Management of Correctness Problems in UML Class
Diagrams – Towards a Pattern-based Approach*

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ABSTRACT
UML is now widely accepted as the standard modeling language for software construction. The
Class Diagram is its core view, having well formed semantics and providing the backbone for
any modeling effort. Class diagrams are widely used for purposes such as software specification,
database and ontology engineering, meta-modeling, and model transformation. The central role
played by class diagrams emphasizes the need for strengthening UML modeling tools with
features such as recognition of erroneous models and the detection of errors’ sources.
Correctness of UML class diagrams refers to the capability of a diagram to denote a finite but not
empty reality. This is a natural, unquestionable requirement. Nevertheless, incorrect diagrams
are often designed, due to the interaction of contradicting constraints and the limitations of
current tools. In this paper we clarify the notion of class diagram correctness, discuss various
approaches for detecting correctness problems, and propose a pattern-based approach for

* This work was supported in part by the Paul Ivanir Center for Robotics and Production Management at Ben-
Gurion University of the Negev.
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identifying situations in which correctness problems occur, and for providing explanations and repair advices.

**Keywords:** Modeling Languages, incorrectness patterns, UML class diagrams, consistency, finite satisfiability, redundancy, incomplete design, reasoning, detection, identification, model driven engineering

**INTRODUCTION**

The Unified Modeling Language (UML) is the de facto standard for system development, as it was developed and adopted by the Object Management Group (OMG-UML, 2009). It consists of several diagrammatic languages, each describing a different view of object-oriented software; a system model consists of a collection of such diagrams. The most important view of UML is the static/structural specification which describes a structural abstraction of the real world. This view is expressed by class diagrams, which consist of *classes* and their *descriptors*, *associations* among them, and *constraints* imposed on both classes and associations. Among the nine visual UML models, class diagrams appear to be the most clear, intuitive and well defined.

Dobing and Parsons (2006) found that the Class Diagram view is the most frequently used (73%) in their examination of the usage of UML. It was found to be useful in clarifying technical understanding, and for maintaining software documentation. The major usage of UML class diagrams is to specify, visualize, and document systems’ static view. They also serve as a basis for generating implementation artifacts such as code skeleton (Martin, 2006) and database schemata (Blaha et al., 1994), as a means for knowledge representation such as specifying ontologies (Cranefield, 2001; Falkovych et al., 2003; Gasevic et al. 2004; Kabilan & Johannesson, 2004; Kogut, 2002; Timm & Gannod, 2005), and for defining meta-models of other programming, modeling, and specification languages.

Class diagrams are models written by people, and therefore, usually suffer from modeling problems like *inconsistency, redundancy, and abstraction errors*. Inexperienced designers tend to create erroneous models, but even experienced ones cannot anticipate the implication of a change on an overall model (Sunye et al., 2001). Indeed, Lange et al. (2006) show that model defects often remain undetected, even if practitioners check the model attentively. These
problems are empowered when a model originates from different resources. Combined sources are usually overlapping, and the integration yields redundant inconsistent models (Ackermann & Turowski, 2006; Huzar et al., 2004). It is a common belief that such problems can best be solved at the level of models (Jackson & Rinard, 2004).

In view of the wide spread usage of UML class diagrams and the difficulties of producing high quality models, it is essential to equip UML CASE tools with reasoning capabilities for identifying problems within models (Berardi et al., 2005; Cadoli et al., 2004; Hartman, 2001; Jackson & Rinard, 2004). These capabilities can help in detecting design problems, identifying the reasons for these errors, suggesting possible solutions, and providing advice for design improvements (Unhelkar, 2005). The quality of models is especially important for the emerging Model Driven Engineering (MDE) approach, in which software is developed by repeated transformations of models (Stahl et al., 2006).

The major correctness features of class diagrams involve consistency and finite satisfiability. They guarantee the natural, unquestionable requirement, that a diagram can denote a finite but non empty reality: Consistency accounts for non-emptiness, and finite satisfiability accounts for finiteness. Emptiness is caused by contradicting constraints, such as designing a subclass of two necessarily disjoint classes. Non-finiteness is caused by interaction among multiplicity (cardinality) constraints, which restrict the number of interactions between objects of related classes. Contradicting multiplicity constraints, or a contradiction between multiplicity constraints to other constraints, like generalization and association class constraints, impose class multiplicity requirements that can be satisfied only by empty or infinite classes. For example, the class diagram in Figure 1 is not finitely satisfiable, since in every legal instance of this diagram the sets denoted by the classes CatalyzedReaction, Enzyme, Protein, Molecule, Chemical, and GeneralReaction, are either all empty or all infinite.

In this paper we define the consistency and finite satisfiability problems in class diagrams, describe current approaches for detection of these problems and identification of their cause, and suggest a pattern-based approach for creating explanations and repair advices. The paper opens with a presentation of the class diagram model, its semantics and correctness problems, which is followed by a survey of existing methods for management the correctness of class diagrams. The core of the paper is devoted to a presentation of the pattern-based approach for detecting
correctness problems within class diagrams, and for providing explanations and repair advices. The summary section draws the line for future research.

**UML CLASS DIAGRAMS**

A class diagram is a structural abstraction of a real world phenomenon. The model consists of *basic elements, descriptors and constraints*. Basic elements are classes and associations, descriptors are class and association *attributes*, and constraints are restrictions imposed on these elements. The constraints are (1) *multiplicity (cardinality) constraints on associations*, with or without *qualifiers*; (2) *class hierarchy constraints*; (3) *generalization set constraints*; (4) *association class constraints*; (5) *inter-association constraints*; (6) *aggregation constraints*; and (7) *multiplicity constraints on attributes*. The syntax and informal semantics are described in OMG-UML (2009) and Rumbaugh et al. (2005).

