A MODIFIED APPROACH TO GRADED MODALITY COMPARISON

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1. ABSTRACT

Grades of modality have been suggested (Kratzer, 1981) to be represented semantically using quantification over worlds within sets established by (i) the propositions which are known to be true (the modal base) and (ii) proximity to a perceived ideal (the ordering source). It has been pointed out that this representation fails to capture a grammatical situation, in which two claims are considered probable, but one is perceived as more possible than the other.

In this paper, I modify Kratzer's definition of Comparative Possibilities in a way that better captures our notion of comparison, and accounts for these excluded situations.

2. GRADES OF MODALITY – KRATZER'S APPROACH

Kratzer's analysis assumes six distinct modal operators, five of which relate to a single proposition and the sixth compares between two. One natural-language example for each would suffice for our cause:

(1) **Necessity**: an example from English would be the word "<u>necessarily</u>": "A living human being necessarily has two lungs".

(2) **Human Necessity**: "<u>probably</u>"; "He's so conceited, he's probably an only child".

(3) **Human Possibility**: "<u>can well be</u>"; "It's April now, so it can well be that the ocean water isn't too cold for a swim".

(4) **Possibility**: "<u>possibly</u>"; "*Psycho* was possibly the best suspense movie ever made".

(5) **Slight Possibility**: "a slight chance of"; "I got up at 7:15, but there's still a slight chance of me making it to my 8:00 class on time".

(6) **Comparative Probability**: "p is more probable than q"; "it is more likely that Eve will eat her artichoke than Adam will eat his".

   The model proposed by Kratzer is based on a set of worlds W, each world being defined by the propositions which are true in it. Two functions operate when a modal sentence is uttered in its entirety: the function f stems from the "in view of" clause and defines the <u>modal base</u>, a nonempty set of worlds denoted $\cap f(w)$ containing the worlds in which all the facts known in our world w are true. g is an <u>ordering source</u> acting as an incomplete order relation between worlds: l is the ideal world, the one in which all forces of nature and society work as predicted. A world u is said to be <u>nearer than or as near as</u> v (and denoted $u \leq_g v$) if it is perceived as more resembling this ideal (we will assume this perception is intuitive and not suggest a formalization for it).

   Kratzer now looks at the set of worlds where what is known is true, $\cap f(w)$, and proposes formal representations for the aforementioned grades of modality which may be simplified in the following manner: to $\cap f(w)$ we add a nonempty set of worlds G, which includes worlds "sufficiently close" to l. The size of G is determinable by context, however it must hold that no close world is "left out", that is if u and v are two worlds where $u \leq_g v$, if $v \in G$ then $u \in G$ (a formal suggestion towards building this set G is outlined in the appendix). In addition, it must be the case that $\cap f(w) \cap G$ is nonempty, as some world must always be perceived as the most ideal of the worlds where what is known is true. Having defined G we rephrase Kratzer's representations:

(7) **Necessity**: p is necessary in a world w iff it is true in all worlds within $\cap f(w)$.

(8) **Human Necessity**: p is a human necessity in w iff it is true in all worlds within $\cap f(w) \cap G$.

(9) **Human Possibility**: p is a human possibility in w iff it is true in some world within $\cap f(w) \cap G$.

(10) **Possibility**: p is possible in w iff it is true in some world within $\cap f(w)$.

(11) **Slight Possibility**: p is a slight possibility in w iff it is true in some world within $\cap f(w)$-G, and in no world within $\cap f(w) \cap G$.

(12) **Comparative Probability**: p is more probable than q in w iff the following two conditions hold:

   a. For every world u in $\cap f(w)$ where q is true, there exists a world v in $\cap f(w)$ such that $v \leq_g u$ and p is true in v.

   b. There exists some world v in $\cap f(w)$ where p is true, such that no world u in $\cap f(w)$ where q is true in u satisfies $u \leq_g v$.

