Size-Change Termination Analysis in k-Bits

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At the Bottom Line

- Data-structure for large sets of size change graphs
- Set-based operations: union, composition, and test (for termination condition)

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Plan of the Talk

- Introduction to size-change termination
- A Constraint view (sets of solutions)
- From graphs to sets of graphs
- Representation using BDD's

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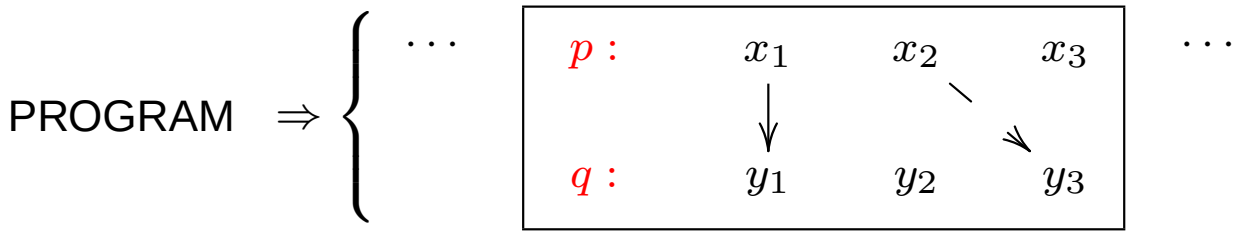
Approximation of Termination

Programs are mapped to sets of size change graphs



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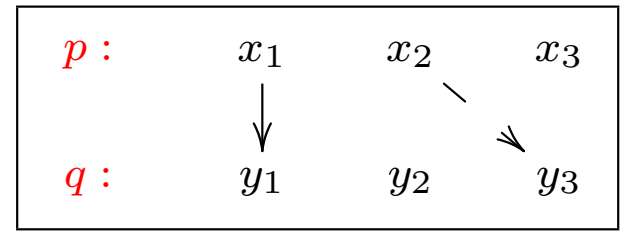
PROGRAM $\Rightarrow$ { $\cdots$  $\cdots$

Size-Change Graphs

$x_i > y_j, x_i \geq y_j,$

$x_i \in \bar{x}, y_j \in \bar{y}$

(Lee,Jones,Ben-Amram 2001)

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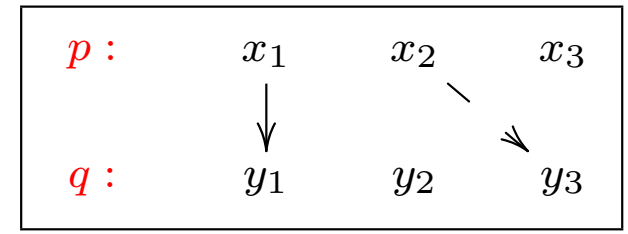
PROGRAM $\Rightarrow$ { $\cdots$  $\cdots$

Size-Change Graphs	Monotonicity Constraints
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(Lee,Jones,Ben-Amram 2001)	(Lindenstrauss,Sagiv 1989)

$p(\bar{x}) \leftarrow x_1 > y_1, x_2 \geq y_3,\ q(\bar{y}).$

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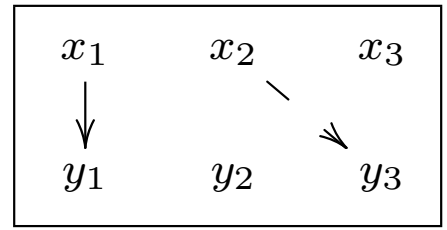
PROGRAM $\Rightarrow$ { $\cdots$  $\cdots$

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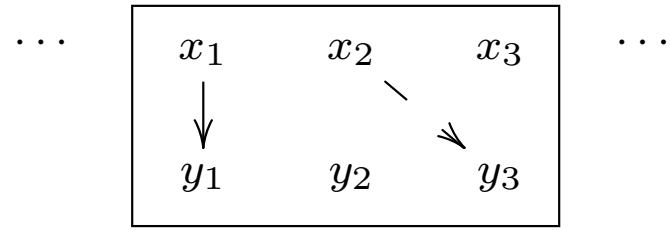
PROGRAM $\Rightarrow$ 

