Genetic Programming

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Videos from Koza's GP4 DVD

Overview: Part I

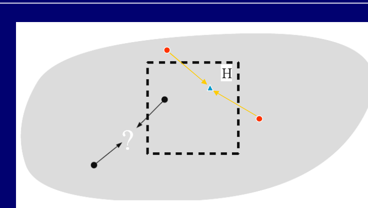
- Introduction
- Basics
- Examples
- More advanced techniques
- Applications

Overview: Part II

- Theory in theory
- GA schema theory
- GP schema theory
- Lessons and implications
- Conclusions

- $m(H,t)$ is number of individuals in the schema H at generation t,
- M is the population size,
- $p(H,t)$ is the selection probability for strings in H at generation t,
- p_m is the mutation probability,
- $O(H)$ is the schema order, i.e. number of defining bits,
- p_c is the crossover probability,
- $L(H)$ is the defining length, i.e. distance between the furthest defining bits in H,
- N is the bitstring length.

- Idea:



- Features:
  - The theorem includes an *expected value*
  - It provides a *lower bound*
  - So, it is difficult to make accurate predictions

- The factor σ differs in the different formulation of the schema theorem:
  – $\sigma=1-m(H,t)/M$ in (Holland, 1975),
  – $\sigma=1$ in (Goldberg, 1989),
  – $\sigma=1-p(H,t)$ in (Whitley, 1994).
- In 1997 Stephens and collaborators produced an exact formulation for $\alpha(H,t)$: an *"exact" schema theorem*.

How can we get an exact schema theorem?

- Let us assume that only reproduction and (one-offspring) crossover are performed.
- Because these two operators are mutually exclusive, for a generic schema H we have:

$$\alpha(H,t) = \Pr[\text{An individual in H is obtained via reproduction}] + \Pr[\text{An offspring matching H is produced by crossover}]$$

- Reproduction is performed with probability p_r and crossover with probability p_c (with $p_r+p_c=1$), so

$$\alpha(H,t) = p_r \times \Pr\left[\text{An individual in } H \text{ is selected for cloning}\right] + p_c \times \Pr\left[\begin{array}{c}\text{The parents and the crossover points}\\ \text{are such that the offspring matches } H\end{array}\right]$$

where $\Pr\left[\text{Selecting an individual in } H \text{ for cloning}\right] = p(H,t)$

- The process of crossover point selection is independent from the actual primitives in a parent.
- The probability of choosing a particular crossover depends only on the actual size of the parent.
- E.g., the probability of choosing any crossover point in

  1 1 0 1 0 1

  is identical to the probability of choosing any crossover point in

  0 0 0 1 1 0

$$\Pr\left[\begin{array}{c}\text{The parents and the crossover points}\\ \text{are such that the offspring matches } H\end{array}\right]$$

$$= \sum_{\substack{\text{For all crossover}\\ \text{points } i}} \Pr\left[\begin{array}{c}\text{The parents are such that if crossed over}\\ \text{at position } i \text{ the offspring matches } H\end{array}\right]$$

$$= \sum_{\substack{\text{For all crossover}\\ \text{points } i}} \Pr\left[\begin{array}{c}\text{Choosing crossover}\\ \text{point } i\end{array}\right] \times \Pr\left[\begin{array}{c}\text{Selecting parents such that if}\\ \text{crossed over at point } i \text{ produce an offspring in } H\end{array}\right]$$

- Let us assume that crossover points are selected with uniform probability:

$$\Pr\left[\begin{array}{c}\text{Choosing crossover}\\ \text{point } i\end{array}\right] = \frac{1}{\text{Number of bits} - 1}$$

$$\Pr\left[\begin{array}{c}\text{Selecting parents such that if}\\ \text{crossed over at point } i \text{ produce an offspring in } H\end{array}\right]$$

$$= \Pr\left[\begin{array}{c}\text{Selecting a first parent such that if crossed over at point } i\\ \text{provides the necessary material to create an offspring in } H\end{array}\right]$$

$$\times \Pr\left[\begin{array}{c}\text{Selecting a second parent such that if crossed over}\\ \text{at point } i \text{ provides the remaining necessary material}\end{array}\right]$$

Stephens and Waelbroeck's Exact GA Schema Theory (1997)

- For a binary GA with one point crossover applied with probability p_{xo} (and assuming p_m=0)

$$E[m(H,t+1)/M] = (1-p_{xo})p(H,t) + \frac{p_{xo}}{N-1}\sum_{i=1}^{N-1} p(L(H,i),t)p(R(H,i),t)$$

$$\Pr(L(H,i),t) = \Pr\left[\begin{array}{c}\text{Selecting a first parent such that if crossed over at point } i\\ \text{provides the necessary material to create an offspring in } H\end{array}\right]$$

$$\Pr(R(H,i),t) = \Pr\left[\begin{array}{c}\text{Selecting a second parent such that if crossed over}\\ \text{at point } i \text{ provides the remaining necessary material}\end{array}\right]$$

- *L(H,i)* is obtained by replacing the elements of *H* to the *right* of position *i* with "don't care" symbols
- *R(H,i)* is obtained by replacing the elements of *H* to the *left* of position *i*+1 with "don't care" symbols

- For example, if *H*=1*111, then *L(H,1)*=1****, *R(H,1)*=**111, *L(H,3)*=1*1**, *R(H,3)*=***11.
- For the schema *11, the theorem gives:

$$\begin{aligned} E[m(*11,t)/M] &= (1-p_{xo})p(*11,t) + \\ &\quad \frac{p_{xo}}{2}(p(***,t)p(*11,t) + p(*1*,t)p(**1,t)) \\ &= (1-\frac{p_{xo}}{2})p(*11,t) + \frac{p_{xo}}{2}p(*1*,t)p(**1,t), \end{aligned}$$

since $p(***,t)=1$.