Conformant Planning
First Step toward Planning Under Uncertainty
Conformant Planning

- Basic assumption in classical planning: the initial state is fully known
- What if we don’t know everything about the initial state?
- Conformant planning -- like classical planning, but instead of a single possible initial state, a set of possible initial states
- Other forms of uncertainty:
  - Uncertainty about the effect of actions (non-deterministic, stochastic)
  - Some conformant planning algorithms can deal with non-deterministic effects
- Related issues:
  - Observability: can we observe information about the current state?
  - Conformant planning: no observations during plan execution
Conformant Planning: the Trouble with Incomplete Info

Problem: A robot must move from an uncertain $I$ into $G$ with certainty, one cell at a time, in a grid $nxn$

- Conformant and classical planning look similar except for uncertain $I$ (assuming actions are deterministic).
- Yet plans may be quite different: best conformant plan above must move robot to a corner first! (in order to localize)

H. Palacios & H. Geffner, Solving Conformant Planning Using a Classical Planner (Sometimes), AAAI-06
Conformant Planning

- Conformant Planning problem \( \langle P, A, I, G \rangle \)
  - I is an arbitrary formula, and any state \( s \) that satisfies I is a possible initial state
  - A can be non-deterministic. Later we will focus on deterministic effects

- Model -- identical to classical planning (possibly non-deterministic) automaton with multiple initial states.

- Solution -- a plan that is guaranteed to take us from any initial state to some goal state, no matter what the effect of actions is.

- Language -- like strips except:
  - Initial state described by a formula -- any assignment satisfying it is a legal state
  - Non-determinism can be captured by disjunctive effects: \( p \lor \neg p \)
Belief States

- Central concept: **belief state** --- the set of possible (world) states

- Initial belief state: \( \{s \mid s \models I\} \)

- If our current belief state is \( b \) and we apply action \( a \), then we reach a new belief state \( b' = \{a(s) \mid s \models b\} \)
Example:

GoSouth  GoEast  GoEast  GoEast  GoSouth  GoNorth
Search in Belief Space

- Conformant planning can be viewed as the problem of finding a path in belief space

- Initial state: initial belief state

- Goal state: any belief state $b$ such that $s \in b \Rightarrow s \models g$

- Actions: $a(b) = \{a(s) \mid s \models b\}$

- In general, a belief state could require an exponentially large (in # of state variables) description
Remark:
- the search space is $Pow(S)$
- $S$ contains 15 states,
- $Pow(S)$ contains 32767 belief states!
Complexity

- We can verify that a classical plan is true in time linear in plan length and # of propositions
- Verifying that a conformant plan is correct may be intractable
  - Initial state: initial belief state
  - Goal state: any belief state $b$ such that $s \in b \Rightarrow s \models g$
  - Actions: $a(b) = \{ a(s) \mid s \models b \}$
  - In general, a belief state could require an exponentially large (in # of state variables) description
Generating Conformant Plans

- Two main issues:
  - How do we represent belief states efficiently?
    - Small size desirable
  - Ability to quickly detect goal satisfaction
  - Ability to quickly detect which action is applicable
  - How can we generate good heuristic estimates?
Special Case

- Standard STRIPS actions

- Initial state: the value of some propositions is known, the value of others is completely unknown (no constraints of the form \( p \lor q \))

- Solution:???
Representing Belief States

1. Explicit representation: Maintain a set of states

   • All operations require time linear in number of possible states
   • All operations are conceptually simple
   • The number of possible states can be very large
   • Does not work in practice

2. Symbolic representation: Maintain formula $\phi$ over state propositions

   • $s$ is a possible state iff it satisfies $\phi$
   • Key issue: how do we represent $\phi$

     • Different choices affect the computational and conceptual difficulty of different operations (update, verification of goal/preconditions) and the size of the formula
Alternative Symbolic Representations

- Logical formula w/o constraints

- Conjunctive Normal Form: Conjunction of Disjunctions
  - (pvqvr) & (-pvwvd) & (-wvqvs)
  - Checking whether a precondition/goal holds require solving un-sat problem

- Disjunctive Normal Form: Disjunction of Conjunctions
  - (p&q&r) v (-p&w&d) v (-w&qs&)
  - Checking whether a condition holds is easy
  - The number of conjuncts can grow rapidly

- Binary Decision Diagrams
Binary Decision Diagrams

- A data structure used for compactly representing boolean functions
- Made popular by work on program verification
- Based on recursive Shannon expansion

\[ f = x f(x) + x' f(x'), \]

- Canonical representation
  - reduced ordered BDDs (ROBDD) are canonical (= there is only one way to represent any function given a fixed variable order)
Recursive Shannon Expansion for
\[ f = ac + bc \]
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Recursive Shannon Expansion for $f = ac + bc$

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\[ g = bc \]
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BDD operations