3.   THE PROBLEM

Kratzer's definitions seem intuitive and account well for many relations and implications, as those stated by Kratzer herself, for example[1]:

(13)

    a. If p is necessary, it is humanly necessary (because p is true in all worlds, and so in all worlds in $\cap f(w) \cap G$); and if p is humanly necessary, it is humanly possible (because it is true in some world in $\cap f(w) \cap G$, as this set is nonempty); and if p is humanly possible, it is possible (because it is true in some world). None of these entailments works the other way around, and the first two satisfy their comparative counterparts (a necessary proposition is more probable than a humanly necessary proposition, which is more probable than a humanly possible proposition).[2]

    b. There is no problem with p and ¬p both being humanly possible: each may be true in a different $\cap f(w) \cap G$ world, and so both satisfy the condition for human possibility. p and ¬p may not, however, be both humanly necessary: for that to happen, each world in $\cap f(w) \cap G$ must satisfy p ∧ ¬p, a contradiction in any model.

    c. If p is slightly possible, then ¬p is humanly necessary: this stems from both definitions in that by p being slightly possible, it follows p is not true for any world in $\cap f(w) \cap G$, hence ¬p is true in all of them, and so satisfies the condition for human necessity.

    d. If p is humanly possible and q is slightly possible, then p is more probable than q: there is a world *v* in $\cap f(w) \cap G$ where p is true. Since all worlds where q is true are not in $\cap f(w) \cap G$, *v* is closer (in the strong sense) than all of them to the ideal, satisfying both conditions of definition (12).

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[1] It should be noted that the same ordering source and set of "sufficiently close" worlds G must be used within every situation, a plausible demand when considering discourse wellformedness.

[2] I believe the third comparative counterpart does not always work. Consider the following: "it is possible that Jeffery like bees, and it can well be that Maude keeps a bee as a pet, but it's more probable that they both just enjoy bees from a distance." In this consistent discourse, the possible proposition is more probable than the humanly possible proposition.

Consider, however, the following discourse:

(14)

    a. It is probable that Paul will dance tonight.

    b. It is probable that Quentin will dance tonight.

    c. It is more probable that Paul will dance tonight than that Quentin will
       dance tonight.

(Keep in mind that "p is probable" is considered a <u>human necessity</u> marker. Also, from now on the proposition "Paul will dance tonight" will be referred to as "p", and "Quentin will dance tonight" will be referred to as "q".)

Empirically speaking, the three sentences above are consistent: they can all be considered true when uttered together in an appropriate context. However, Kratzer's formal semantics fail to capture this situation: Under any choice of worlds for G, in all worlds which are <u>sufficiently close</u> to the ideal q is true (according to (b) and to the definition of human necessity) and so for every world v where p is true, there exists a world u which satisfies $u \leq_g v$ where q is true: if v is sufficiently close to the ideal, then v is this world ($\leq_g$ is reflexive); and if not, then surely some "sufficiently close" world is closer to the ideal than v is, and in that world q is true. The conclusion is that condition (12) cannot be satisfied[3], and so sentence (c) is not predicted to be consistent with (a) and (b).

In addition, this exact same argument may be followed using (a') to substitute for (a), where (a') is "Paul will necessarily dance tonight", or "p is necessary"; or in fact, whenever any two propositions which are <u>ideally true</u> (that is, true in the most ideal world in $\cap f(w) \cap G$), are being compared. Kratzer's definitions do not allow for any such comparison.

We have found a serious empirical flaw in Kratzer's theory.

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[3] For every v we can find an at-least-as-close u such that q is true in u, and so no v exists where: p is true, and in no world u which is at least as close as v q is true. This is the negation of condition (b).

4.   A New Definition For Comparative Probability

I propose the following definition:

(15)   **Comparative Probability**: p is more probable than q iff:

    a. A world v exists where p is true and q is false in v.

    b. For every world u in ∩f(w) where q is true and p is false, there exists a world v in ∩f(w) such that $v \leq_g u$, and p is true and q is false in v.

This definition captures the consistency of the three sentences in (14), without losing the successful predictions such as those in (13).