![Class Diagram](image)

Figure 1. A class diagram describing a partial ontology in the molecular biology domain

Figure 1 is an example of a class diagram, which partially specifies ontology in the molecular biology domain. It demonstrates the above constraints, apart from multiplicity constraints on attributes (no attributes in the diagram). The constraints are inter-woven in a complex way: Class *DNASegment* is involved in an association class constraint, class hierarchy, generalization set constraint and a multiplicity constraint. Its subclasses are also involved in aggregation constraints.
The semantics of a class diagram is given by its legal instances. An instance of a class diagram is an assignment of: (1) set extensions1 to classes; (2) relations among these classes to associations; and (3) value mappings to attributes. A legal instance is an instance that satisfies all constraints in the diagram. For example, in Figure 1, the Chemical class represents the set of chemicals within a cell, and the association between Chemical and Reaction denotes a relation (a set of links) between the Chemical extension and Reaction extension. In a legal instance of Figure 1, every Element object must also be a Chemical object, and be related to at least one Reaction object.

Constraints are used to restrict the otherwise unrestricted extensions of the components of a class diagram. Class and association constraints restrict the set and relation extensions of classes and associations, respectively. Attribute constraints restrict attribute values in terms of types and multiplicity. Association multiplicity constraints specify the number of objects of one class that can be associated with one object from the other class.

Hierarchy constraints specify subset relations between the extensions of classes or associations. In Figure 1, the classes DNASequence, Gene, Chromosome and the Genome form a Generalization Set (GS), with super-class DNASequence, and subclasses Gene, Chromosome and Genome are presented. It states that the subclass extensions are subsets of the super-class extension. The GS is constrained by the GS constraint \{complete, disjoint\}. GS constraints describe (1) disjointness (or a lack of – overlap) among the subclasses, and (2) completeness (or lack of – incomplete) of covering the super-class. The \{complete, disjoint\} constraint on the above GS indicates that in every legal instance, the extensions of the Gene, Chromosome, and Genome classes are disjoint, and their union covers the extension of the DNASequence class.

An association class constraint unifies the extension of the association class with that of the related association. In Figure 1, the extension of the DNASequence class coincides with that of the unary BasePair association.

Inter association constraints can enforce hierarchy or disjointness among associations. These include a XOR constraint and four hierarchy constraints: subsets, union, redefinition and association specialization. For example, the redefinition constraint, which was added in UML 2, enables overriding of association ends (roles). Figure 2a, q2 redefines the association end r2,

1 The term set extension refers to the set of objects (class instances) that instantiate a class, and to the set of links (association instances) that instantiate an association.
meaning that instances of class $B$ can be $q_2$-related to instances of class $D$, and cannot be $r_2$-related to instances of class $D$. Thus, the object diagram in Figure 2b is illegal, while the object diagram in Figure 2c is legal. More details about inter association constraints appear in OMG-UML (2009).

Aggregation constraints enforce whole-part relations. They are anti-symmetric and transitive. Anti-symmetry means that if an object $x$ is part of object $y$, then $y$ cannot be part of $x$. Transitivity means that if $x$ is part of $y$, and $y$ is part of $z$, then $x$ is part of $z$ (Rumbaugh et al., 2005). Altogether, aggregation prohibits cycles: an object cannot be part of itself (directly or indirectly). Composition is a strong form of aggregation, describing physical aggregation. Composition requires that an object may be part of at most one composite at a time (Rumbaugh Rumbaugh et al., 2005; OMG-UML 2009).

Correctness of class diagrams involves consistency and finite-satisfiability. A class is consistent if it has a non-empty extension in some legal instance; a class diagram is consistent if all of its classes are consistent$.^2$ A class diagram instance is finite if all class extensions are finite. A class is finitely-satisfiable if it has a non-empty extension in a legal finite instance; a class diagram is

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$^2$ Berardi et al. (2005), introduce a weaker notion of consistency. They define a class diagram as consistent if it has a legal instance with at-least one non-empty class extension. But this definition misses the point of consistency, since it implies that every class diagram that has an unconstrained class is consistent. In fact, every realistic class diagram that we have checked proved to have such a class. In the following diagram, the Seminar class is unconstrained and can be freely instantiated, implying that every class diagram that includes it is consistent.
finitely satisfiable if all of its classes are finitely satisfiable\(^3\). The class diagram in Figure 3 is inconsistent since the \{\textit{disjoint}\} constraint on the generalization set \{\textit{Chemical, Molecule, Compound}\} enforces the class \textit{MacroMolecule} to be empty, implying that all of its subclasses are also empty.

![Class diagram with an inconsistency problem.](image)

Figure 3. A class diagram with an inconsistency problem.

It can be shown that if a class diagram is consistent, then there exists a legal instance in which all class extensions are non-empty, and if the class diagram is finitely satisfiable, then there is a legal finite instance in which all class extensions are non-empty (Lenzerini, M. & Nobili, 1990; Marae, 2007; Marae et al., 2008). Therefore, in order to check consistency it is sufficient to search for a single legal instance, in which all classes are non-empty, and in order to check finite satisfiability it is sufficient to show that this instance is also finite. Note that finite satisfiability requires consistency.

**Complexity:**

Berardi et al. (2005) show that deciding consistency of UML class diagrams is EXPTIME-complete. This proof is obtained by providing consistency and finite satisfiability preserving reductions to/from hard Description Logics (DLs): From ALC, which is the least expressive description logic, and into ALCQI, which is the most expressive description logic that is supported by tools (Haarslev & Möller, 2001; Horrocks, 1998; Tsarkov & Horrocks, 2006). Artale et al. (2007) refine these results by considering fragments of ER diagrams. As for finite satisfiability, it was shown that for ALCQI, it is EXPTIME-complete (Carsten et al., 2005). Since for ALC, finite satisfiability and consistency coincide (Schild, 1992), it follows that finite satisfiability of UML class diagrams is also EXPTIME-complete (Berardi et al., 2005; Carsten et al., 2005; Schild, 1992).