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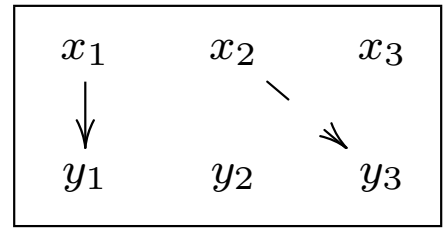
PROGRAM $\Rightarrow$ { $\cdots$  $\cdots$

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$$p(\bar{x}) \leftarrow x_1 > y_1, x_2 \geq y_3,\ p(\bar{y}).$$

- For size change graphs (and monotonicity constraints) termination is decidable.
- It is not about solving the Halting Problem!

Size-Change Graphs by Example

```
int  Ack(int x, int y) {
        if (x==0) return y + 1
        else if (y==0) return Ack(x − 1, 1)
        else return Ack(x − 1, Ack(x, y − 1))
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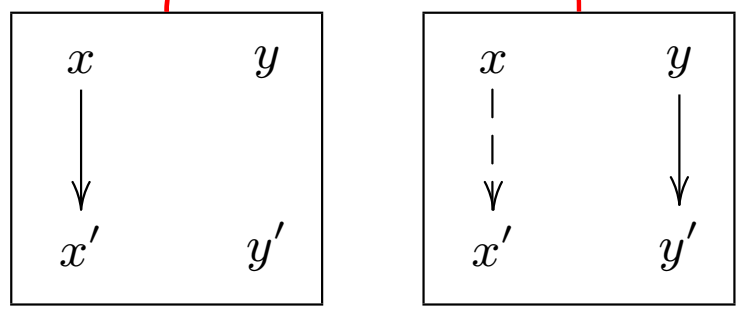


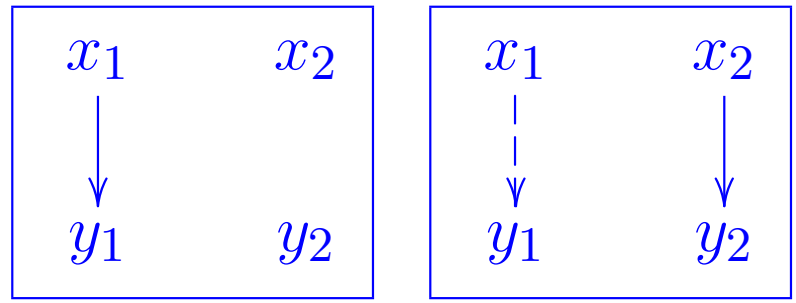
Proving Termination

Classic approach – find a *ranking function* f:

$\exists f\ \forall loop.\ f$ decreases on the $loop$

$Ack(x_1, x_2)$

$f(x, y) = \langle x, y \rangle$



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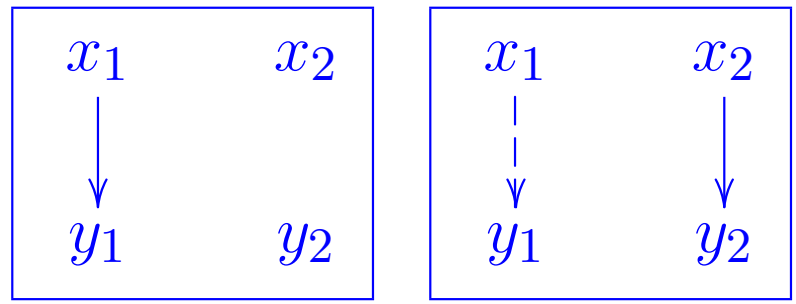

