

with(LinearAlgebra) :

We aim to compute the matrix exponential of the following matrix

$$A := \text{Matrix}([\ [a, b, c], [d, e, f], [0, 0, 0]])$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix} \quad (1)$$

The exponential is straight forward but gives an expression that require complex arithmetic

$$B := \text{simplify}(\text{MatrixExponential}(A, t))$$

$$\begin{aligned} & \left[\left[-\left(2(ae - bd) \left((a - e \right. \right. \right. \right. \right. \\ & \left. \left. \left. \left. \left. -\sqrt{a^2 - 2ae + 4bd + e^2} \right) e^{\frac{1}{2}(e + a - \sqrt{a^2 - 2ae + 4bd + e^2})t} \right. \right. \right. \right. \\ & \left. \left. \left. \left. \left. -e^{\frac{1}{2}(e + a + \sqrt{a^2 - 2ae + 4bd + e^2})t} \left(a - e + \sqrt{a^2 - 2ae + 4bd + e^2} \right) \right) \right) \right) \right) \right) / \\ & \left(\sqrt{a^2 - 2ae + 4bd + e^2} \left(e + a - \sqrt{a^2 - 2ae + 4bd + e^2} \right) \left(e + a \right. \right. \\ & \left. \left. + \sqrt{a^2 - 2ae + 4bd + e^2} \right) \right), -\left(4b \left(e^{\frac{1}{2}(e + a - \sqrt{a^2 - 2ae + 4bd + e^2})t} \right. \right. \\ & \left. \left. - e^{\frac{1}{2}(e + a + \sqrt{a^2 - 2ae + 4bd + e^2})t} \right) (ae - bd) \right) / \left(\sqrt{a^2 - 2ae + 4bd + e^2} \left(e \right. \right. \\ & \left. \left. + a - \sqrt{a^2 - 2ae + 4bd + e^2} \right) \left(e + a + \sqrt{a^2 - 2ae + 4bd + e^2} \right) \right), \\ & -\left(2 \left(\left(\sqrt{a^2 - 2ae + 4bd + e^2} (bf - ce) - ce^2 + (ac + bf) e + b (af \right. \right. \right. \\ & \left. \left. - 2cd) \right) e^{\frac{1}{2}(e + a - \sqrt{a^2 - 2ae + 4bd + e^2})t} + \left(\sqrt{a^2 - 2ae + 4bd + e^2} (bf - ce) \right. \right. \\ & \left. \left. + ce^2 + (-ac - bf) e - b (af - 2cd) \right) e^{\frac{1}{2}(e + a + \sqrt{a^2 - 2ae + 4bd + e^2})t} \right) \end{aligned} \quad (2)$$

$$\begin{aligned}
& -2\sqrt{a^2-2ae+4bd+e^2} (bf-ce) \Big) \Big/ \left(\sqrt{a^2-2ae+4bd+e^2} (e+a \right. \\
& \left. -\sqrt{a^2-2ae+4bd+e^2}) (e+a+\sqrt{a^2-2ae+4bd+e^2}) \right) \Big], \\
& \left[-\left(4d \left(e^{\frac{1}{2}(e+a-\sqrt{a^2-2ae+4bd+e^2})t} \right. \right. \right. \\
& \left. \left. -e^{\frac{1}{2}(e+a+\sqrt{a^2-2ae+4bd+e^2})t} \right) (ae-bd) \right) \Big/ \left(\sqrt{a^2-2ae+4bd+e^2} (e \right. \\
& \left. +a-\sqrt{a^2-2ae+4bd+e^2}) (e+a+\sqrt{a^2-2ae+4bd+e^2}) \right), \\
& \left(2(ae-bd) \left((a-e \right. \right. \\
& \left. \left. +\sqrt{a^2-2ae+4bd+e^2}) e^{\frac{1}{2}(e+a-\sqrt{a^2-2ae+4bd+e^2})t} \right. \right. \\
& \left. \left. -e^{\frac{1}{2}(e+a+\sqrt{a^2-2ae+4bd+e^2})t} (a-e-\sqrt{a^2-2ae+4bd+e^2}) \right) \right) \Big/ \\
& \left(\sqrt{a^2-2ae+4bd+e^2} (e+a-\sqrt{a^2-2ae+4bd+e^2}) (e+a \right. \\
& \left. +\sqrt{a^2-2ae+4bd+e^2}) \right), \left(2 \left(\left(\sqrt{a^2-2ae+4bd+e^2} (af-cd) \right. \right. \right. \\
& \left. \left. +a^2 f+(-cd-ef) a+d(2bf-ce) \right) e^{\frac{1}{2}(e+a-\sqrt{a^2-2ae+4bd+e^2})t} \right. \\
& \left. \left. +\left(\sqrt{a^2-2ae+4bd+e^2} (af-cd) -a^2 f+(cd+ef) a+(-2bf \right. \right. \right. \\
& \left. \left. +ce) d \right) e^{\frac{1}{2}(e+a+\sqrt{a^2-2ae+4bd+e^2})t} -2\sqrt{a^2-2ae+4bd+e^2} (af \right. \\
& \left. \left. -cd) \right) \right) \Big/ \left(\sqrt{a^2-2ae+4bd+e^2} (e+a-\sqrt{a^2-2ae+4bd+e^2}) (e+a \right. \\
& \left. +\sqrt{a^2-2ae+4bd+e^2}) \right) \Big], \\
& \left[\begin{array}{l} 0, 0, 1 \end{array} \right] \Big]
\end{aligned}$$

