Introduction to

Numerical Analysis

CS 201-1-3011

Computer Science Department, BGU
שקפים לפרק ייצוג וקרוב פונקציונלי.
\[ L(\alpha) = L_1(\alpha) + L_2(\alpha) = \frac{w_1}{\sin(\alpha)} + \frac{w_2}{\sin(\pi - \beta - \alpha)} \]

\[ L'(\alpha) = -\frac{w_1 \cos(\alpha)}{\sin^2(\alpha)} - \frac{w_2 \cos(\alpha + \beta)}{\sin^2(\alpha + \beta)} = 0 \]
\[ L'(\alpha) = \frac{w_1 \cos(\alpha)}{\sin^2(\alpha)} - \frac{w_2 \cos(\alpha+\beta)}{\sin^2(\alpha+\beta)} = 0 \]

\[ \sin, \cos, \ldots \]
<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1900</td>
<td>76,212,168</td>
</tr>
<tr>
<td>1910</td>
<td>92,228,496</td>
</tr>
<tr>
<td>1920</td>
<td>106,021,537</td>
</tr>
<tr>
<td>1930</td>
<td>123,202,624</td>
</tr>
<tr>
<td>1940</td>
<td>132,164,569</td>
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<tr>
<td>1950</td>
<td>151,325,798</td>
</tr>
<tr>
<td>1960</td>
<td>179,323,175</td>
</tr>
<tr>
<td>1970</td>
<td>203,211,926</td>
</tr>
<tr>
<td>1980</td>
<td>226,545,805</td>
</tr>
<tr>
<td>1990</td>
<td>248,709,873</td>
</tr>
<tr>
<td>2000</td>
<td>281,421,906</td>
</tr>
<tr>
<td>2010</td>
<td>308,745,538</td>
</tr>
</tbody>
</table>

![Graph showing US population increase over time](image-url)
קרוב

$\sin(x)$ על אי בודד
קרוב של $f(x) = \sin(x)$ על אי בודד.
קרוב

\[ f(x) = \sin(x) \] על אי בודד
The Runge effect occurs when a function with rapid oscillations at the edges of the interval is approximated by polynomials. The function shown is:

\[ f(x) = \frac{1}{1 + 12x^2} \]

As the degree of the polynomial increases, the approximation becomes more accurate in the middle of the interval but exhibits oscillatory behavior near the edges. This effect is particularly pronounced with higher degrees (N=11 and N=15 in the images).
The Chebyshev polynomials (of the first kind) are defined recursively as follows:

\[
\begin{align*}
T_0(x) &= 1 \\
T_1(x) &= x \\
T_{n+1}(x) &= 2xT_n(x) - T_{n-1}(x) & (n \geq 1)
\end{align*}
\]

The explicit forms of the next few \( T_n \) are readily calculated:

\[
\begin{align*}
T_2(x) &= 2x^2 - 1 \\
T_3(x) &= 4x^3 - 3x \\
T_4(x) &= 8x^4 - 8x^2 + 1 \\
T_5(x) &= 16x^5 - 20x^3 + 5x \\
T_6(x) &= 32x^6 - 48x^4 + 18x^2 - 1
\end{align*}
\]
Runge Effect

$f(x) = \frac{1}{1+12x^2}$
 CSI ריכך והאינטגרציה בצורת 'גייז'ב

$\frac{1}{1+12x^2}$

Chebyshev N=3

Chebyshev N=7

Chebyshev N=11

Chebyshev N=15