First, the successful predictions: (13b)-(13c) are unchanged, since the relevant definitions have not been redefined. (13a) has its "comparative counterparts":

    i.   If p is necessary and q is humanly necessary, then p is more probable than q: since q is only humanly necessary, there exists a world where it is false. In this same world, p is true, and so condition (a) is met. Since in no world is p false, the restriction in condition (b) is over an empty set and so the condition is trivially met.

    ii.   If p is humanly necessary and q is humanly possible, then p is more probable than q: condition (a) is met since some "sufficiently close" world satisfies "p is true and q is false". Condition (b) is met since all worlds where q is true and p is false are not "sufficiently close" and so for each of them a closer world may be found, which is indeed "sufficiently close" and where p is true and q is false. It does not even matter how close to the ideal lies the closest world where q is true: q may even be ideally true and yet this comparison will be predicted (Kratzer does not address this option, but her definitions will not work in this case).

    iii.   If p is humanly possible and q is possible, then p is not always more probable than q: and indeed, condition (b) cannot be satisfied in the plausible situation where in the ideal world, q is true and p is false.

As for (13d): if p is humanly possible and q is slightly possible, then p is more probable than q: any "sufficiently close" world is such that p is true and q is false in it, satisfying condition (a); also, every world u in which q is true is not

sufficiently close, so a sufficiently close world v can be found which is closer than u, and in v p is true and q is false (due to the "if"-clause), satisfying condition (b).

We now turn to (14), which was malpredicted by Kratzer's definitions: in all worlds "sufficiently close" to the ideal, both statements (a) and (b) are true, meaning both Paul and Quentin dance tonight. However, these worlds are of no concern to us when following definition (15), since none of them validates the restriction of condition (15) and so are not bound by its consequence. Now, in a situation where (14) be true, it must be the case that in some world, p is true and q isn't (otherwise they would be one and the same), verifying condition (15). Condition (b) is satisfied in the same way as it would be for any other case of modality comparison. This property of my definition serves to include all similar cases (where two ideal propositions are compared) as well.

Having shown its empirical advantage, allow me now to explain the rationale behind this new definition: while Kratzer's original definition looked at each of the propositions p and q as separate entities, each defining its corresponding set of worlds (where it is true) and thus comparable only by means of comparing their sets, my definition takes p and q <u>together</u> and divides all worlds into two sets: one where both truth values (of p and q) are the same, and its complement, where the truth values are opposite (we'll call them "same-valued" worlds and "opposite-valued" worlds, respectively). It then looks only at the latter, makes sure it is nonempty (to assure comparability) via condition (a)[4], and checks for the world closest to the ideal, which of the two propositions is true in it (condition (b))[5]. This proposition is the one predicted to be more probable. I feel that ignoring the worlds of same truth values, regardless of what these values are, reflects a better perception of what underlies our concept of comparison: <u>observation of differences</u>. In addition, I note that using my definition, since

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[4] Condition (a) actually does more than just insuring: it also removes situations where all opposite-valued worlds are "q is true and p is false", thus manifesting the obvious intuition that a proposition which is a proper subset of another (in terms of worlds where it is true) cannot be more probable than its superset.

[5] With appropriate corrections for the model of ordering source used. For example, if not all relevant worlds are comparable, the definition looks at the worlds closest to the ideal. See appendix for further thoughts on this model.

condition (b) compares worlds which are necessarily not the same world, the weak reflexive order "$\leq_g$" may be changed into a strong order "$<_g$" without any adaptations for comparative probability, and so better reflect the wording "more probable than" (or "less probable than") naturally used for comparisons.

We see why each condition is necessary: without condition (a), a proposition may be counter-intuitively said to be more probable than itself. This is due to the fact that a proposition is never both true and false, resulting in an empty quantification of condition (b): since no world exists in which p is true and p is false, the restriction is never verified and the condition is fulfilled, contrary to intuition. This situation requires an explicit demand that this is not the case, or in a broader sense – that p and q are not statements that always have the same truth value.

Condition (b) is obviously necessary as it provides the actual tool for comparison: inspired by Kratzer's conditions, it requires each "q and not p" world to have a "p and not q" world closer to the ideal than it is, insuring that the opposite-valued worlds closest to the ideal are worlds where only the more probable proposition is true.