Two things are worth noting about this expression:

1. The last row is simply [0, 0, 1], so we will not worry about this aspect of the matrix
2. The expression $\sqrt{a^2 - 2ae + 4bd + e^2}$ appears through out the solution. This can get complex valued. We will handle the complex and real settings independently.

We introduce a real variable x which will correspond to the square root in question. Then in the complex case, we work with $I*x$, and in the real case we work with x

assume(x, real)

First we deal with the complex case and replace the square root with $I*x$

$$\begin{aligned}
 M := & \text{simplify}(\text{subs}(\sqrt{a^2 - 2ae + 4bd + e^2} = I*x, B[1..2, 1..3])) \\
 & \left[\left[\frac{-2(ae - bd) \left((Ix\sim - a + e) e^{-\frac{1}{2}(Ix\sim - a - e)t} + e^{\frac{1}{2}(e + a + Ix\sim)t} (a - e + Ix\sim) \right)}{\sqrt{a^2 - 2ae + 4bd + e^2} (Ix\sim - a - e) (e + a + Ix\sim)}, \right. \right. \\
 & \left. \frac{4b \left(e^{-\frac{1}{2}(Ix\sim - a - e)t} - e^{\frac{1}{2}(e + a + Ix\sim)t} \right) (ae - bd)}{\sqrt{a^2 - 2ae + 4bd + e^2} (Ix\sim - a - e) (e + a + Ix\sim)}, \left((-4cd + 2f(e + a \right. \right. \\
 & \left. \left. + Ix\sim)) b - 2ec(Ix\sim - a + e) e^{-\frac{1}{2}(Ix\sim - a - e)t} + ((4cd + 2f(Ix\sim - a - e)) b \right. \right. \\
 & \left. \left. - 2(a - e + Ix\sim) ec) e^{\frac{1}{2}(e + a + Ix\sim)t} - 4I(bf - ce) x\sim \right) / \right. \\
 & \left. \left(\sqrt{a^2 - 2ae + 4bd + e^2} (Ix\sim - a - e) (e + a + Ix\sim) \right) \right], \\
 & \left[\frac{4d \left(e^{-\frac{1}{2}(Ix\sim - a - e)t} - e^{\frac{1}{2}(e + a + Ix\sim)t} \right) (ae - bd)}{\sqrt{a^2 - 2ae + 4bd + e^2} (Ix\sim - a - e) (e + a + Ix\sim)}, \right. \\
 & \left. - \left(2(ae - bd) \left((a - e + Ix\sim) e^{-\frac{1}{2}(Ix\sim - a - e)t} + (Ix\sim - a \right. \right. \right. \\
 & \left. \left. \left. + e) e^{\frac{1}{2}(e + a + Ix\sim)t} \right) \right) / \left(\sqrt{a^2 - 2ae + 4bd + e^2} (Ix\sim - a - e) (e + a \right. \right. \\
 & \left. \left. + Ix\sim) \right), \left((-2a^2f + (2cd - 2f(Ix\sim - e)) a + 2(-2bf + (Ix\sim \right. \right. \\
 & \left. \left. + e) c) d) e^{-\frac{1}{2}(Ix\sim - a - e)t} + (2a^2f + (-2cd - 2(Ix\sim + e) f) a + 2d(2bf \right. \right.
 \end{aligned} \tag{3}$$

$$\left. \left. \left. \left((I\tilde{x} - e)c \right) e^{\frac{1}{2}(e+a+I\tilde{x})t} + 4I\tilde{x}(af - cd) \right) / \left(\sqrt{a^2 - 2ae + 4bd + e^2} (I\tilde{x} - a - e) (e + a + I\tilde{x}) \right) \right] \right]$$

