))$ *rounds, for every* $u \in V$, $G_u = H_{p_{\max}}(t')$.

Proof: Note that all the messages actually passed along the edges of $H_{p_{\max}}(t')$ since all the nodes transmit at $R_{\max}$. There are $2n$ distinct messages (`hello` and `neighbors`). Only the `neighbors` messages are forwarded (once at each node).

Therefore there are at most $O(|E_{p_{\max}}(t')| \cdot n)$ messages received. Clearly, after $\Delta(H_{p_{\max}}(t'))$ rounds, no messages are forwarded and each node u has constructed some image of the underlying topology G_u. We argue that $G_u = H_{p_{\max}}(t')$. Since the graph $H_{p_{\max}}(t')$ is strongly connected, then G eventually receives all the `neighbors` messages. So we only need to show that all these messages will be eventually handled. In other words, if $\mathit{Unknown} = \emptyset$ then $\mathit{Forwarded} = V$. Suppose by contradiction that at some point $\mathit{Unknown} = \emptyset$ and $\mathit{Forwarded} \subset V$.

Since $H_{p_{\max}}(t')$ is strongly connected, there exists $(v,w) \in E_{p_{\max}}(t')$so that $v \in \mathit{Forwarded}$ and $w \notin \mathit{Forwarded}$. Therefore, w was added to *Unknown* at some point, either line 10 or 17. Note that a node is removed from *Unknown* (line 18) only if it is added to *Forwarded*. Therefore w is never removed from *Unknown*, and as a result $\mathit{Unknown} \neq \emptyset$. A contradiction. □

Remark 1. We notice that the lines 6-20 can be executed without any assumption of synchronized clocks, since we can use synchronizers [2]. However, this will affect running time and/or message complexity of the algorithm.

Once every node holds an image of the underlying topology it can disseminate its current location and mobility plan through $H_{p_{\max}}(t')$ by using one of the existing algorithms [10], [44].

Remark 2. Using the standard *gossiping* mechanism [12], dissemination of current layout and mobility plans as well as topology construction can be performed without frame-based MAC protocol in unknown ad hoc network [26]. In the gossiping problem, each node v in the network initially holds a message m_v, and we want to distribute all messages m_v to all nodes in the network. However, in this case the complexity of the algorithm (in terms of time and message complexities) is much higher and also heavily depends on the size of the information we need to transmit between nodes of the network.

The first phase described in this section is executed periodically with an interval of $t_f - t_s$ time units between two consecutive executions; every time starting from scratch, where each node is only aware of its own current location and future mobility plan. Once the data is disseminated, each node has all the required information to carry out the second phase, which is the actual topology construction algorithm, and decide on its own power assignment. Each nodes carries out the topology control algorithm and decides on its own power assignment. The topology constructed in the second phase is valid for the time interval $[t_s, t_f]$.

3 BOUNDED HOP STRONG CONNECTIVITY

Once every node $u \in V$ has acquired the mobility plans of all the other nodes (as described in Section 2), it is able to compute the value of $d_{u,v}(t)$ for every $v \in V$ and $t \in [t_s, t_f]$. We assume that the computation time is negligible compared to the message transmission time. After that each node executes the algorithms proposed in this section, which in turn defines its transmission power. In what follows we first propose an approximation algorithm for Problem 1.3 (SBH), and then show a polynomial time optimal solution for Problem 1.4 (DBH).

3.1 Static bounded hop communication graph

In this section we propose an approximation algorithm for the SBH problem. We wish to find a power assignment p', which induces a low cost static h-bounded communication graph, with high network lifetime and low hop-diameter.

We need some definitions. Let $G_V = (V, E_V)$ be an undirected complete graph. For any $t \in [t_s, t_f]$, let $w_t(u,v) = (d_{u,v}(t))^2$, for every $(u,v) \in E_V$, a weight function over the edge set E_V. Note that $w_t(u,v)$ matches the amount of energy required to transmit from u to v, at time t. For any weight function w, defined on a weight set E_V, the weight of a graph $H = (V, E_H)$, $E_H \subseteq E_V$, is $w(H) = \sum_{(u,v) \in E_H} w(u,v)$. Let $MST(w)$ be a minimum weight spanning tree of G_V based on a weight function w.

For any two nodes $u, v \in V$, in order that an edge (u,v) would exist in every $H_p(t)$, $t \in [t_s, t_f]$, the power assigned to u should be at least the square of the maximum distance between u and v

during $[t_s,t_f]$.We define a weight function w' which reflects this amount of energy for any pair of nodes,

$$w'(u,v) = \max_{t\in[t_s,t_f]} (d_{u,v}(t))^2, \text{ for every } u,v \in V.$$

The following lemma shows that w' satisfies the weak triangle inequality.[2]

Lemma 3.1. *For any $u,v,z \in V$, it holds*

$$w'(u,v) \leq 2(w'(u,z)+w'(z,v)).$$

Proof: Let $t_0 \in [t_s,t_f]$ be the moment so that $w'(u,v) = (d_{u,v}(t_0))^2$. Due to the triangle inequality in the Euclidean space, for any $u,v,z \in V$ and $t \in [t_s,t_f]$, it holds $d_{u,v}(t) \leq d_{u,z}(t)+d_{z,v}(t)$. Therefore,

$$\begin{aligned} w'(u,v) = (d_{u,v}(t_0))^2 &\leq (d_{u,z}(t_0)+d_{z,v}(t_0))^2 \\ &\leq 2((d_{u,z}(t_0))^2+(d_{z,v}(t_0))^2) \\ &\leq 2(w'(u,z)+w'(z,v)). \end{aligned}$$