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\(^3\) Lenzerini and Nobili (1990) and Thalheim (1993) used the term strong satisfiability for this notion.
SURVEY OF CORRECTNESS HANDLING TECHNIQUES FOR CLASS DIAGRAMS

Inconsistency and lack of finite satisfiability are considered erroneous design; the first, because an inconsistent class diagram does not have a non-empty extension, and the latter because there is no finite and non-empty extension. Inconsistency is caused by contradictory constraints that cannot be simultaneously satisfied. Hartman (2001) specifies three levels of reasoning concerning these problems: Problem Detection, Cause Identification, and Advice\(^4\). Problem detection refers to notification that a problem exists. Cause identification means pointing to the problem source (like advanced IDE compilers), and advice amounts to suggesting a solution. Most reasoning approaches provide problem detection alone.

Consistency of class diagrams has been handled by translation to other reasoning frameworks. The most notable approach is the translation of UML class diagrams into a description logic and activation of a description logic reasoner for determining consistency (Berardi et al., 2005; Cadoli et al., 2004). Other approaches combine reasoning over class diagrams with OCL reasoning. We are not aware of direct techniques for reasoning about class diagram consistency.

Reasoning on finite satisfiability of entity relationship and class diagrams has attracted much attention. The problem was independently identified by Lenzerini and Nobili (1990) and by Thalheim (1993), and referred to entity relationship diagrams. Subsequently, the methods have been extended to various fragments of UML class diagrams.

There are two main approaches: the linear inequalities approach and the detection graph approach. The first approach reduces the finite satisfiability problem to the problem of finding a solution to a system of linear inequalities. The second approach identifies infinity causing cycles in the diagram, and possibly suggest repair transformations. All methods apply only to fragments of UML class diagrams.

The Linear Inequalities Approach

The fundamental method of Lenzerini and Nobili (1990) and Thalheim (1993) is defined for an entity relationship diagram that includes entity types (classes), n-ary relationship types

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\(^4\) Hartman uses the terminology Problem Recognition, Cause Detection, Problem Repair.
(associations), and multiplicity constraints\(^5\). The method consists of a transformation of the multiplicity constraints into a set of linear inequalities whose variables stand for the size of the entity and relationship types in a possible instance. The binary association in Figure 4 yields the following inequalities:

- \( C_1 > 0; \ C_2 > 0; \ r \geq 0; \)
- For \( \min_1 \neq 0 \): \( r \geq \min_1 \cdot C_1; \) for \( \max_1 \neq \infty \): \( r \leq \max_1 \cdot C_1; \)
- For \( \min_2 \neq 0 \): \( r \geq \min_2 \cdot C_2; \) for \( \max_2 \neq \infty \): \( r \leq \max_2 \cdot C_2; \)

The rationale behind these inequalities is that in order to satisfy these constraints there must be at least \( \min_2 \cdot C_1 \) and \( \min_1 \cdot C_2 \), and at most \( \max_2 \cdot C_1 \) and \( \max_1 \cdot C_2 \) links in the relationship \( r \), since every \( C_1 \) instance is related to at least \( \min_1 \) and at most \( \max_1 \) instances of \( C_2 \), and vice versa for \( C_2 \). In addition, for every entity \( C \) or relationship of type \( R \), the inequalities \( c > 0 \) and \( r \geq 0 \) are inserted. The size of the inequality system is polynomial in the size of the diagram.

The main result is that the entity relationship diagram is finitely satisfiable if and only if the inequality system has a solution. Since linear programming is solvable in polynomial time in the size of the problem encoding, finite satisfiability for this fragment of class diagrams can be decided in polynomial time.

Calvanese and Lenzerini (1994) extend the inequalities based method to apply to diagrams with class hierarchy constraints. The expansion introduces a variable for every possible class intersection among subclasses of a super-class, and splits relationships accordingly. Therefore, the size of the resulting system of inequalities is exponential in the size of the class diagram.

Marae and Balaban extend the method of Lenzerini and Nobili (1990) and Thalheim (1993) to apply to diagrams with class hierarchy constraints, GS constraints, qualifier constraints, and association class constraints (Maraee, 2007; Marae & Balaban, 2007; Marae & Balaban, 2008; Marae et al., 2008). The extension is summarized in the \textit{FiniteSat} efficient algorithm for

\(^5\) Lenzerini and Nobili (1990) use the membership semantics for cardinality constraints (consult Balaban and Shoval (2002) for semantics of cardinality constraints). For non-binary relationships, this is not the standard semantics of cardinality constraints, neither in the entity relation model nor in the class diagram model.
deciding finite satisfiability in UML class diagrams. The scope of the algorithm is defined by the structure of the class hierarchy in a knowledge base, rather than by a fragment of the language.

**Example:** The application of *FiniteSat* to the class diagram in Figure 5 yields the inequality system below. We use the variables *ds* for *DNASegment*, *gn* for *Gene*, *ch* for *Chromosome*, *gm* for *Genome* and *ds-gn* for the *DNASegment-Genome* association.

- **Multiplicity constraints:** *ds-gn ≥ 1·ds*, *ds-gn ≤ 1·ds*, *ds-gn ≥ 1·gn*, *ds-gn ≤ 1·gn*
- **Class hierarchy constraints:** *ds ≥ gn*, *ds ≥ ch*, *ds ≥ gn*
- **GS constraint:** *ds > gn+ch+gm*
- **Non emptiness inequalities:** *ds, dn, ch, dm, dg > 0, ds-gn≥0*

![Figure 5. Non-finite satisfiability due to a generalization set constraint](image)

This system has no solution since the multiplicity inequalities imply *ds = gn*, while the GS constraint and the non emptiness inequalities require that *ds > gn*. Therefore, *FiniteSat* returns False.