We then split the exponentials and apply Euler's theorem, $\exp(I^*x) = \cos(x) + I^*\sin(x)$

$$K := \text{simplify} \left(\text{subs} \left(e^{-\frac{1}{2}I\tilde{x}t} = \left(\cos \left(-\frac{1}{2}\tilde{x}t \right) + I \sin \left(-\frac{1}{2}\tilde{x}t \right) \right), \text{subs} \left(e^{\frac{1}{2}I\tilde{x}t} = \left(\cos \left(\frac{1}{2}\tilde{x}t \right) + I \sin \left(\frac{1}{2}\tilde{x}t \right) \right), \left(\text{subs} \left(e^{\frac{1}{2} \cdot (e+a+I\tilde{x}) \cdot t} = e^{\frac{a+e}{2} \cdot t} \cdot e^{\frac{1}{2}I\tilde{x}t}, \text{subs} \left(e^{-\frac{1}{2} \cdot (I\tilde{x}-a-e) \cdot t} = e^{\frac{a+e}{2} \cdot t} \cdot e^{-\frac{1}{2}I\tilde{x}t}, M \right) \right) \right) \right) \right)$$

$$\left[\left[\frac{2(ae - bd) \left((a - e - \tilde{x}) e^{\frac{1}{2}(e+a-\tilde{x})t} - e^{\frac{1}{2}(e+a+\tilde{x})t} (a - e + \tilde{x}) \right)}{\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - \tilde{x}) (e + a + \tilde{x})}, \right. \right. \tag{4}$$

$$\left. \left. \frac{4b \left(e^{\frac{1}{2}(e+a-\tilde{x})t} - e^{\frac{1}{2}(e+a+\tilde{x})t} \right) (ae - bd)}{\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - \tilde{x}) (e + a + \tilde{x})}, \left((4cd - 2f(e+a + \tilde{x})) b - 2ce(a - e - \tilde{x}) e^{\frac{1}{2}(e+a-\tilde{x})t} + (-4cd + 2f(e+a - \tilde{x})) b + 2ce(a - e + \tilde{x}) e^{\frac{1}{2}(e+a+\tilde{x})t} + 4(bf - ce)\tilde{x} \right) / \left(\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - \tilde{x}) (e + a + \tilde{x}) \right) \right], \right.$$

$$\left[\frac{4d \left(e^{\frac{1}{2}(e+a-\tilde{x})t} - e^{\frac{1}{2}(e+a+\tilde{x})t} \right) (ae - bd)}{\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - \tilde{x}) (e + a + \tilde{x})}, \right.$$

$$\left. \frac{2(ae - bd) \left((a - e + \tilde{x}) e^{\frac{1}{2}(e+a-\tilde{x})t} - e^{\frac{1}{2}(e+a+\tilde{x})t} (a - e - \tilde{x}) \right)}{\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - \tilde{x}) (e + a + \tilde{x})}, \right.$$

$$\left. \left(\left(2a^2f + (-2cd - 2f(e - \tilde{x}))a + 4d \left(bf - \frac{1}{2}c(e + \tilde{x}) \right) \right) e^{\frac{1}{2}(e+a-\tilde{x})t} + \left(-2a^2f + (2cd + 2f(e + \tilde{x}))a - 4 \left(bf - \frac{1}{2}c(e - \tilde{x}) \right)d \right) e^{\frac{1}{2}(e+a+\tilde{x})t} - 4\tilde{x}(af - cd) \right) / \left(\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - \tilde{x}) (e + a + \tilde{x}) \right) \right] \right]$$

