Parameterized Algorithms (20226191) - HW1

Submission deadline: 3.1.18 (Wednesday).

The homework should be submitted in class. It can be submitted by groups of size up to 2. Copying is not allowed. Submissions must be typed (handwritten submissions are not allowed). In each question, it is required to write a formal proof that the proposed algorithm is correct, and to analyze its running time.

1. In the Graph Motif problem, we are given a graph $G = (V, E)$, a set of colors $C$, a coloring function $\text{col} : V \rightarrow C$, and an integer $k$. Note that $|C|$ can be larger than $k$. The task is to decide whether $G$ has a colorful tree on $k$ vertices, that is, whether there exists a subtree $T$ of $G$ on $k$ vertices such that for all distinct vertices $u, v \in V(T)$, $\text{col}(u) \neq \text{col}(v)$. Design a deterministic algorithm for this problem that runs in time $(3e)^{2k+o(k)}n^{O(1)}$.

2. Design a randomized algorithm for Graph Motif that runs in time $2^{2k}n^{O(1)}$.

3. In the Connected Vertex Cover problem, we are given a graph $G = (V, E)$ and an integer $k$. The task is to decide whether there exists a subset $U \subseteq V$ of size $k$ such that $U$ is a vertex cover of $G$ and $G[U]$ is a connected graph. Design a deterministic algorithm for this problem that runs in time $6^k n^{O(1)}$.

4. In the Subset Sum problem, we are given a multiset $M$ of integers, and an integer $t$. The task is to decide whether there is a multiset $S \subseteq M$ such that the sum of the numbers in $S$ is $t$. Design a deterministic algorithm for this problem that runs in time $k^{O(k)}n^{O(1)}$, where $k$ is the number of distinct numbers in $M$.

5. Let $\mathcal{A}$ be a deterministic algorithm for Vertex Cover that runs in time $1.274^k n^{O(1)}$. Let $c$ be some number between 1 and 2. Use $\mathcal{A}$ to design a $c$-approximation parameterized algorithm for Vertex Cover that runs in time $1.274^{(2-c)k} n^{O(1)}$. 