Size change termination – one loop at a time

- for a "price" it is sufficient to prove termination for each individual loop
- there is also a "prize": simpler termination conditions and easier to automate

One Loop at a Time

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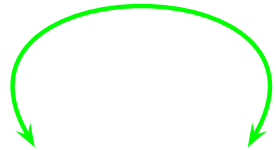
One Loop at a Time



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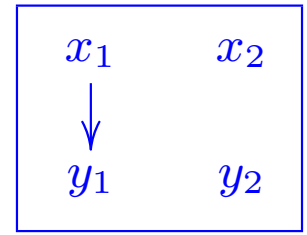

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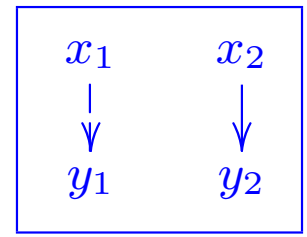

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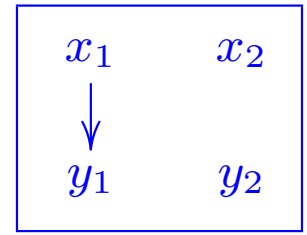

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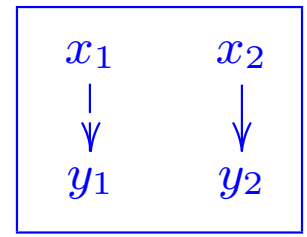

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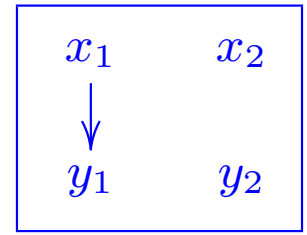

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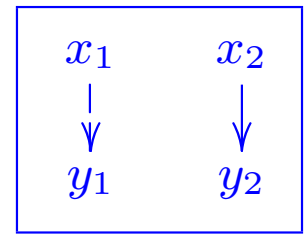