The last inequality follows from the definition of w'. □

In [22] the authors show that given a complete graph $G = (U,E)$ with n nodes, a weight function w that satisfies a weak triangle inequality, and a parameter λ, $1 \leq \lambda \leq n-1$, it is possible to construct in polynomial time a spanning tree T of G, so that $\Delta(T) \leq n/\lambda+\log\lambda$, and the weight of T is at most $O(\lambda)$ times the weight of the minimum weight spanning tree of G. In addition the weight of an edge in T is at most λ^2 times the maximum weight of an edge in the minimum weight spanning tree of G. This construction is based on a Hamiltonian cycle. The construction of Hamiltonian circuit can be done very efficiently in a distributed fashion using standard leader election techniques proposed by Awerbuch [3]. Once the leader in the tree is found using $O(n)$ time with $O(n\log n)$ messages, the distributed algorithm for finding Hamiltonian circuit behaves exactly as the centralized description below. The algorithm is applied to a tree T and an edge $e = (u,v)$ of T. Removing the edge e divides the tree into two subtrees T_1 and T_2. In each subtree the algorithm selects an arbitrary edge $e_1 = (u,w)$ (for T_1) and $e_2 = (x,v)$ (for T_2), and recursively computes a Hamiltonian cycle of T_1 and T_2 that includes the edge e_1 and e_2, respectively. The circuit consists of the cycles in T_1 and T_2 without two edges e_1 and e_2. The two resulting paths are glued together using e and the edge connecting other endpoints of two edges e_1 and e_2. This can be done by the convergecast process through the nodes towards the leader.

We use this construction to obtain a spanning tree $T' = (V,E')$ of G_V with a weight function w', which has similar properties; this is possible since w' satisfies the weak triangle inequality (Lemma 3.1). Let e' and e_{MST} be the maximum weight edges in T' and $MST(w')$, respectively. The next theorem summarizes the properties of T'.

Theorem 3.2 ([22])**.** *For any λ, $1 \leq \lambda \leq n-1$, and a weight function w' which satisfies the triangle inequality, it is possible to construct a spanning tree T' of G_V so that $w'(e') \leq \lambda^2 w'(e_{MST})$, $\Delta(T') \leq n/\lambda+\log\lambda$, and $w'(T') \leq O(\lambda)w'(MST(w'))$.*

2. A weight function w holds a weak triangle inequality, if there exists some constant value $\mu > 1$, so that for any three nodes x,y,z, $w(x,y) \leq \mu(w(x,z)+w(z,y))$.

We are now ready to define the power assignment p'. Let $p'(u) = \max_{(u,v)\in E'} w'(u,v)$, for every $u \in V$. The hop-diameter of the induced communication graph and the cost of p' are derived in the following lemma.

Lemma 3.3. *The power assignment p' induces a static h-bounded communication graph for $h = n/\lambda+\log\lambda$ and $c(p') \leq 2w'(T')$.*

Proof: Let T'_D be the directed version of T' (each undirected edge appears as two directed edges). If an undirected edge (u,v) is in T', then the two directed edges, (u,v) and (v,u), appear in $H_{p'}(t)$ for every $t \in [t_s,t_f]$, since $p(u) \geq w'(u,v)$ and $p(v) \geq w'(v,u)$ (which ensures that u and v are within the transmission range of each other for the whole time interval $[t_s,t_f]$). Therefore, for every $t \in [t_s,t_f]$, T'_D is a subgraph of $H_{p'}(t)$. Clearly, p' induces a static h-bounded communication graph for $h = n/\lambda+\log\lambda$ as the hop-diameter of T'_D is at most $n/\lambda+\log\lambda$. Finally,

$$\begin{aligned} c(p') = \sum_{u\in V} \max_{(u,v)\in E'} w'(u,v) &\leq \sum_{u\in V} \sum_{(u,v)\in E'} w'(u,v) \\ &\leq 2 \sum_{(u,v)\in E'} w'(u,v) = 2w'(T'). \end{aligned}$$

This completes our proof. □

In the next two lemmas we derive the lower and upper bounds for the cost and network lifetime, respectively, of a power assignment which induces a static h-bounded communication graph, for any $h \geq 1$.

Lemma 3.4. *Let p_h^C be the minimum cost power assignment which induces a static h-bounded communication graph for some parameter $h \geq 1$. Then, it holds $c(p_h^C) \geq w'(MST(w'))$.*

Proof: It is easy to see that $c(p_h^C) \geq c(p_{n-1}^C)$, as every graph which is h-bounded, $h \geq 1$, is also $(n-1)$-bounded. We prove $c(p_{n-1}^C) \geq w'(MST(w'))$.

Let p be some power assignment which induces a static $(n-1)$-bounded communication graph. Therefore, there exists a directed graph $H_p = (V,E_p)$, so that H_p is strongly connected and for every $t \in [t_s,t_f]$, H_p is a subgraph of $H_p(t)$. From the definition of H_p, if $(u,v) \in E_p$, then $p(u) \geq w'(u,v)$.