* When the two outgoing edges of a node point to the same node, remove it
BDD operations

- When the two outgoing edges of a node point to the same node, remove it

\[ f = a' \, g(b) + a \, g(b) = g(b) \]

\[ (f_a + f_{a'} = 1) \]
BDD operations

* When the two outgoing edges of a node point to the same node, remove it

\[ f = a' g(b) + a g(b) = g(b) \]
\[ (f_a + f_a' = 1) \]
BDD Operations

• Merge duplicate nodes
BDD Operations

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\[ f_1 = a' g(b) + a h(c) = f_2 \]
BDD Operations

- Merge duplicate nodes

\[ f_1 = a' g(b) + a h(c) = f_2 \]

\[ f = f_1 = f_2 \]
BDD Construction

- You can start with a decision tree and merge: example $f=ac+bc$

- Reduced, ordered, BDD:
  - Reduced -- no additional reductions can be applied
  - Ordered -- the order of variables in a path from the root to a leaf is fixed
BDD Construction

- You can start with a decision tree and merge: example \( f = ac + bc \)

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Truth table

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BDD Construction

- You can start with a decision tree and merge: example $f = ac + bc$

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### Decision tree

- Reduced, ordered, BDD:
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BDD Construction (continued)
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1. Merge terminal nodes
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2. Merge duplicate nodes
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BDD Construction (continued)

1. Merge terminal nodes
2. Merge duplicate nodes
3. Remove redundant nodes
BDD Construction (continued)

1. Merge terminal nodes
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\[ f = (a+b)c \]
BDDs Support Efficient Logical Manipulations

- Negating a function (very simple??)
- Conjoining two functions
- Disjoining two functions
- Others
- Operations utilize the recursive definition of the function
Implicit Representation

- This is also a representation via a formula, but with different propositions
- Essentially, this is the same formula generated by a SAT-encoding
- A state \( s \) is possible currently if there is a satisfying assignment that assigns the propositions at time \( t \) the same values as \( s \).
  - Update is very easy
  - Checking whether a condition holds now requires verifying that a formula is unsatisfiable
  - The formula can be simplified during run-time
Searching in Belief Space

• All current planners use forward search

• Main problem: heuristics are difficult to generate

• Size heuristic: \( hs(b) = -1 \times |\{s : s \in b\}| \)

• Pushes toward belief states with more certainty

• That’s about it ... not strong enough.
The Translation-Based Approach

- In classical planning, if we know the initial state, we know the current state simply from the description of the actions.
- Basic idea: maintain a copy of each proposition for each possible initial state.
  - $p_{i_1}, p_{i_2}, \ldots, p_{i_k}$
  - And also a “general” copy: $p$
- Generate actions that update all copies.
  - If $p \rightarrow q$ is an original effect of $a$, add $p_{i_j} \rightarrow q_{i_j}$ for every $1 \leq j \leq k$.
- This way, we know what’s true now as a function of what was true initially.
- We can also deduce that if $p_{i_j}$ holds now for every $1 \leq j \leq k$, then $p$ holds.
  - This way, we can know whether some precondition or goal condition holds.
- So far, pretty wasteful because we may have exponentially many initial states.
The Translation-Based Approach

- We can use this idea to generate a new classical planning problem
- Propositions: $p, p/i_j$ for every possible proposition $p$ and every possible initial state $i_j$
- Actions:
  - the original actions, with effects modified as described before
  - special inference actions: $p/i_1 \land p/i_2 \land ... \land p/i_k \rightarrow p$ for every proposition $p$
- Initial state: $p/i_j$ is true iff $p$ holds in possible initial state $i_j$
- Goal state: $g$ (as in the original problem)
- We get a classical planning problem, and we can solve it with a classical planner
- No need for special heuristics!
The Translation-Based Approach

- Actually, in the literature:
- Propositions: Kp, Kp/i is used
  - Kp -- p is known
  - Kp/i -- p is known given i
  - More generally: Kp/t -- p is known given some condition t on the initial state
- K is used in logics of knowledge: Something is known if it holds in all possible states.
  - This is captured by: Kp/i₁ ∧ Kp/i₂ ∧ ... ∧ Kp/iₖ → Kp
- The planner is reasoning about our state of knowledge
The Translation-Based Approach

- Main problem: many possible initial states
- Possible solution: use tags (conditions) that are more general
- This is not always possible, but in many problem it works
  - When it doesn’t work, we’re in trouble -- why?
- Example: two variables: p1, p2, ..., pk. Both unknown initially.
  - \(2^k\) possible initial states
  - Suppose that the goal is p1 & ... & pk, and \(a_i\) has a conditional effect: \(-p_i \rightarrow p_i\)
  - According to previous slides, we need \(2^k\) possible tags
  - We can work with \(2^k\) tags -- one for each value of each variable
  - Reason -- the effect on tags is independent