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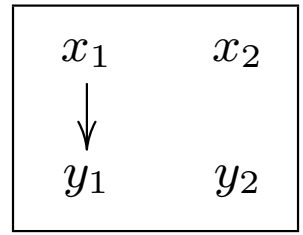

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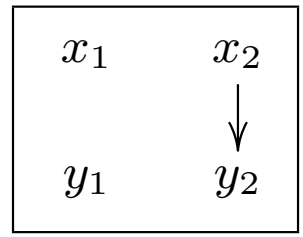

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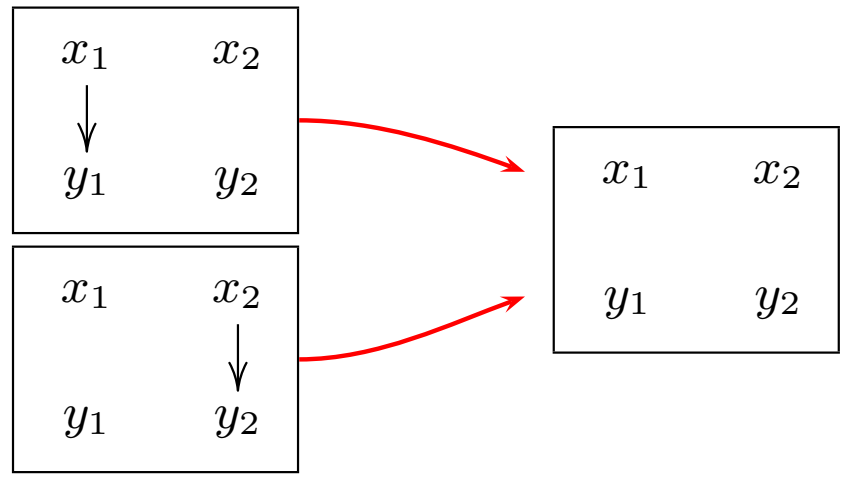

- The composition $\mu_1(\bar{x}, \bar{y}) \circ \mu_2(\bar{x}, \bar{y})$ is a size-change graph entailed by $\exists \bar{z}.\mu_1(\bar{x}, \bar{z}) \wedge \mu_2(\bar{z}, \bar{y})$.
- Loop descriptions are closed under composition:

$$\mu_1 \in G^* \wedge \mu_2 \in G^* \ \Rightarrow\ \mu_1 \circ \mu_2 \in G^*$$

Idempotence

A size-change graph μ is idempotent if $\mu \circ \mu = \mu$.

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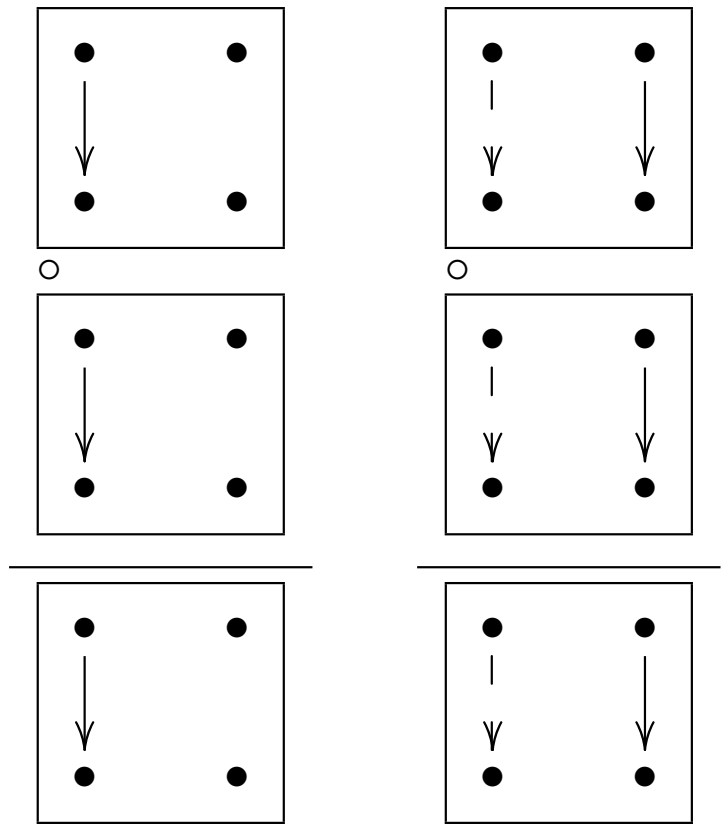
Example: the two graphs of $Ack(x, y)$ are idempotent



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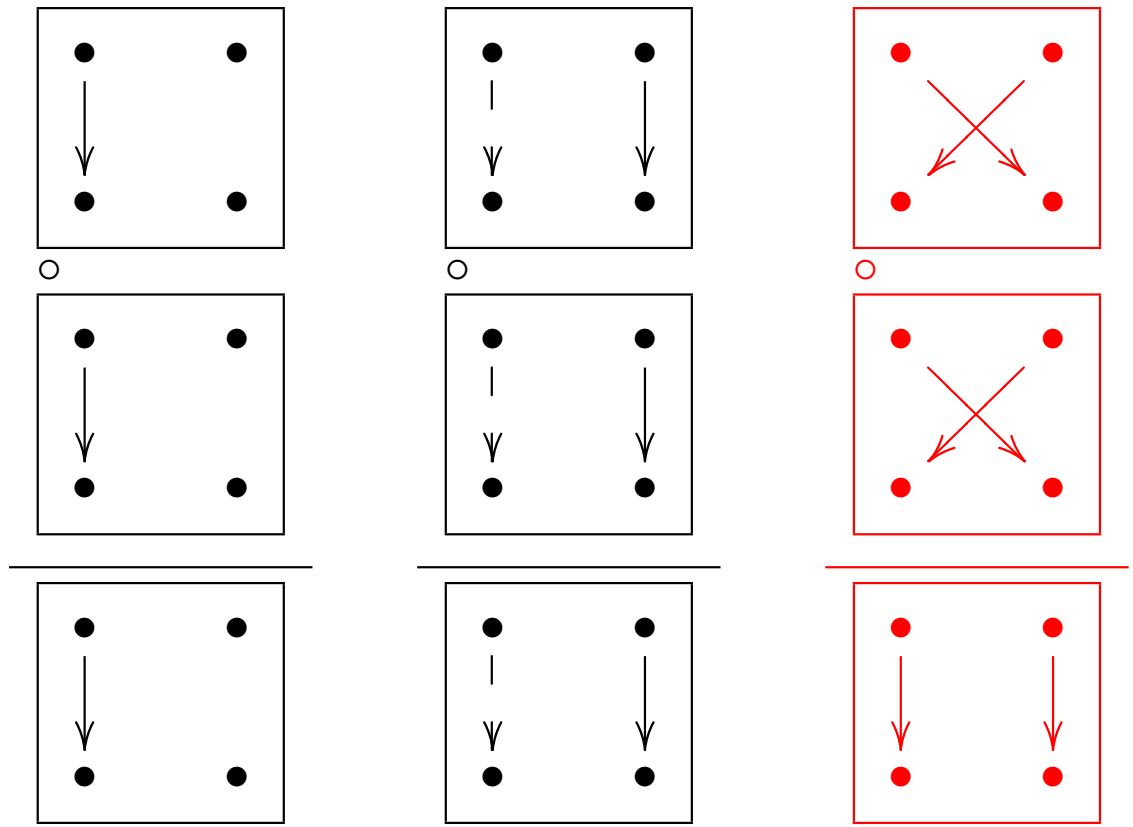
Example: a non-idempotent graph



Correctness of Local Approach