$$\begin{aligned}
& (\text{simplify}(K[1].\text{Vector}([u, v, 1]) \cdot \sqrt{a^2 - 2ae + 4bd + e^2} (e + a - x\sim) (e + a + x\sim)) - 4 \\
& \cdot (b \cdot f - c \cdot e) \cdot x) \\
& (4b^2 dv + ((-4av - 2du - 2f)e + (2du - 2f)a - 2x\sim du - 2fx\sim + 4cd)b \quad (5) \\
& - 2e(au + c)(a - e - x\sim)) e^{\frac{1}{2}(e+a-x\sim)t} + (-4b^2 dv + ((4av + 2du \\
& + 2f)e + (-2du + 2f)a - 2x\sim du - 2fx\sim - 4cd)b + 2e(au + c)(a - e \\
& + x\sim)) e^{\frac{1}{2}(e+a+x\sim)t}
\end{aligned}$$

For some reason Maple has not replaced $\sqrt{a^2 - 2ea + 4db + e^2}$ in the denominator with $I \cdot x$ so we do this manually here

$$\begin{aligned}
L & := K \cdot \frac{\sqrt{a^2 - 2ea + 4db + e^2}}{I \cdot x} \\
& \left[\left[\frac{2I(ae - bd) \left((a - e - x\sim) e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} (a - e + x\sim) \right)}{x\sim (e + a - x\sim) (e + a + x\sim)}, \quad (6) \right. \right. \\
& \frac{4Ib \left(e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} \right) (ae - bd)}{x\sim (e + a - x\sim) (e + a + x\sim)}, \\
& - \frac{1}{x\sim (e + a - x\sim) (e + a + x\sim)} \left(I \left(((4cd - 2f(e + a + x\sim))b - 2ce(a - e \right. \right. \\
& - x\sim)) e^{\frac{1}{2}(e+a-x\sim)t} + ((-4cd + 2f(e + a - x\sim))b + 2ce(a - e \\
& + x\sim)) e^{\frac{1}{2}(e+a+x\sim)t} + 4(bf - ce)x\sim \left. \left. \right) \right) \left. \right], \\
& \left[\frac{4Id \left(e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} \right) (ae - bd)}{x\sim (e + a - x\sim) (e + a + x\sim)}, \right. \\
& - \frac{2I(ae - bd) \left((a - e + x\sim) e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} (a - e - x\sim) \right)}{x\sim (e + a - x\sim) (e + a + x\sim)}, \\
& - \frac{1}{x\sim (e + a - x\sim) (e + a + x\sim)} \left(I \left(\left(2a^2 f + (-2cd - 2f(e - x\sim))a + 4d \left(bf \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{1}{2}c(e + x\sim) \right) \right) e^{\frac{1}{2}(e+a-x\sim)t} + \left(-2a^2 f + (2cd + 2f(e + x\sim))a - 4 \left(bf \right. \right. \right.
\end{aligned}$$

$$\left. \left. \left. \left. -\frac{1}{2} c(e-x\sim) \right) d \right) e^{\frac{1}{2}(e+a+x\sim)t} -4x\sim (af-cd) \right) \right) \right]$$

Finally, we get rid of the imaginary numbers in the denominator by expanding it

$$E := \text{expand}((Ix\sim - a - e) (e + a + Ix\sim)) \quad -a^2 - 2ae - e^2 - x\sim^2 \quad (7)$$

And we arrive at the final result for the complex case

$$\text{result1} := \frac{L \cdot (Ix\sim - a - e) (e + a + Ix\sim)}{E}$$

$$\left[\left[\frac{2I(ae - bd) \left((Ix\sim - a + e) e^{-\frac{1}{2}(Ix\sim - a - e)t} + e^{\frac{1}{2}(e + a + Ix\sim)t} (a - e + Ix\sim) \right)}{(-a^2 - 2ae - e^2 - x\sim^2) x\sim}, \right. \right. \quad (8)$$

$$\left. - \frac{4Ib \left(e^{-\frac{1}{2}(Ix\sim - a - e)t} - e^{\frac{1}{2}(e + a + Ix\sim)t} \right) (ae - bd)}{(-a^2 - 2ae - e^2 - x\sim^2) x\sim}, \right.$$

$$\left. - \frac{1}{(-a^2 - 2ae - e^2 - x\sim^2) x\sim} \left(I \left((-4cd + 2f(e + a + Ix\sim)) b - 2ec(Ix\sim - a + e) \right) e^{-\frac{1}{2}(Ix\sim - a - e)t} + ((4cd + 2f(Ix\sim - a - e)) b - 2(a - e + Ix\sim) ec) e^{\frac{1}{2}(e + a + Ix\sim)t} - 4I(bf - ce) x\sim \right) \right) \right],$$