Correctness of the *FiniteSat* depends on the structure of class hierarchies in the given class diagram. For that purpose, class hierarchy constraints are viewed as a graph (directed or not), whose nodes represent classes and its edges represent class hierarchy constraints, directed from super-classes to their subclasses. Three class hierarchy structures are analyzed: (1) *Tree class hierarchies*: A tree structure as in Figure 1; (2) *Acyclic class hierarchies*: The undirected graph is acyclic (a tree), as in Figure 6a; (3) *Cyclic class hierarchies*: The undirected graph is cyclic, as in Figure 6b, implying unrestricted multiple inheritance.
The correctness results for \textit{FiniteSat} are as follows:

1. If the class hierarchy structure in a class diagram \( CD \) does not include cycles with a disjoint or complete GS constraint, then \( CD \) is finitely satisfiable if and only if the inequality system constructed by \textit{FiniteSat} is solvable.

2. If the class hierarchy structure in a class diagram \( CD \) includes cycles with a disjoint or complete GS constraint, then \( CD \) is not finitely satisfiable if the inequality system constructed by \textit{FiniteSat} is unsolvable.

\textbf{Example: [FiniteSat Limitation]} The class diagram in Figure 7a is not finitely satisfiable, since it implies the semantic inter-relations shown in Figure 7b. Yet, \textit{FiniteSat} yields the solvable inequality system in Figure 7c. The reason for the failure of \textit{FiniteSat} lies in the projection of the disjoint constraint from the \( A \) GS to the \( E \) GS, which is not recorded in the inequality system. Recently, \textit{FiniteSat} was strengthened with propagation of disjoint and complete GS constraints (Maraee & Balaban, 2009).

\begin{equation}
\begin{aligned}
C^f \cap D^f &= \emptyset \\
C^f \cup D^f &\subseteq E^f \\
D^f &\subseteq E^f \\
|E^f| &= |D^f|
\end{aligned}
\end{equation}

\( a \geq b, a \geq c, a \geq d \)

\( e \geq c, e \geq d \)

\( r = d, r = e \)

\( a, b, r > 0 \)

\textbf{The Detection Graph Approach}

A method for detection of the cause for non finite satisfiability is suggested in Hartmann (2001), Lenzerini & Nobili (1990) and Thalheim (1993). The method is based on construction of a directed graph, termed the \textit{identification graph}, whose nodes stand for classes and associations, and its edges connect association nodes with their end class nodes. The edges are weighted by
the multiplicity constraints, as shown in Figure 8. The weight of a path is the product of the weights of its edges.

![Figure 8. The detection graph of a binary association](image)

The identification graph is used for identifying causes for non finite satisfiability of a class diagram. Cycles whose weight is less than 1, termed critical cycles, point on non finite satisfiability. Moreover, a critical cycle singles out a non-finitely satisfiable set of multiplicity constraints. Figure 9b shows the identification graph for Figure 9a, and a critical cycle that singles out the unary predecessor-successor association as the cause of a finite satisfiability problem.

![Figure 9. An identification graph with a critical cycle](image)

Maraee et al. (2008) have extended these results to apply to class diagrams with class hierarchy, qualifier and association class constraints. Detecting finite satisfiability in presence of GS constraints is still an open issue.

Hartman (2001) handles finite satisfiability in diagrams with binary multiplicity constraints from all three aspects of problem detection, cause identification and advice. For cause identification, he suggests an algorithm for determining all non-finitely satisfiable classes, based on identification of minimal distances in critical cycles. His advice approach involves automatic corrections.

**Reasoning about Class Diagrams with Constraints**
The expressivity of class diagrams is limited to class level interaction and constraints. The Object Constraint Language OCL (OMG-OCL, 2006; Warmer & Kleppe, 2003) is intended to extend a UML model (mainly class diagrams) with symbolic constraints. Cabot et al. (2008) describe a CSP-based tool for reasoning about finite satisfiability of class diagrams that are extended with OCL constraints. USE is a tool for validation of UML/OCL models (Gogolla et al., 2001; Richters & Gogolla, 2000). Alloy is a Z based tool for automated analysis of object structured software specification (Jackson, 2002). Together with the recent UML2Alloy tool (Anastasakis et al., 2007), it provides a UML/OCL analysis tool. Most UML/OCL tools do not separate reasoning about visual constraints from symbolic OCL constraints. Therefore, since OCL, as an unrestricted first order logic language, is undecidable, Cabot et al. (2008) and Jackson (2002) perform incomplete bounded verification.

**EXPLAINING AND REPAIRING CORRECTNESS PROBLEMS USING A PATTERN-BASED APPROACH**

The goal of a model development tool is to help the designer in developing a correct and high quality model. For this purpose, the tool should point to design problems, explain their cause and suggest possible repairs. The detection and cause identification methods, surveyed in the previous section, do not function as advice and explanatory tools. Consider, for example, the non finite satisfiability problem in Figure 1. The identification graph method (Maraee et al., 2008) detects the critical cycle \textit{CatalyzedReaction, Enzyme, Protein, Molecule, Chemical, Reaction, CatalyzedReaction}. But the critical status of this cycle may be caused by several interactions of constraints:

1. The \textit{Enzyme-Chemical} class hierarchy and the multiplicity constraints in the rest of the critical cycle.
2. The \textit{Reaction-CatalyzedReaction} class hierarchy and the multiplicity constraints in the rest of the critical cycle.
3. The multiplicity constraints in the critical cycle.
The reason lies in the second interaction: The intended direction of class hierarchy, from the subclass *CatalyzedReaction* to the super-class *Reaction* is reversed. This final conclusion is domain dependent, and can be made by the designer, based on personal knowledge, or by consulting appropriate domain ontologies. Therefore, a desirable approach seems to involve proposing possible explanations, and letting the designer take the final repair decision (in contrary to the approach of Hartman (2001)).