Let G be a set of size-change graphs. If every recursive (idempotent) $\mu \in G^*$ has a ranking function then any program described by G terminates. (Dershowitz *et al*, 2001) (Lee,Jones,Ben-Amram 2001)

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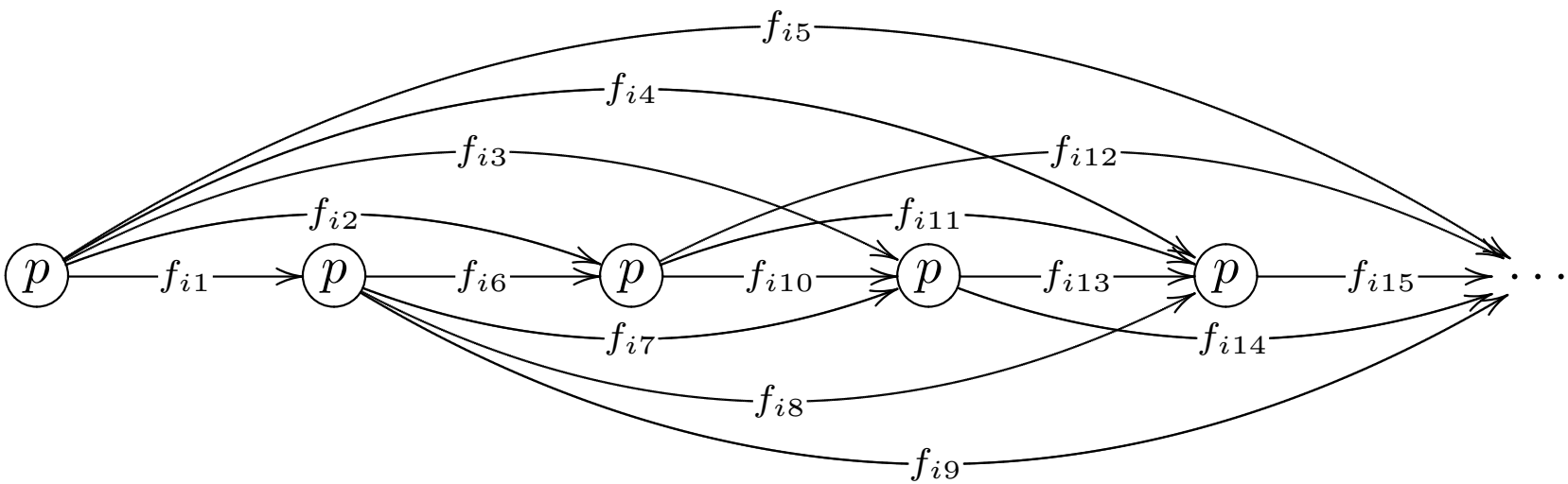

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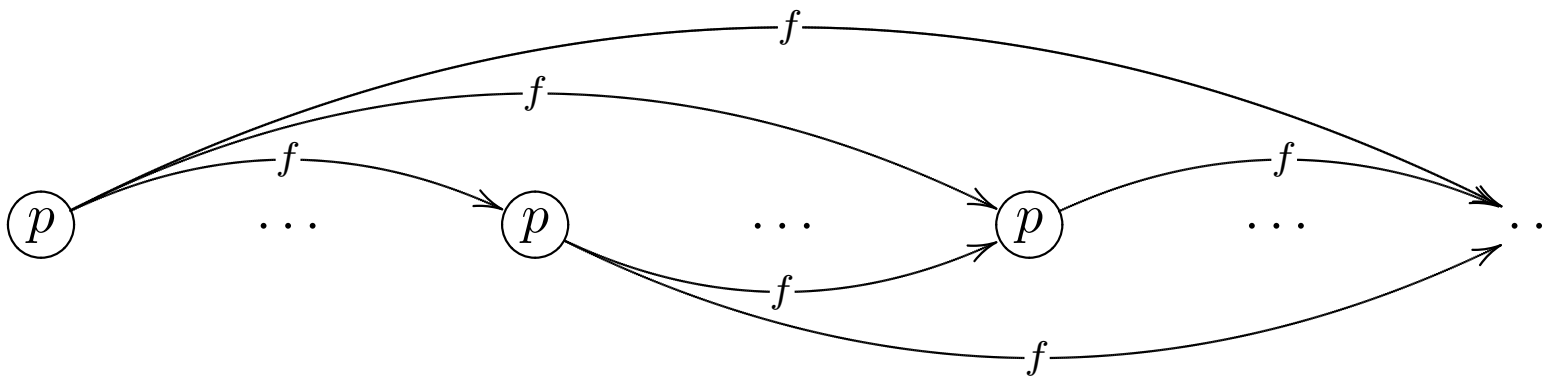

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Completeness for Size-Change Graphs

For an idempotent SCG if there is any ranking function f then there is one of the form $f(\bar{x}) = x_i$. (Lee *et al*, 2001)

The termination algorithm:

1. Compute the closure G^*
2. Compute the subset of idempotent graphs $I \subseteq G^*$
3. For each $\mu(\bar{x}, \bar{y}) \in I$ check that $\bigvee_i (\mu(\bar{x}, \bar{y}) \rightarrow (x_i > y_i))$

Example: $Ack(x, y)$ is terminating