Choose an arbitrary node $r \in V$ as the root. For $u \in V$, $u \neq r$, let P_u be a simple directed path from u to r in H_p. Denote by $E(P_u)$ the set of directed edges in P_u. The union of the edges in all the paths, $E = \bigcup_{u\in V,u\neq r} E(P_u)$ forms a directed tree $T = (V,E)$, rooted at r, where all the edges are directed toward the root. For every edge (u,v) in T, the power assigned to u is at least $w'(u,v)$ (since $E \subseteq E_p$). As there is only one outgoing edge from every node (toward the root), we obtain $w'(T) = \sum_{(u,v)\in E} w'(u,v) \leq \sum_{u\in V,u\neq r} p(u) \leq c(p)$. Let T_U be the undirected version of T obtained by omitting the edge directions. Clearly T_U is a spanning tree of G_V, and $w'(T) = w'(T_U) \geq MST(w')$.

Since we chose p to be any power assignment which induces a static $(n-1)$-bounded communication graph, we therefore conclude $c(p_{n-1}^C) \geq w'(T) \geq w'(MST(w'))$. □

Recall that the network lifetime is defined as the time it takes the first node to run out of its battery charge. For equal initial battery charges b and a power assignment p, the network lifetime $l(p)$ is defined as $l(p) = \min_{v\in V} b/p(v)$.

Lemma 3.5. *Let p_h^L be the maximum network lifetime power assignment which induces a static h-bounded communication graph for some parameter $h \geq 1$. Then, $l(p_h^L) \leq b/w'(e_{MST})$.*

Proof: Since p_h^L induces a static h-bounded communication graph, there exists a directed h-bounded graph $H_{p_h^L}$. It is a well known fact that for any spanning tree $ST = (V, E_{ST})$ of G_V, $\max_{e \in E_{ST}} w'(e) \geq w'(e_{MST})$. Let $e = (u,v)$ be the maximum weight edge in $H_{p_h^L}$. As in the proof of Lemma 3.4, $p(u) \geq w'(e)$. From the definition of network lifetime, $l(p_h^L) \leq b/p(u)$. Therefore, since $H_{p_h^L}$ is strongly connected, $w'(e) \geq w'(e_{MST})$. We conclude $l(p_h^L) \leq b/w'(e) \leq b/w'(e_{MST})$. $\square$

Note that the bounds shown in Lemmas 3.4 and 3.5 do not depend on the value of h. We can now state the main result of this section based on Theorem 3.2, and Lemmas 3.3, 3.4, and 3.5.

Theorem 3.6. *Given n mobile wireless nodes V, and a parameter λ, $1 \leq \lambda \leq n-1$, it is possible to construct in polynomial time a power assignment p_h that induces a static h-bounded communication graph, $h = n/\lambda + \log \lambda$, so that $c(p_h) \in O(\lambda c(p_h^C))$ and $l(p_h) \geq l(p_h^L)/\lambda^2$, where p_h^C and p_h^L are optimal (in terms of cost and network lifetime, respectively) power assignments that induce a static h-bounded communication graph.*

The tightness of the tradeoff follows from the fact that for the linear layout of n nodes located on the line with unit distances between neighboring nodes, the diameter of h is possible only if the edge of weight $(\frac{n-1}{h})^2$ exists. Putting $h = n/\lambda$ proves the result.

3.2 Dynamic bounded hop communication graph

In the case that the wireless nodes share the same initial battery charge b, maximizing the network lifetime is equivalent to minimizing the maximum power assigned to any node. The authors in [42] noted that if the required optimization is to minimize the maximum power assigned, it is possible to assign the same power level to all nodes. Hence, all we need to do is to choose the minimum power level which induces an h-bounded communication graph, for any given $h \geq 1$.

For non-mobile nodes, given a power level x it is easy to test whether the induced communication graph is h-bounded in polynomial time. Furthermore, it makes sense to test only those power levels x, for which there exists a pair of nodes at a distance exactly $\sqrt{x}$ of each other, otherwise the power level can be decreased. Thus, there are at most $\binom{n}{2}$ possible power levels. In Figure 3(b) the same topology as in Figure 3(a) is induced with a lower power level.

Adopting the above scheme to mobile nodes is challenging for two reasons. First, given a specific power level, it is difficult to test if the induced communication graph is dynamically h-bounded in the whole time interval $[t_s, t_f]$. Second, it is unclear what power levels should be considered, as the distance between any pair of nodes might change constantly.

In the rest of this section we assume that every node moves with constant speed in a single direction along a straight line during $[t_s, t_f]$. We show that given a power level x, it is possible to test in polynomial time whether the power assignment induces a dynamic h-bounded communication graph, and also show that

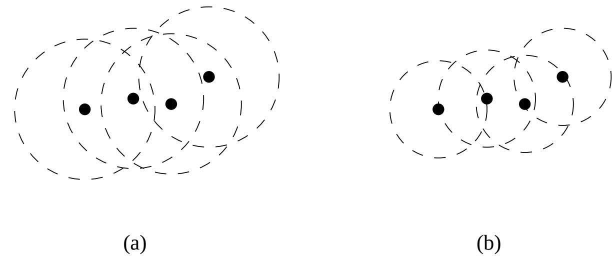


Fig. 3. Power efficiency: (a) an inefficient power level; (b) an efficient power level, same topology.

the number of possible power levels is at most $O(n^4)$. Let p_x be a power assignment, where for every $u \in V$, $p_x(u) = x$.