We suggest bridging the gap between current correctness handling methods and desirable explanations and advice by using a pattern based approach. This idea is influenced from the *design patterns* (Gamma et al, 1994) paradigm. Design patterns function as advices for solving typical problems that can occur in various contexts. They fulfill an educational role: awareness to design patterns yields better solutions. By way of analogy, patterns of correctness problems characterize typical situations in which correctness problems arise, analyze the causes, and suggest possible solutions. Their educational role is to increase the awareness of designers to avoid contradictory constraints that cause correctness problems.

The patterns based approach proposed in this paper reminds the *anti-patterns* (Brown et al, 1998) and the *refactoring* (Fowler, 1999) techniques for software improvement. Anti-patterns present bad solutions to typical problems (that possibly cause more problems than they solve), followed by suggestions of desirable solutions. In that sense, anti-patterns are an extension of design patterns. The patterns presented in this paper can be viewed as anti-patterns that point to negative designs, and suggest various repairs.

Refactoring is a change of the internal structure of software that preserves its functional behavior. Refactorings are usually applied to already running, correct code. Modern IDEs (Integrated Development Environments) like Eclipse and IntelliJ IDEA offer some automated refactorings at the code level. Recently, there is also research at the direction of model level refactoring (Mens and Tourwe, 2004, Mens et al., 2007). France et al. (2003) propose an approach for refactoring automation, where a metamodel is used for writing refactorings as pattern transformations (Kim, 2008). The approach proposed in this paper, also consists of patterns and transformations aimed at model improvement. The major difference is that correctness handling patterns aim at repairing problems, and might change the meaning of models.
We present *incorrectness patterns*, which characterize typical cases of erroneous design. Each pattern describes a correctness problem, caused by problematic interaction of constraints, and can be identified by characteristic structures within a class diagram. A pattern is also associated with advices for repairing the identified problem. Each pattern includes proofs that justify the identification structure and the repair advices.

We distinguish four correctness problem types – *inconsistency*, *non-finite satisfiability*, *redundancy*, and *incomplete design* – which represent two aspects of *correctness*. The first two are problems of *formal correctness*, while the last two are problems of *design quality*, which is another form of correctness: A low quality design is formally correct, but does not meet some criteria for desirable design. Such criteria include for example duplication and missing specifications. In this paper two patterns are described in detail, and few others are presented in a condensed format, including only concrete examples.

**Patterns of Inconsistency and Non-Finite Satisfiability Problems**

Inconsistency is caused by contradictory constraints that cannot be simultaneously satisfied. Finite satisfiability problems are caused by cycles of conflicting multiplicity constraints. The cycles might involve other constraints, like class hierarchies, generalization sets, association classes, inter-association and OCL constraints.

**Lack of Finite Satisfiability patterns:**

1. **Multiplicity constraint interaction:**

**Pattern Name:** Pure Multiplicity Cycle (PMC)

**Pattern Description:** A cycle of associations with multiplicity constraints might introduce a finite satisfiability problem.

**Pattern Identification Structure:**

A minimal association cycle in which all multiplicity constraints are different from (0, *). Figure 10 shows the pattern’s identification structure.

Figure 10. The PMC Pattern

1. All classes are different.
2. All multiplicity constraints are different from (0,*).
The identification structure can be symbolically described as follows:

\[
[C_i_{\text{min}}:C_i_{\text{max}} \rightarrow C_{i+1}_{\text{min}}:C_{i+1}_{\text{max}}], \ldots, [C_n_{\text{min}}:C_n_{\text{max}} \rightarrow C_1_{\text{min}}:C_1_{\text{max}}]
\]

In this expression, \([C_i,C_{i+1}]\) represents an association between \(C_i\) and \(C_{i+1}\), with the designated multiplicities. The symbolic representation can be formally described using regular expression notation, where \("*"\) denotes unbounded repetition, \("n..m\) denotes bounded repetition, \("+\) denotes alternatives, and \(",\) denotes sequencing (concatenation):

\[
[C_i_{\text{min}}:C_i_{\text{max}} \rightarrow C_{i+1}_{\text{min}}:C_{i+1}_{\text{max}}]^{1..n-1}, \ [C_n_{\text{min}}:C_n_{\text{max}} \rightarrow C_1_{\text{min}}:C_1_{\text{max}}]
\]

**Concrete Examples:**

**Example 1:** Figure 11 presents a class diagram with a minimal association cycle that does not include a \((0,\ast)\) multiplicity constraint, and causes a finite satisfiability problem. To see that, note that each course has a single successor and at least two predecessors. Therefore, if the number of courses in a legal instance is \(C\), and the number of \(\text{predecessor-successor}\) links is \(D\), then \(D\) must satisfy: \(D = C \ast 1\) and \(D \geq C \ast 2\), implying the inequality: \(C \geq C \ast 2\), that can be satisfied only by an empty or an infinite extension of the class \(Course\).

![Figure 11. Non-finite satisfiability due to a multiplicity constraint interaction](image)

**Example 2:** A class diagram with a non-minimal association cycle that includes a \((0,\ast)\) constraint, and has a finite satisfiability problem:

![Figure 12. A non-minimal association cycle that includes a \((0,\ast)\) constraint](image)

In Figure 12 the finite satisfiability problem is caused by the minimal cycle created by the \(s\) and \(t\) associations.
Example 3: A class diagram with an association cycle that includes a multiplicity constraint \((0, n)\) for \(n \neq *\), and a constraint \((m, *)\), for \(m \neq 0\), and causes a finite satisfiability problem. Figure 13 present an example of a class diagram with such a cycle.