5. SUMMARY

We have seen that an empirical problem arising from Kratzer's definitions of graded modality may be solved by amending a single definition, that of modality comparison. While not compromising any previously well-predicted intuitions, it offers a more natural perspective on the notion of comparing the probability of two propositions.

6. REFERENCES

Kratzer, Angelika. 1981. The Notional Category of Modality. In *Words, Worlds and Contexts: New Approaches in Word Semantics.* Eikmeyer and Rieser, Eds.

APPENDIX.   A SUGGESTED INTERPRETATION OF WORLD COMPARISON

Throughout my work on this paper I tried to imagine what the behavior of the ordering source g must be like in order for it to come to terms with intuition and predicted behavior of propositions and modality. I now wish to share these thoughts, as well as a geometrical model for representation of worlds, inspired by these thoughts. The model provides a visualization of the creation of G, the set of "sufficiently close" worlds to a given ideal world l, given an ordering source g.

The mindset which guided me is that not all worlds are comparable to one another. However, all worlds are comparable to the ideal (as they are "less close" than it is, by definition) and so all worlds can be divided into sets, each containing worlds which are comparable to one another and only to one another, and furthermore – <u>may be ordered</u> in "idealness" within each set. I should reiterate that this is only one possible interpretation of the world ordering; however it is the one that fits my perception best and so is the one I present.

The following diagram visualizes this model: the ideal world l is placed at the center of a plane, and infinitely many straight arrows are directed at it. Each arrow represents one of these ordered sets, and is represented by a unique angle. Every world is placed on the plane, so by definition it lies on one of these arrows (conversely, one may choose to define the locations of the worlds after the arrow sets have been given as priors). The ordering source g works within the bounds of each arrow: if u and v are two worlds lying on the same arrow, we equate the statement "$u{\leq_g}v$" with the statement "u lies closer to the origin, or as close, as v"[6]. Note this only covers the cases where both worlds lie on the same arrow, as specified in the rationale above. In the diagram below, u and v are comparable with each other, but neither is comparable with w.

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[6] For equality (when both $u{\leq_g}v$ and $v{\leq_g}u$) to be possible, we allow two worlds to occupy the same point on the plane.

(16)   Introducing the ordering plane:
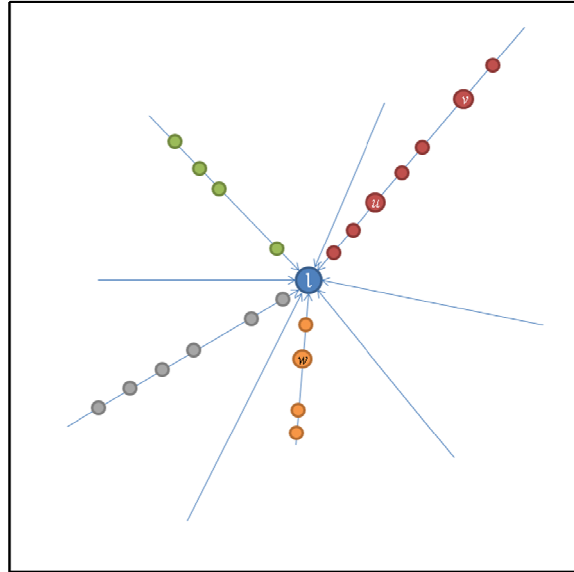
Next I introduce the notion of <u>rescaling</u>: when an arrow is rescaled, each of its worlds becomes closer to the ideal (or further from it) by a fixed ratio. This preserves the ordering of worlds within the arrow as well as the perceived distance between these worlds, and so is independent of g and may be freely used for purposes of our model. In the next diagram, the "orange arrow" has been rescaled.