The distance between any two nodes, each moving with constant speed in a single direction along a line, can be either constant or first decrease to some minimum value and then constantly increase (see Figure 4 for the exposition of different types), as summarized in the following observation.

Observation 3.7. *If during the time interval $[t_s, t_f]$ every node moves in a single direction along a straight line with constant speed then there exists $t' \in [t_s, t_f]$ for any pair of nodes $u, v \in V$ so that the distance function $d_{u,v}(t)$ is monotone non-increasing in $[t_s, t']$ and monotone non-decreasing in $[t', t_f]$.*

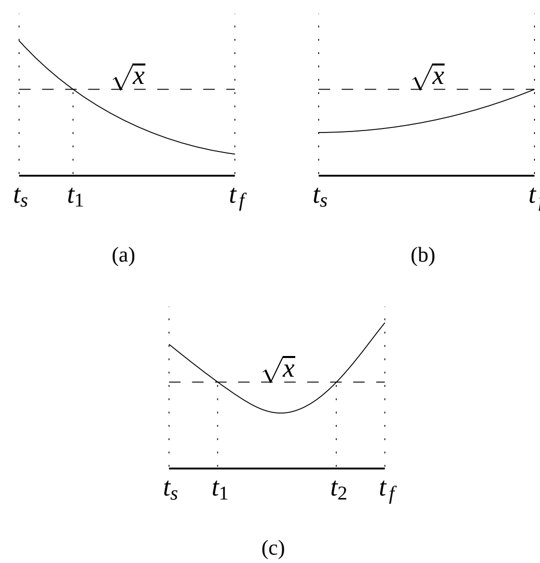


Fig. 4. Distance function types and different cases of the $C_x^{u,v}$ set: (a) $C_x^{u,v} = \{t_1\}$; (b) $C_x^{u,v} = \emptyset$; (c) $C_x^{u,v} = \{t_1, t_2\}$.

3.2.1 Verifying the hop-diameter

Given a power level x, we would like to verify that the communication graph induced by $p_x(u) = x$, for every $u \in V$, is h-bounded. The general idea is to verify the hop-diameter in a finite set of critical time points, and then based on these verifications to conclude for the whole time interval $[t_s, t_f]$.

For any $u, v \in V$, let $[t_s^{u,v}, t_f^{u,v}] \subseteq [t_s, t_f]$ be a non-empty time interval (if exists) so that $d_{u,v}(t) \leq \sqrt{x}$ for $t \in [t_s^{u,v}, t_f^{u,v}]$. If such an interval exists, we define the set of critical time points, $C_x^{u,v}$, to be

$$C_x^{u,v} = \{t_s^{u,v}, t_f^{u,v}\} \setminus \{t_s, t_f\}.$$

Otherwise, $C_x^{u,v} = \emptyset$. That is, $C_x^{u,v}$ is a set of time points, in the open time interval (t_s, t_f), when the nodes u and v change their connectivity status. Note that $C_x^{u,v} = C_x^{v,u}$. Let $C_x = \bigcup_{u,v \in V} C_x^{u,v}$. We claim that it is sufficient to verify the hop-diameter only for the time points in $C_x \cup \{t_s, t_f\}$.

Lemma 3.8. *Given a power level x, the power assignment p_x induces a dynamic h-bounded communication graph if and only if for each $t \in C_x \cup \{t_s, t_f\}$, $H_{p_x}(t)$ is h-bounded.*

Proof: Let $t_s = t_0 \leq t_1 \leq \ldots \leq t_m = t_f$ be the time points in $C_x \cup \{t_s, t_f\}$ sorted in ascending order. Let us focus on a single arbitrary interval $[t_i, t_{i+1}]$, where $0 \leq i \leq m-1$. From the definition of C_x, for any pair of nodes $u, v \in V$ and any time point t', $t_i < t' < t_{i+1}$, it holds $d_{u,v}(t') \neq \sqrt{x}$. Thus, there are no edge changes inside the open time interval (t_i, t_{i+1}). Also, due to the definition of C_x, edge cannot exist only at t_i. Therefore, the graphs $H_{p_x}(t_i)$ and $H_{p_x}(t')$ are identical for every t', $t_i \leq t' < t_{i+1}$.

The *only if* case is trivial, so we concentrate on the *if* case. Suppose that for every i, $0 \leq i \leq m$, $H_p(t_i)$ is h-bounded. From the above, the graphs remain unchanged within each interval $[t_i, t_{i+1})$, $0 \leq i \leq m-1$. Therefore, $H_{p_x}(t)$ is h-bounded for every $t \in [t_s, t_f]$. $\square$

3.2.2 Possible power levels

Although the nodes constantly change their location, there is a finite set of possible power levels which should be tested. The irrelevant power levels are either those which do not supply full coverage for a specific node, or those which supply excessive coverage and may be reduced.