![Class diagram with an association cycle](image)

**Figure 13. Non-finite satisfiability due to a multiplicity constraint interaction**

**Pattern justification:**

The above examples justify the following requirements in the pattern identification structure:

1. Example 1 shows that a minimal association cycle without a \((0, *)\) multiplicity constraint can cause a finite satisfiability problem. Example 3 shows that a finite satisfiability problem can occur even when the cycle includes \((0, n)\) for \(n \neq *\), or \((m, *)\), for \(m \neq 0\), multiplicity constraints.

2. Example 2 shows that dropping the minimality requirement might overlook some problems.

We still need to justify the "no \((0, *)\) multiplicity constraint” requirement. That is, we have to show that a minimal association cycle with a \((0, *)\) multiplicity constraint cannot cause a finite satisfiability problem. The proof relies on properties of the identification graph of the cycle (see Figure 9, Lenzerini & Nobili (1990), and Maraee et al. (2008)), and on the two propositions below. It shows that if a minimal association cycle \((AC)\) includes a \((0, *)\) multiplicity constraint, then its identification graph \(IG_{AC}\) does not include a critical cycle.

**Proposition 1:** Let \(AC\) be a minimal association cycle, and \(IG_{AC}\) be its identification graph. Then a cycle in \(IG_{AC}\) that includes an edge for a 0 minimum constraint or a * maximum constraint is not critical.

**Proof:** The weight of such edges in \(IG_{AC}\) is \(\infty\).

**Proposition 2:** Let \(AC = [c_{min, max}, c_{min, max}]^{\leq n-1} \), \(c_{min, max}, c_{min, max}\) be a minimal association cycle, such that its identification graph \(IG_{AC}\) includes a critical cycle. Let \(IG_{cycle}\) be a minimal critical cycle in \(IG_{AC}\), i.e., a critical cycle that cannot be pruned. Then, the
edges of $IG_{cycle}$ correspond to alternating minimum-maximum constraints in $AC$: $\max'_{1}, \min_{1}, \max'_{2}, \min_{2}, ..., \min_{n}$, or $\max_{n}, \min'_{n}, \max_{n-1}, \min'_{n-1}, ..., \min'_{1}$.

**Proof:** A cycle in $IG_{AC}$ results either from traversing an association path in $AC$ forwards and backwards, or from traversing an association cycle in $AC$ in a single direction. But, $IG_{cycle}$ cannot include successive edges for a multiplicity constraint ($\min_{i}, \max_{i}$), since such edges create a cycle whose weight is greater than 1, and therefore can be pruned, in contradiction to the minimality of $IG_{cycle}$. Therefore, $IG_{cycle}$ results from traversing an association cycle in $AC$ in a single direction. Since $AC$ is minimal (does not include inner cycles), $IG_{cycle}$ traverses through all associations in $AC$, implying its claimed structure.

The following two claims provide the justification for the pattern identification structure:

**Claim 1:** A minimal association cycle $AC$ that includes a $(0,*)$ constraint cannot create a finite satisfiability problem.

**Proof:** Proposition 2 implies that every minimal cycle in $IG_{AC}$, that traverses through all associations of $AC$, includes an edge, either for the 0 or for the * constraints, and therefore cannot be critical.

**Pattern verification:** The pattern identification structure is not tight. That is, it characterizes a necessary but not sufficient condition on the multiplicity constraints in an association cycle. In order to find out whether a PMC causes a finite satisfiability problem, there is a need to further check the conditions introduced in Claim 2.

**Claim 2:** A minimal association cycle $AC$ (as in Proposition 2) without $(0,*)$ creates a finite satisfiability problem if and only if one of the following conditions holds:

$$\max'_{1} \cdot \max'_{2} \cdot \ldots \cdot \max'_{n} \leq \min_{1} \cdot \min_{2} \cdot \ldots \cdot \min_{n}$$

$$\max_{n} \cdot \max_{n-1} \cdot \ldots \cdot \max_{1} \leq \min'_{n} \cdot \min'_{n-1} \cdot \ldots \cdot \min'_{1}$$

**Proof:** By Proposition 2, the edges of a minimal critical cycle in $IG_{AC}$ correspond to alternating minimum-maximum constraints in $AC$: $\max'_{1}, \min_{1}, \max'_{2}, \min_{2}, ..., \min_{n}$, or $\max_{n}, \min'_{n}, \max_{n-1}, \min'_{n-1}, ..., \min'_{1}$. The conditions in the claim capture the conditions that the cycle is critical.

**Repair advice:** Consider increasing a maximum constraint or decreasing a minimum constraint, such that one of the conditions in Claim 2 holds. For example, in Figure 11, the minimum predecessor requirement for a course can be decreased, or alternatively the maximum successor.
requirement can be increased as shown in Figure 14. The repair solves the finite satisfiability problem since the new inequalities for $D$ and $C$ (see the explanation to Figure 11 in page 16) are $D \geq C*1$ and $D \leq C*2$, which are solvable.

2. **Interaction of multiplicity and class hierarchy constraints:**

**Pattern Name:** Multiplicity Hierarchy Cycle (MHC).

**Pattern Description:** A cycle of associations with multiplicity constraints and class hierarchy constraints might introduce a finite satisfiability problem.

**Pattern Identification Structure:**
A minimal cycle of associations and class hierarchy constraints, in which all multiplicity constraints are different from $(0,*)$ and all class hierarchy constraints are in the same direction. Figure 15 shows the pattern’s identification structure.

