Hidato Solving

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Introduction

- Original Hidato
- Generalization of Hidato
- Solvers
  1. Naive solver
  2. Matching solver
  3. SAT Reduction solver
Original Hidato

- Given is a grid of $n$ cells, some of the grid’s cells are labeled by an integer between 1 and $n$.
- The goal is to find the label of each cell, so that there is a path of adjacent cells from label 1 to label $n$.

[Diagram of Hidato grid with labeled cells]
Our project considers the following generalization:

- Given is a graph on $n$ vertices, where some of the vertices are labeled by distinct integers between 1 and $n$.
- The goal is to find a Hamiltonian path which passes through the labeled vertices at the times (labels) specified at those vertices.
Generalization of Hidato
Solvers implemented in our project:

1. Naive solver
2. Matching solver
3. SAT Reduction solver
Naive solver

1. Split to intervals

2. Find all the possible paths for each interval

3. Combine these paths to a full Hamiltonian path (if such exists)
Given $G=(V,E)$, we define the bipartite graph $G'=(V \cup L,E')$:

- $V$ is $G$’s group of vertices
- $L$ is the group of labels, i.e. $L = \{1, 2, .., |V|\}$
- $E' = \{(v_i, l_j) \mid v_i \in V, l_j \in L, l_j \text{ is } v_i \text{’s label in a solution for } G\}$
What is a solution for $G$ in this context?
A perfect matching in $G'$

The algorithm searches for a perfect matching by eliminating $<\text{vertex},\text{label}>$ ordered pairs from the matching options
Matching solver - continued

Methods to reduce options:

- Let $i$ be a label whose group of optional vertices is $V_i$. The group of options of labels $i - 1$ and $i + 1$ may only contain neighbors of vertices from $V_i$.

<table>
<thead>
<tr>
<th>Label</th>
<th>Vertices options</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>${V_1, V_2, V_3}$</td>
</tr>
<tr>
<td>2</td>
<td>${V_4}$</td>
</tr>
<tr>
<td>3</td>
<td>${V_1, V_2, V_3}$</td>
</tr>
<tr>
<td>4</td>
<td>${V_1, V_2, V_3}$</td>
</tr>
</tbody>
</table>
Matching solver - continued

**Methods to reduce options:**

- Let $i$ be a label whose group of optional vertices is $V_i$. The group of options of labels $i - 1$ and $i + 1$ may only contain neighbors of vertices from $V_i$.

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</tr>
<tr>
<td>3</td>
<td>${V_1, \times, V_3}$</td>
</tr>
<tr>
<td>4</td>
<td>${\times, V_2, \times}$</td>
</tr>
</tbody>
</table>
Matching solver - continued

Methods to reduce options - continued:

- Let $V$ be a group of $m$ distinct vertices, and let $L$ be a group of $m$ distinct labels such that $\forall l \in L: l.\text{verticesOptions} \text{ is } V$.

Thus $\forall l \notin L: l.\text{verticesOptions} \cap V = \emptyset$.

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</tr>
<tr>
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</tr>
<tr>
<td>4</td>
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$V = \{V_1, V_2\}$  
$L = \{2, 4\}$

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Methods to reduce options - continued:

maximally-matchable edge: An edge that is included in some maximum matching

- Find the non maximally-matchable edges in the bipartite graph, $G'$, and remove them

And if no more options can be eliminated? Guess a pair and try to solve or contradict
SAT Reduction solver

- Transform the graph instance into a CNF expression
- Call the miniSAT solver
- Transform miniSAT’s output to a solution for the graph instance
Reduction to CNF:

- **Variables:** $n^2$ variables - each vertex has $n$ options for its label

  \[ \forall v_i \in G(n) : \{x_{v_i,1}, x_{v_i,2}, ..., x_{v_i,n}\} \]

  i.e.

  - $x_{v_i,j} \iff v_i$’s label is $j$
  - $\neg x_{v_i,j} \iff v_i$’s label is not $j$
SAT Reduction solver - continued

- Clauses (partial):
  
  1. Every vertex in the graph must have an integer (a.k.a. label) ranging from 1 to n

\[ \forall v_i \in G(n) : x_{v_i,1} \lor x_{v_i,2} \lor ... \lor x_{v_i,n} \]

  2. No vertex has two different labels

\[ \forall v_i \in G(n), \forall j_1, j_2 \in \{1, 2, .., n\}, j_1 \neq j_2 : \neg x_{v_i,j_1} \lor \neg x_{v_i,j_2} \]

  3. No two vertices have the same label

\[ \forall v_i, v_k \in G(n), v_i \neq v_k, \forall j \in \{1, 2, .., n\} : \neg x_{v_i,j} \lor \neg x_{v_k,j} \]
Questions?