There are three types of power levels which should be considered. Intuitively, the first type L_1 allows nodes to remain within the reach of each other for the whole time interval $[t_s, t_f]$. That is the power level matches the definition of w' in Section 3.1,

$$L_1 = \left\{ x : \exists u, v \in V, \max_{t \in [t_s, t_f]} d_{u,v}(t) = \sqrt{x} \right\}.$$

The second type is due to the following simple logic; if something changes at one place, something has to change at some other place, otherwise the power level is either insufficient or exaggerated. In other words, the second type of relevant power levels consists of values which match the squared distance of a pair of nodes, shared at the same time by at least two pairs.

$$L_2 = \left\{ \begin{array}{ll} x : & \exists t \in [t_s, t_f], \exists u, v, z, y \in V, (u,v) \neq (z,y), \\ & d_{u,v}(t) = d_{z,y}(t) = \sqrt{x} \end{array} \right\}.$$

The third type covers all possible power levels at time points t_s and t_f, similar to the non-mobile case.

$$L_3 = \left\{ x : \exists u, v \in V, d_{u,v}(t_s) = \sqrt{x} \text{ or } d_{u,v}(t_f) = \sqrt{x} \right\}.$$

Denote by $L = L_1 \cup L_2 \cup L_3$. We are ready to present the next lemma.

Lemma 3.9. *Let x be the minimum possible (optimal) power level so that p_x induces a dynamic h-bounded communication graph, where for every $u \in V$, $p_x(u) = x$. Then, $x \in L$.*

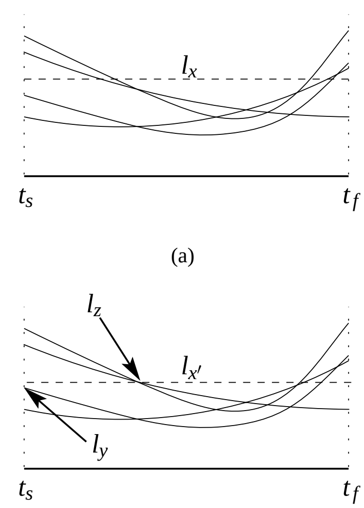
Proof: Suppose by contradiction that $x \notin L$. There are two possible cases to consider.

Case 1: The induced graph remains unchanged for the entire time interval $[t_s, t_f]$. that is for every $t_1, t_2 \in [t_s, t_f]$ the graphs $H_{p_x}(t_1)$ and $H_{p_x}(t_2)$ are identical. Then, from the definition of L for every pair of nodes $u, v \in V$ and $t \in [t_s, t_f]$, $d_{u,v}(t) \neq \sqrt{x}$. Therefore, if $d_{u,v}(t_s) < \sqrt{x}$, then it is also $d_{u,v}(t) < \sqrt{x}$ for any $t \in [t_s, t_f]$.

Let N_u be the set of nodes which are within the transmission range $\sqrt{x}$ from u in $[t_s, t_f]$ (note that this set does not change for the whole time interval). Let,

$$x' = \max_{u \in V, v \in N_u, t \in [t_s, t_g]} d^2_{u,v}(t).$$

It is easy to verify that $x' < x$ and that $p_{x'}$ induces a dynamic h-bounded communication graph. A contradiction that x is optimal.



(b)

Fig. 5. The Illustration of Lemma 3.9, Case 2: (a) Distance functions of all pairs; (b) Lowering the l_x bar to obtain x'.

Case 2: The induced graph changes during the time interval $[t_s, t_f]$. Let us look at some distance function $d_{u,v}(t)$ in time interval $[t_s, t_f]$, and let $l_x = \sqrt{x}$ be a horizontal line corresponding to the transmission range threshold. Whenever $d_{u,v}(t)$ is not above l_x, the edges (u,v) and (v,u) both exist in the communication graph. Once $d_{u,v}(t)$ crosses l_x, a topology change occurs and nodes u, v either connect or disconnect. This resembles the definition of $C_x^{u,v}$.

We now draw all the distance functions (there are $\binom{n}{2}$ such functions) on the same coordinates system (see Figure 5(a)). The topology of the communication graph can be described as follows.

Top-1 Initially, at time t_s, an edge (u,v) exists in the graph $H_{p_x}(t_s)$ if $d_{u,v}(t_s)$ is not above l_x.

Top-2 Then, each time some distance function crosses l_x, a topology change occurs and either a new edge is added or an existing is removed.[3] This resembles the definition of C_x.

Top-3 Finally, at time t_f, an edge (u,v) exists in the graph $H_{p_x}(t_f)$ if $d_{u,v}(t_f)$ is not above l_x.

For some power level x', let $l_{x'} = \sqrt{x'}$. We say that a power level x' preserves the topology properties of a power level x if the following conditions hold

Cond-1 Graph $H_{p_x}(t_s)$ is identical to $H_{p_{x'}}(t_s)$, and graph $H_{p_x}(t_f)$ is identical to $H_{p_{x'}}(t_f)$ (Top 1 and Top 3)

Cond-2 The distance functions cross l_x and $l_{x'}$ at exactly the same order (Top 2).

Since p_x induces a dynamic h-bounded communication graph, it is easy to see that if a power level x' preserves the topology properties of power level x, then $p_{x'}$ also induces a dynamic h-bounded communication graph. Before we proceed, we make some observations.