![Figure 15. The MHC Pattern](image)
The pattern describes possibly interleaved chains of association and class hierarchy constraints (in the same direction). The pattern can be precisely described by the regular expression notation, which supports alternation and sequencing. First, we introduce an expression that captures the class hierarchy constraint: \( C \leftarrow D \) stands for \( C \) is a subclass of \( D \). The expression \([C_{\text{min}}:maxt}, \ C_{i+1}[\text{min}:maxt}] \) denotes either an association or a class hierarchy constraint. The expression \([C_{\text{min}}:maxt}, \ C_{i+1}[\text{min}:maxt}] \) denotes a sequence of up to \( n \) alternating association or class hierarchy constraints. The overall pattern is captured by the expression:

\[
\left[[C_{\text{min}}:maxt}, \ C_{i+1}[\text{min}:maxt}] \right] \ + \ C_{i} < C_{i+1} \right]^{1..n}
\]

Concrete Examples:
Figure 16a presents a multiplicity constraint cycle that involves class \( \text{Graduate} \), which is a subclass of class \( \text{Academic} \), and whose instances must be related to \( \text{Academic} \) instances. Therefore, assuming that \( G \) and \( A \) are the number of graduates and academics, respectively, the number of \( \text{student-advisor} \) links in every legal instance must be both, \( G*1 \) and \( A*2 \), implying \( G = A*2 \). In addition, the extensions of \( \text{Graduate} \) and \( \text{Academic} \) must satisfy \( G \leq A \), since \( \text{Graduate} \) is a subclass \( \text{Academic} \). These constraints can be satisfied only by empty or infinite extensions. Figure 16b presents a finite satisfiability problem which is a reduced version of the finite satisfiability problem in Figure 1.
**Pattern justification:**
The above examples show that an association–class-hierarchy cycle can cause a finite satisfiability problem. It is left to show that the minimality of the cycle and the “no (0,*) multiplicity constraint” are necessary conditions. As in the previous pattern, the proof relies on the identification graph of the cycle (Maraee et al., 2008). The identification graph is constructed in two steps:

1. First, every class hierarchy constraint in the cycle is replaced by an association with multiplicity constraints (0, 1) and (1, 1) on the sub-class and super-class sides, respectively. The intuition is that every object of the sub-class is also an object of the super-class, but not necessarily vice-versa. Maraee & Balaban (2007) have shown that this transformation preserves the consistency and finite-satisfiability properties.

2. This replacement creates a plain association cycle, for which the identification graph can be constructed, following Lenzerini & Nobili (1990). For this cycle, the PMC pattern already justifies the “no (0,*) multiplicity constraint” requirement.

Since the class hierarchy to association translation does not insert a (0,*) multiplicity constraint, we conclude that if the original association cycle is minimal and does not include a (0,*) multiplicity constraint, the cycle might cause a finite satisfiability problem, whilst the presence of a (0,*) multiplicity constraint guarantees that no finite satisfiability problem can be caused by the cycle.

**Pattern verification:**
In order to find out whether an MHC may cause a finite satisfiability problem, there is a need to check the conditions in Claim 2 described above.

**Repair advice:**
Consider relaxation of the multiplicity constraints, as in the PMC repair advice. A different advice might be to switch the direction of a class hierarchy constraint in the cycle. The latter repairs the finite satisfiability as shown in the following proposition.

**Proposition 3:** A minimal cycle of association with multiplicity constraints and class hierarchy constraints in opposite direction, cannot introduce a finite satisfiability problem.
**Proof:** Every class hierarchy constraint introduces a "\(\infty\)" label edge in the identification graph directed from the subclass to the super class. Based on Proposition 2, there cannot be critical cycles.

Applying this advice to Figure 16b yields Figure 17, in which the direction of the CatalyzedReaction-Reaction hierarchy is switched (which is semantically correct), and does not have a finite satisfiability problem.

![Figure 17. A repaired class diagram of Figure 16b](image)

3. **Interaction of multiplicity constraints via an association class constraint (condensed):**

Figure 18 presents an association class Contract that is constrained by contradictory multiplicity constraints. In every legal instance, the number \(C\) of contracts is as twice as the number of employees, \(E\), since every employee is linked to two departments. Yet, the number of contracts is equal to the number of employees, as dictated by the manages association. That is, \(C = E \times 2\), and \(E = C\), implying \(E = E \times 2\) which can be achieved only by either empty or infinite extensions for all three classes. This example shows that the interaction of multiplicity constraints via an association class can cause a finite satisfiability problem.

![Figure 18. Non-finite satisfiability due to multiplicity and association class constraints](image)

4. **Interaction of multiplicity and Generalization Set constraints (condensed):**
In Figure 19 the \{disjoint, incomplete\} constraint suggests that the \textit{DNASegment} extension properly includes the \textit{Gene}, \textit{Chromosome} and the \textit{Genome} extensions. Yet, \textit{DNASegment} instances are mapped in a 1:1 manner to \textit{Genome} instances, implying that the sets have the same size. The only solution for proper set inclusion with equal size is that the sets are either empty or infinite.

![Diagram of set constraints](image)

**Figure 19. Non-finite satisfiability due to multiplicity and generalization set constraints**

5. \textit{Interaction of multiplicity and inter-association hierarchy constraints (condensed)}:

In Figure 20 the subset constraint between the \textit{largeComposition} and the \textit{composition} associations tightens the multiplicity constraint of \textit{Member} in \textit{largeComposition} into 2..3, which causes a finite satisfiability problem. For the \textit{management} association, if \(M, L, Man\) are the number of instances of \textit{Member}, \textit{large} and \textit{management}, respectively, then \(Man = L*4, Man = M*1\) imply \(M = L*4\). For the \textit{largeComposition} association, if \(LC\) is the number of its links, then \(L*2 \leq LC \leq L*3, LC = M*1\) imply \(L*2 \leq M \leq L*3\). Replacing \(M\) by \(L*4\) yields the inequality \(L*2 \leq L*4 \leq L*3\), that can be satisfied only by either empty or infinite extensions for \textit{Large}, and \textit{Member}.