$$I = G^* = \left\{ \begin{array}{|cc|} \hline x & y \\ \downarrow & \\ x' & y' \\ \hline \end{array} \quad \begin{array}{|cc|} \hline x & y \\ \vdots & \downarrow \\ x' & y' \\ \hline \end{array} \right\}$$

Completeness for Size-Change Graphs

For any SCG if there is a ranking function f then there is one of the form $f(\bar{x}) = \sum_{i \in I} x_i$. (Codish *et al*, ICLP2005)

The termination algorithm:

1. Compute the closure G^*
2. For each $\mu(\bar{x}, \bar{y}) \in G^*$ check that $\mu(\bar{x}, \bar{y}) \models \bigvee_i (x_i > y_i)$

Example:

$$G = \left\{ \begin{array}{|cc|} \hline x_1 & x_2 \\ \searrow & \swarrow \\ y_1 & y_2 \\ \hline \end{array} \right\} \qquad G^* = \left\{ \begin{array}{|cc|} \hline x_1 & x_2 \\ \searrow & \swarrow \\ y_1 & y_2 \\ \hline \end{array} \quad \begin{array}{|cc|} \hline x_1 & x_2 \\ \downarrow & \downarrow \\ y_1 & y_2 \\ \hline \end{array} \right\}$$

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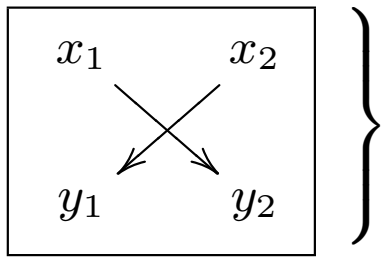