Fact-1 Since $x \notin L_I$, then there does not exist a constant distance function $d_{u,v}(t) = \sqrt{x}$.

Fact-2 Since $x \notin L_2$, only one distance function can cross or tangent to l_x at any time point.

Fact-3 Since $x \notin L_3$, each distance function is either strictly above or strictly below l_x at times t_s and t_f.

We next use the monotonicity characteristics of the distance functions (Observation 3.7) and the above facts to derive a power level $x' < x$ so that x' preserves the topology properties of x. From Facts 1 and 3, there exists a power level $y < x$ so that graph $H_{p_x}(t_s)$ is identical to $H_{p_y}(t_s)$, and graph $H_{p_x}(t_f)$ is identical to $H_{p_y}(t_f)$ (complying with Cond 1). From Fact 2 we conclude that there exists a power level $z < x$ so that the distance functions cross l_x and l_z at exactly the same order (complying with Cond 2).[4] Intuitively, this is achieved by lowering the l_x bar until it first encounters one of the following: (1) a distance function on one of the vertical lines t_s or t_f, (2) a constant distance function, (3) a intersection between two distance functions. In figure Figure 5(b) we can see the l_y power level, as an intersection between t_s and one of the distance functions, while $l_z = l_{x'}$ is an intersection between two distance functions.

Clearly x' preserves the topology properties of x, and therefore $p_{x'}$, induces a dynamic h-bounded communication graph. A contradiction to the optimality of x. □

3.2.3 Running time analysis

To implement the scheme above we first need to compute L. Then for each value $x \in L$ we compute C_x and compute the hop-diameter of $H_{p_x}(t)$ for every $t \in C_x \cup \{t_s, t_f\}$. We choose the minimum value of x for which all the graphs $H_{p_x}(t)$, $t \in C_x \cup \{t_s, t_f\}$ are h-bounded. We base our time analysis on the following lemma.

Lemma 3.10. *In polynomial time it is possible to compute L, C_x, for any $x > 0$, and the hop-diameter of $H_{p_x}(t)$ for every $t \in C_x \cup \{t_s, t_f\}$. It also holds $|L| = O(n^4)$ and $\max_{x \in L} |C_x \cup \{t_s, t_f\}| = O(n^2)$.*

Proof: Clearly $|L_1|$ and $|L_3|$ can be computed in polynomial time, and $|L_1| = O(n^2)$, $|L_3| = O(n^2)$. Since all the nodes travel at constant speeds without direction changes, each pair of distance functions intersects at most 3 times. Therefore, we can compute L_2 in polynomial time and $|L_2| = O(n^4)$.

From Observation 3.7 it is easy to see that for any x, $|C_x^{u,v}| \le 2$ and therefore $\max_{x \in L} |C_x \cup \{t_s, t_f\}| = O(n^2)$.

To compute the hop-diameter of a graph we apply the BFS algorithm from an arbitrary node r and then run another BFS from the most distant node from r. This can be done in time linear to the number of edges, i.e. $O(n^2)$ time. □

Clearly, Lemma 3.10 implies the main theorem of this section.

Theorem 3.11. *Given n mobile wireless nodes V, and a parameter h, $1 \le h \le n-1$, if during the time interval $[t_s, t_f]$ every node moves in a single direction along a straight line with constant speed then it is possible to construct in polynomial time a power assignment p_h that induces a dynamic h-bounded communication graph, and $l(p_h) = l(p_h^L)$, where p_h^L is a maximum network lifetime power assignment that induces a dynamic h-bounded communication graph.*

3.2.4 Running time optimization

According to Lemma 3.10 the computation of L can take $\Omega(n^4)$ time. The most efficient method to search for the optimum power level x is to have a sorted array of values in L and then perform a binary search over L. However, to obtain a sorted array of values in L may require $\Omega(n^4 \log n)$ time even if we use binary search trees (e.g. AVL trees). Therefore, the overall running time may take $\Omega(n^4 \log n) + \Omega(\log n)\sigma(n)$, where $\sigma(n)$ is the validation time for a given power level x, which according to Lemma 3.10 is $O(n^2)$.

In what follows we show how to substantially decrease the search range of relevant power levels which will reduce the overall running time to $O(n^2)$. Recall the definition of w' from Section 3.1 which defines the square of the maximum distance between any two nodes in the time interval $[t_s, t_f]$. Let $d^* = \max_{u,v \in V} \sqrt{w'(u,v)}$ and $u^*, v^* \in V$ be two nodes such $d^* = \sqrt{w'(u^*, v^*)}$. Note that for any $x < \left(\frac{d^*}{n-1}\right)^2$ there exists $t \in [t_s, t_f]$ such that there is no path in $H_{p_x}(t)$ from u^* to v^* since the distance between u^* to v^* cannot be covered by $n-1$ hops of $\sqrt{x}$ each – which is the longest possible (in terms of distance) path in $H_{p_x}(t)$. On the other hand, considering power levels above $(d^*)^2$ is impractical as $H_{p_{w^*}}(t)$ has a hop-diameter of 1 for every $t \in [t_s, t_f]$. Thus, we can limit our search only to the range $\left[\left(\frac{d^*}{n-1}\right)^2, (d^*)^2\right]$.