![Diagram of association constraints](image)

**Figure 20. Non-finite satisfiability due to class hierarchy, association hierarchy, and multiplicity constraints**

6. \textit{Lack of finite satisfiability due to acyclic structure of aggregation and composition (condensed)}:
The asymmetric property of aggregation/composition requires that a part of an assembly cannot aggregate one of its aggregators. Consequently, a legal instance of a class diagram cannot include aggregation cycles. Figure 21 describes a class whose instances aggregate instances of the same class.

The whole-part association defines a one-to-one function from the set Origin to itself, which maps an element of Origin to its single part. If the extension of Origin is a finite non-empty set, the mapping must be cyclic. Therefore, Origin can have only empty or infinite extensions.

![Figure 21. Unsatisfiability due to the asymmetry of aggregation](image)

Wahler et al. (2009) present the No-Cyclic-Dependency pattern (NCD pattern), which prevents cyclic links between objects of classes within a path of associations. The authors show that in order to satisfy this constraint, it is sufficient that two association ends in opposite direction in the association cycle have a zero minimal multiplicity as shown in Figure 22. A repair advice for this pattern can be based on Wahler's NCD pattern.

![Figure 22. An association cycle that satisfies Wahler’s NCD pattern](image)

### Inconsistency patterns (condensed):

1. **Contradictory disjoint generalization set constraints:**

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6 Constraint patterns are parameterized pattern expressions that can be instantiated to form a specific constraint (Ackermann & Turowski, 2006; Wahler 2008).
Figure 23 presents the diamond class hierarchy, in which the disjoint constraint enforces the MacroMolecule class to be empty.

![Figure 23. Inconsistency due to a generalization-set constraint: disjoint](image)

2. **Contradictory disjoint and complete generalization set constraints:**

In Figure 24 the interaction of the disjoint and the complete constraints forces class $B$ to be empty, since an instance of $E$ must be an instance of $C$ or $D$, which are disjoint from $B$.

![Figure 24. Inconsistency due to generalization-set constraints: disjoint and complete](image)

3. **Interaction of redefinition and disjoint constraints:**

In Figure 25, a common instance $e$ of classes $E$ and $F$ must be $r$-related to an instance $x$ of class $A$. But, since $e$ is an object of $F$ and $b1$ redefines $r1$, $x$ must be an instance of $B$. However, $e$ is also an instance of $E$, and since $c1$ redefines $r1$, $x$ is also an instance of the $C$, which violates the disjoint constraint (Costal & Gómez, 2006). Therefore, the interaction of the disjoint and the
redefinition constraints implies that classes $F$ and $E$ must be disjoint. Consequently, in every legal instance, class $G$ must be empty.

Patterns of Redundant and Incomplete Design (condensed)

Redundancy means that the specification can be simplified without affecting its meaning. For example, classes or associations that have the same extension in all instances are redundant. Values that cannot be realized in multiplicity constraints are redundant. In the first case, one of the equivalent elements can be removed. In the latter case, the multiplicity value range can be tightened.

Incomplete design means that implied constraints do not appear in the diagram. It reflects a lack of awareness on the designer’s part, and therefore shows low design quality. In some cases, incomplete design can prevent the detection of correctness problems, and consequently be rejected by modeling tools. Some problems of class diagram redundancy and implicit consequences are identified and discussed by Berardi et al. (2005) and Costal & Gomez (2006).

1. Redundancy due to equivalent classes:

Class redundancy occurs mainly due to class hierarchy. In Figure 26 the class $D$ extension is a subset of $A$ extension due to the class hierarchy constraint, and the sets have the same size, since
$A$ instances are mapped in a 1:1 manner to $D$ instances. If the sets are finite, they must be equal, and therefore the $D$ class is redundant and can be removed.

2. **Redundancy due to equivalent associations**

This case is similar to the previous pattern. Association equivalence occurs due to hierarchy of association classes. In Figure 27, $R$ and $Q$ are associations that constrain the association classes, $P$ and $S$, respectively, and are also constrained by a 1:1 multiplicity constraint. Therefore, $P$ and $S$ have equal extensions in all legal instances, and associations $R$ and $Q$, and one of the association classes $P$ and $S$ are redundant.

3. **Redundancy of multiplicity constraint values:**

In Figure 28, the multiplicity constraint 1..* of the $Member$ class in the $LargeComposition$ association is too loose. The subset constraint between the $LargeComposition$ and the $Composition$ associations, and the 1..3 multiplicity constraint of $Member$ in $Composition$, imply that the maximum multiplicity constraint of $Member$ in $LargeComposition$ can be 3.
4. **Incomplete design:**

Figure 29 shows an incomplete design: The semantics of the *subsets* constraint on association ends requires that it appears on both ends of an association. A modeling tool might add derived constraints so to provide a more precise model.

**SUMMARY**

In this paper we have analyzed correctness problems in UML class diagrams, surveyed existing approaches for handling correctness of UML class diagrams, and proposed a pattern-based approach for identifying correctness problems and for providing explanations and repair advices. The proposed approach aims to bridge the gap between current correctness handling methods and desirable explanations and advice. Our work is motivated by the belief that correctness management is essential for supporting advanced IDE and CASE tools in all application areas of UML class diagrams. In addition, the emerging model driven development approach requires reliable models that are equipped with powerful reasoning capabilities for assuring high quality models.

We intend to develop an on-line catalog of incorrectness patterns. The catalog will accumulate knowledge regarding design problems. The intention is to use the patterns for
detecting design problems, as well as for educational purposes. We intend to further develop the pattern-based approach presented here, examine its computational complexity and programmatic applicability and combine it within an implementation of existing cause identification methods.

We envision that the next generation of modeling tools will apply some correctness management facilities. Tools will employ a mixture of reasoning methods, applying simple scalable methods in an incremental way, and resorting to heavy translation-based reasoning when other methods fail. Our ultimate goal is to develop a model level IDE that combines the patterns catalog within its modeling support.

REFERENCES


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