(17)   The plane (L), and the plane with *w*'s arrow rescaled at a ratio of 2:1 (R):
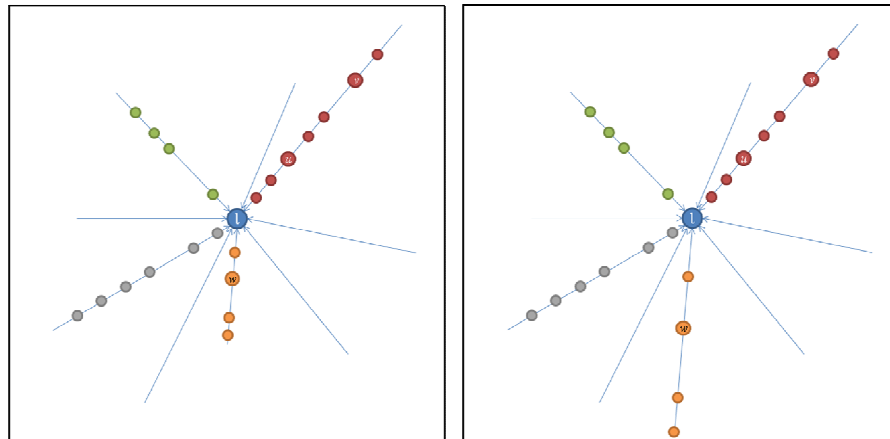
Now, a clean formulation of G, the near-ideal set of worlds, is at hand: G is built by fixing a circle centered at l and rescaling all arrows so that the worlds which are meant to be "close enough", and only them, are within its radius. The following illustration shows how *u* and *w* may change their respective positions with respect to G: they may either be both close to ideal (left) or *w* may be left out (right).

(18)   Two possible creations of G:
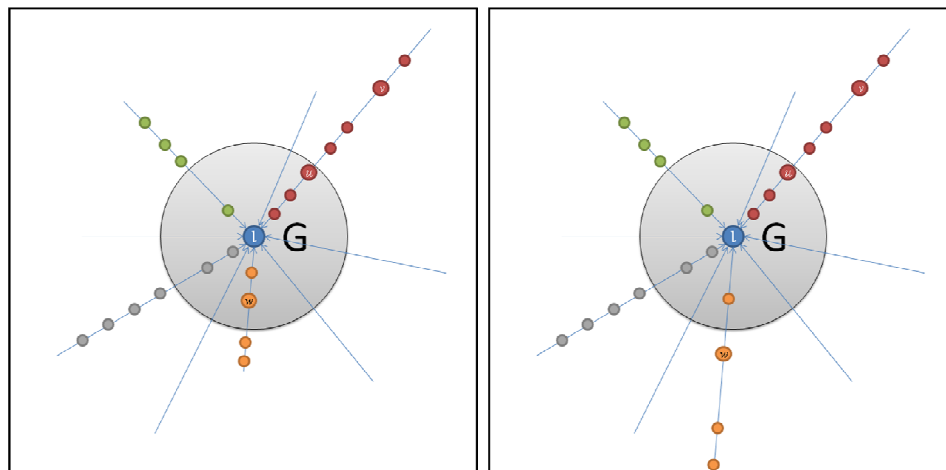
This method of building a set G is independent of g only to a very strict extent (it may not change the given orderings) and so <u>may be contextually driven</u>. This is good for various reasons, for example formulation of Comparative Probability: this notion, as defined in (15), can now be said to construct a "dynamic set G": when the ideal world, the modal base and the ordering source are viewed as constants, two propositions are given: *p* and *q*. An arbitrary radius R around l is set, and <u>for each arrow</u> the closest world where *p* and *q*'s truth values differ is placed (using rescaling) in a distance of R-ε away from l, with ε being a very small positive number. If no such world exists on the arrow, all worlds are set to a distance greater than R from l. G is then defined to be all worlds within distance R of l. Now, if *p* is true in all the worlds forming the bounds of G, it is more probable than *q*. This model prevents the possibility of the creation (given the same ordering source) of a set G' where *q* turns out to be more probable than *p*, according to definition (15), and is in my opinion a fair representation of the intuitions of proposition comparison and world ordering.

A final thought: if this is indeed the model in mind, some plausible situations need to be addressed in terms of comparative probability: for example, if it turns out after such a rescaling that a majority of the worlds in G's bounds are "*p* and not *q*", but some are "*q* and not *p*", do we say the two propositions may not be compared (as is deduced from the current definitions)? Or do we employ some other measure, statistical for example, to allow for one proposition to be more probable than the other after all? This is, of course, beyond the scope of this work.