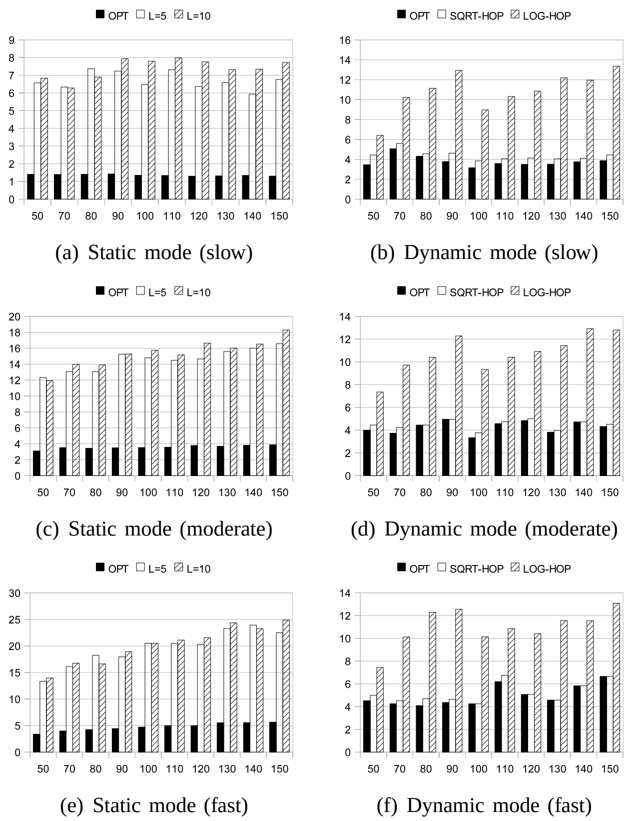
Clearly it is possible to compute d^* in $O(n^2)$ time and then use a standard binary search technique with $O(\log n + k)$ steps, $k >= 0$, to obtain a power level x' which is at most $O(1 + 1/2^k)$ times the optimum. To conclude, we obtain a PTAS with an approximation ratio $O(1 + 1/2^k)$ in $O(n^2 + (\log n + k)n^2) = O((\log n + k)n^2)$ time.

4 Numerical results

We measure the efficiency of our power assignments through extensive simulations of various network scenarios. We consider various network sizes ranging from $n = 50$ to 200 with an increase of 10. For each network size we randomly and uniformly place n

3. If a distance function does not cross, but only a tangent to l_x, then no topology change occurs, since the relevant edge does not appear in the communication graph for a continuous time period.

4. Due to Observation 3.7 we can also conclude that any two values a, b so that $y \le a \le x$ and $z \le b \le x$, comply with conditions Cond 1 and Cond 2, respectively.

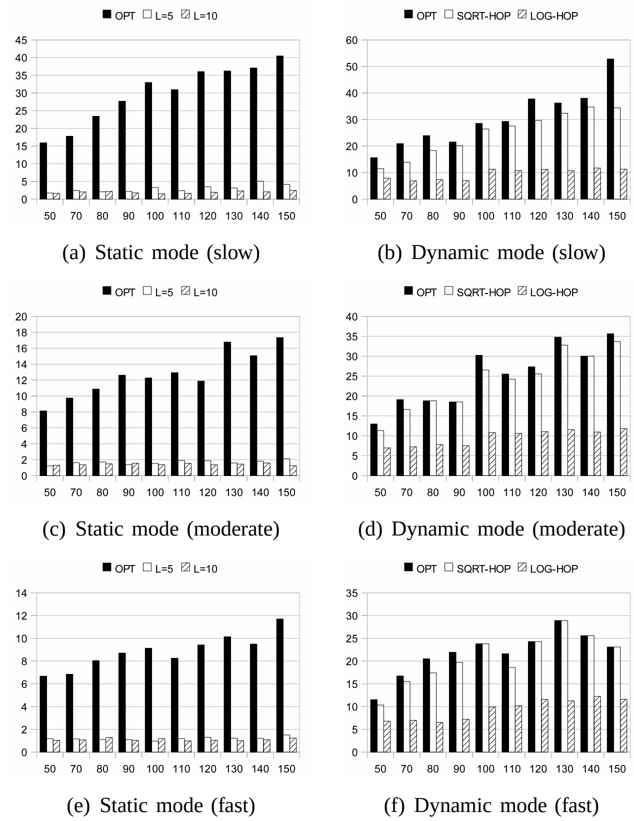


(a) Static mode (slow) (b) Dynamic mode (slow)

(c) Static mode (moderate) (d) Dynamic mode (moderate)

(e) Static mode (fast) (f) Dynamic mode (fast)

Fig. 6. Total energy consumption



(a) Static mode (slow) (b) Dynamic mode (slow)

(c) Static mode (moderate) (d) Dynamic mode (moderate)

(e) Static mode (fast) (f) Dynamic mode (fast)

Fig. 7. Network lifetime

nodes in a unit square and generate mobility patterns. Each node travels along a line segment in a unit square while we distinguish between three types of networks: in *slow networks* a node travels a distance which is in the interval $\left[0, \frac{1}{\sqrt{n}}\right]$; in *moderate networks* the traveled distance is in $\left[\frac{1}{\sqrt{n}}, \frac{\log n}{\sqrt{n}}\right]$; and in *fast networks* the distance is in $\left[\frac{\log n}{\sqrt{n}}, 1\right]$. Each point in the plot is an average of 5 tries.