$$\left[- \frac{4Id \left(e^{-\frac{1}{2}(Ix\sim - a - e)t} - e^{\frac{1}{2}(e + a + Ix\sim)t} \right) (ae - bd)}{(-a^2 - 2ae - e^2 - x\sim^2) x\sim}, \right.$$

$$\left. \frac{2I(ae - bd) \left((a - e + Ix\sim) e^{-\frac{1}{2}(Ix\sim - a - e)t} + (Ix\sim - a + e) e^{\frac{1}{2}(e + a + Ix\sim)t} \right)}{(-a^2 - 2ae - e^2 - x\sim^2) x\sim}, \right.$$

$$\left. - \frac{1}{(-a^2 - 2ae - e^2 - x\sim^2) x\sim} \left(I \left((-2a^2f + (2cd - 2f(Ix\sim - e)) a + 2(-2bf + (Ix\sim + e) c) d) e^{-\frac{1}{2}(Ix\sim - a - e)t} + (2a^2f + (-2cd - 2(Ix\sim + e) f) a + 2d(2bf + (Ix\sim - e) c)) e^{\frac{1}{2}(e + a + Ix\sim)t} + 4Ix\sim(af - cd) \right) \right) \right]$$

We now proceed to the real (non-complex case). That is we replace

$$\sqrt{a^2 - 2ae + 4bd + e^2} \text{ with } x:$$

$$\begin{aligned}
M := & \text{simplify}(\text{subs}(\sqrt{a^2 - 2ae + 4bd + e^2} = x, B[1..2, 1..3])) \\
& \left[\left[-\frac{2(ae - bd) \left((a - e - x\sim) e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} (a - e + x\sim) \right)}{\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - x\sim) (e + a + x\sim)}, \right. \right. \\
& -\frac{4b \left(e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} \right) (ae - bd)}{\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - x\sim) (e + a + x\sim)}, \left((4cd - 2f(e + a \right. \\
& + x\sim)) b - 2ce(a - e - x\sim) e^{\frac{1}{2}(e+a-x\sim)t} + ((-4cd + 2f(e + a - x\sim)) b \\
& + 2ce(a - e + x\sim) e^{\frac{1}{2}(e+a+x\sim)t} + 4(bf - ce)x\sim) \left. \right) / \\
& \left. \left(\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - x\sim) (e + a + x\sim) \right) \right], \\
& \left[-\frac{4d \left(e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} \right) (ae - bd)}{\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - x\sim) (e + a + x\sim)}, \right. \\
& \frac{2(ae - bd) \left((a - e + x\sim) e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} (a - e - x\sim) \right)}{\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - x\sim) (e + a + x\sim)}, \\
& \left(\left(2a^2f + (-2cd - 2f(e - x\sim))a + 4d \left(bf - \frac{1}{2}c(e + x\sim) \right) \right) e^{\frac{1}{2}(e+a-x\sim)t} \right. \\
& + \left(-2a^2f + (2cd + 2f(e + x\sim))a - 4 \left(bf - \frac{1}{2}c(e - x\sim) \right) d \right) e^{\frac{1}{2}(e+a+x\sim)t} \\
& \left. - 4x\sim(af - cd) \right) / \left(\sqrt{a^2 - 2ae + 4bd + e^2} (e + a - x\sim) (e + a + x\sim) \right) \left. \right] \Big]
\end{aligned} \tag{9}$$

Again, Maple does not make the substitution of the square root in the denominator, so we do it manually