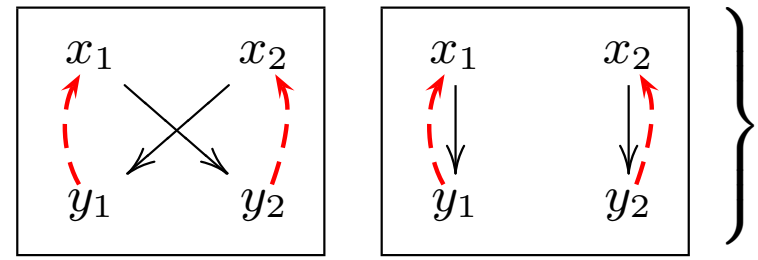

$$\mu(\bar{x}, \bar{y}) \models \bigvee_i (x_i > y_i) \;\equiv\; \neg\Big(\mu(\bar{x}, \bar{y}) \wedge \bigwedge_i (x_i \leq y_i)\Big)$$

Test for Ranking Function

$\bigvee_i \mu(\bar{x}, \bar{y}) \models (x_i > y_i)$ idempotent graphs (Lee *et al*)

$\mu(\bar{x}, \bar{y}) \models \bigvee_i (x_i > y_i)$ all graphs (Codish *et al*)

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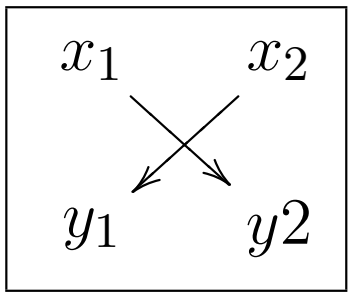

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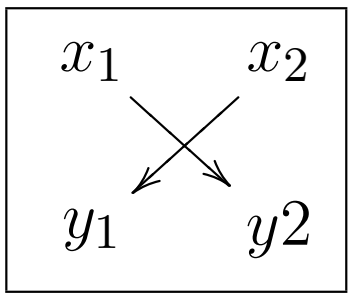

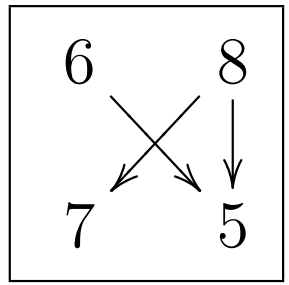

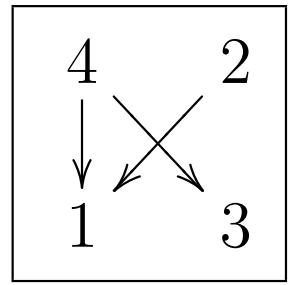

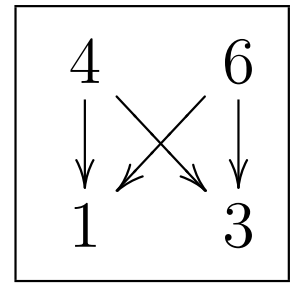

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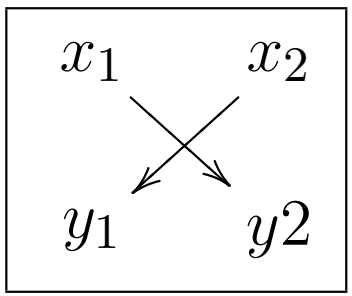

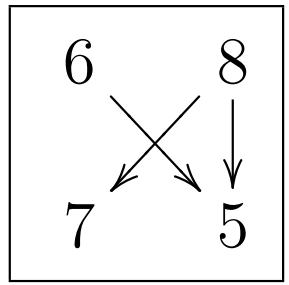

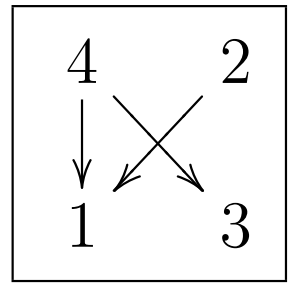

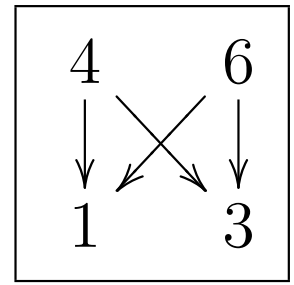

- Later it facilitates set-based test for termination.