For each network size and type we compute two power assignments, denoted **L=5** and **L=10**, for the static mode, which are essentially the construction described in Section 3.1 with $\lambda = 5$ and $\lambda = 10$, respectively. For the dynamic mode we compute the power assignments **LOG-HOP** and **SQRT-HOP** which are the power assignment described in Section 3.2 with $h = \log n$ and $h = \sqrt{n}$. For both modes and three network types we measure the total energy consumption (Figure 6), network lifetime (Figure 7) and the produced hop-diameter (Figure 8). The numerical results show that in practice the approximation ratios are much better than the theoretical ones.

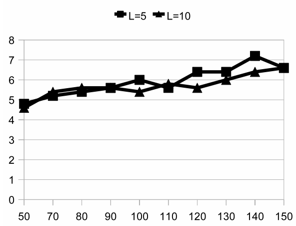
4.1 Total energy consumption

The total energy consumption is depicted in Figure 6. For the static mode we compute the optimal lower bound (OPT) according to Lemma 3.4. The optimal lower bound for the dynamic mode is the cost of the maximum lifetime unit range power assignment such that the network is connected at all times. We can observe that for slow networks, the total energy consumption of the static mode is lower than in the dynamic mode, even though the produced hop-diameters are similar (see Figure 8). However, as the nodes have an increased velocity and they travel to larger distances, the total energy consumption in the static mode grows while it remains stable under the dynamic mode.

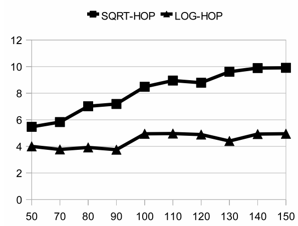
It is interesting to notice that the SQRT-HOP power assignment is almost optimal in terms of energy efficiency. This is due to the fact that in the presence of omnidirectional antennas we get many "short-cuts" which substantially decrease the hop-diameter. Which means that the requirement for a relatively high hop-diameter is easily satisfied and that the basic requirement for strong connectivity induces a communication graph with a low hop-diameter.

4.2 Network lifetime

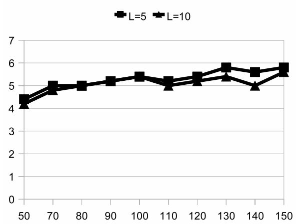
The performance of our power assignments, in terms of network lifetime, is shown in Figure 7. Similarly, the optimal network lifetime for the static mode is computed according to Lemma 3.5. For the dynamic mode the optimal network lifetime is taken as the network lifetime of a strongly connected topology, without the hop-diameter requirement. The simulation results show that the dynamic mode is superior to the static one, especially when the velocity of the nodes increases. In addition we can see that OPT bound in the static mode decreases considerably in fast networks as opposed to slow networks (almost 5 times lower), while in the dynamic mode it is experiences only a slight drop. Note that, as in the case of total energy consumption, the power assignment SQRT-HOP is almost optimal.



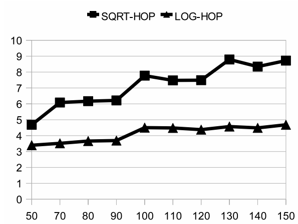
(a) Static mode (slow)



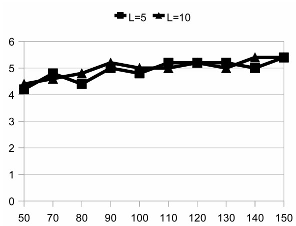
(b) Dynamic mode (slow)



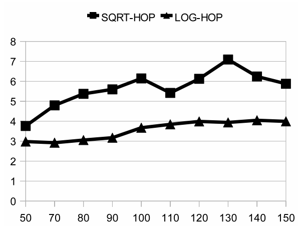
(c) Static mode (moderate)



(d) Dynamic mode (moderate)



(e) Static mode (fast)



(f) Dynamic mode (fast)

Fig. 8. Actual hop-diameter

4.3 Hop-diameter

We measured the actual hop-diameter produced by the power assignments as presented in Figure 8. Interestingly, the hop-diameter remains stable in both modes and for all network types. In addition it is much higher than the theoretical guarantee. As already noted, this is due to the positive side-effect of omnidirectional propagation model which results in multiple "short-cut" links.

5 Conclusions and future work

In this paper we consider a set of n mobile wireless nodes, which have no initial information about each other. The only information a single node holds is its current location and future mobility plan. We proposed a two-phase scheme to solve the stretchable topology control problem under two modes, static and dynamic. The scheme optimize the obtained network for an entire continuous time segment, rather than discrete time(s).

We believe that these realistic settings should be further studied. One of the most interesting future direction is to try and provide a power assignment for the DBH problem which is both cost and network lifetime efficient. Another possible direction is to study the case when different nodes may have different initial power for both modes of topology construction – notice that now the constructions based on minimum spanning tree would not work. It would be also intriguing to perform some simulation tests for the first phase of the scheme and calibrate the various parameters.

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