$$\begin{aligned}
\text{result2} := & \frac{M \cdot \sqrt{a^2 - 2ae + 4bd + e^2}}{x} \\
& \left[\left[-\frac{2(ae - bd) \left((a - e - x\sim) e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} (a - e + x\sim) \right)}{x\sim (e + a - x\sim) (e + a + x\sim)}, \right. \right. \\
& -\frac{4b \left(e^{\frac{1}{2}(e+a-x\sim)t} - e^{\frac{1}{2}(e+a+x\sim)t} \right) (ae - bd)}{x\sim (e + a - x\sim) (e + a + x\sim)}, \left. \right] \tag{10}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{x \sim (e+a-x \sim) (e+a+x \sim)} \left(((4cd-2f(e+a+x \sim)) b - 2ce(a-e \right. \\
& \left. -x \sim)) e^{\frac{1}{2}(e+a-x \sim)t} + ((-4cd+2f(e+a-x \sim)) b + 2ce(a-e \right. \\
& \left. +x \sim)) e^{\frac{1}{2}(e+a+x \sim)t} + 4(bf-ce)x \sim \right), \\
& \left[-\frac{4d \left(e^{\frac{1}{2}(e+a-x \sim)t} - e^{\frac{1}{2}(e+a+x \sim)t} \right) (ae-bd)}{x \sim (e+a-x \sim) (e+a+x \sim)}, \right. \\
& \left. \frac{2(ae-bd) \left((a-e+x \sim) e^{\frac{1}{2}(e+a-x \sim)t} - e^{\frac{1}{2}(e+a+x \sim)t} (a-e-x \sim) \right)}{x \sim (e+a-x \sim) (e+a+x \sim)}, \right. \\
& \frac{1}{x \sim (e+a-x \sim) (e+a+x \sim)} \left(\left(2a^2f + (-2cd-2f(e-x \sim)) a + 4d \left(bf \right. \right. \right. \\
& \left. \left. - \frac{1}{2}c(e+x \sim) \right) \right) e^{\frac{1}{2}(e+a-x \sim)t} + \left(-2a^2f + (2cd+2f(e+x \sim)) a - 4 \left(bf \right. \right. \\
& \left. \left. - \frac{1}{2}c(e-x \sim) \right) d \right) e^{\frac{1}{2}(e+a+x \sim)t} - 4x \sim (af-cd) \left. \right) \left. \right]
\end{aligned}$$

Then this is a perfectly fine solution for the real case.

The resulting code should then compute

$$y := a^2 - 2ae + 4bd + e^2 \tag{11}$$

if $y < 0$, then apply the complex solution. If $y > 0$, apply the real solution, and if $y = 0$, then I guess we can consider the limit $x \rightarrow 0$:

$$\begin{aligned}
& \text{simplify}(\text{limit}(\text{result2}[1, 1], x = 0)) \\
& \frac{2e^{\frac{1}{2}(e+a)t} (at-et+2) (ae-bd)}{(e+a)^2} \tag{12}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{a^2 + 2ae + e^2} \left(2e^{\frac{1}{2}e} e^{\frac{1}{2}a} a^2 e - 2e^{\frac{1}{2}e} e^{\frac{1}{2}a} abd - 2e^{\frac{1}{2}e} e^{\frac{1}{2}a} ae^2 \right. \\
& \left. + 2e^{\frac{1}{2}e} e^{\frac{1}{2}a} bde + 4e^{\frac{1}{2}e} e^{\frac{1}{2}a} ea - 4e^{\frac{1}{2}e} e^{\frac{1}{2}a} db \right) \tag{13}
\end{aligned}$$

$$\begin{aligned}
& \text{simplify}(\text{limit}(\text{result2}[1, 2], x = 0)) \\
& \frac{4tb(ae-bd)e^{\frac{1}{2}(e+a)t}}{(e+a)^2} \tag{14}
\end{aligned}$$

simplify(*limit*(*result2*[1, 3], $x = 0$))

$$\frac{(-2ce^2 + (2bf + 2c(a+2))e + 2(-2cd + f(a-2))b)e^{\frac{1}{2}e + \frac{1}{2}a} + 4bf - 4ce}{(e+a)^2} \quad (15)$$

simplify(*limit*(*result2*[2, 1], $x = 0$))

$$\frac{4td(ae - bd)e^{\frac{1}{2}(e+a)t}}{(e+a)^2} \quad (16)$$

simplify(*limit*(*result2*[2, 2], $x = 0$))

$$-\frac{2e^{\frac{1}{2}(e+a)t}(at - et - 2)(ae - bd)}{(e+a)^2} \quad (17)$$

simplify(*limit*(*result2*[2, 3], $x = 0$))

$$\frac{1}{(e+a)^2} \left(\left(-2ta^2f + (2cdt + 2eft + 4f)a - 4d \left(bft - \frac{1}{2}cet + c \right) \right) e^{\frac{1}{2}(e+a)t} - 4af + 4cd \right) \quad (18)$$