- We have no set-based operation for idempotent graphs.

The Downside

- Size change termination is complete for PSPACE (Lee *et al* 2001). Closure under composition may introduce an exponential number of additional size change graphs.

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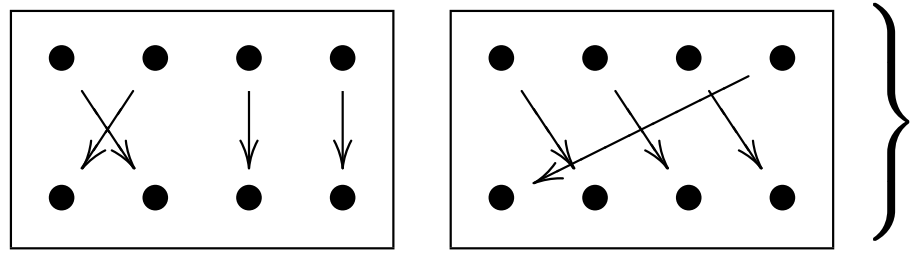

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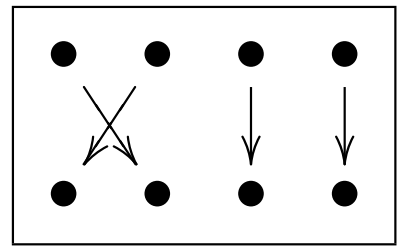

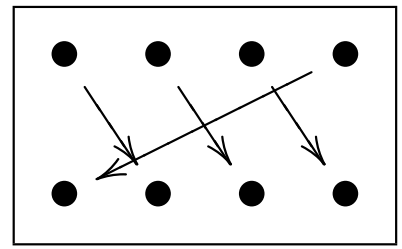

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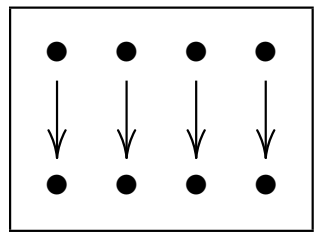

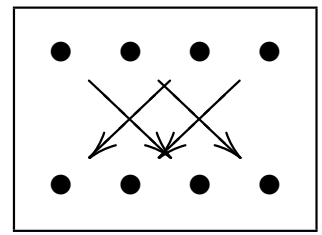

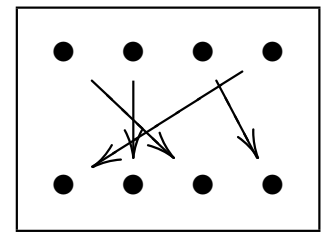

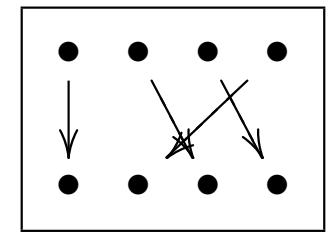

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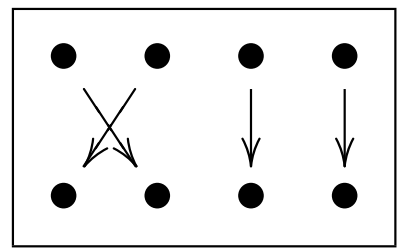

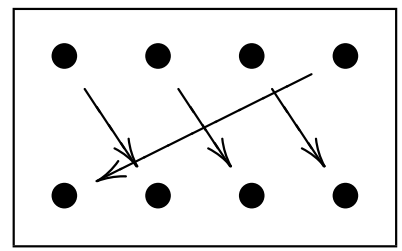

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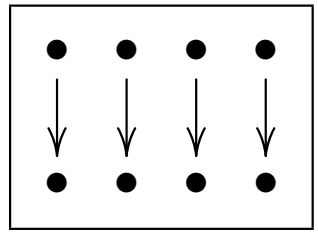

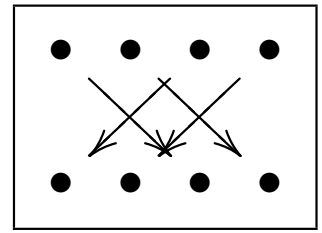

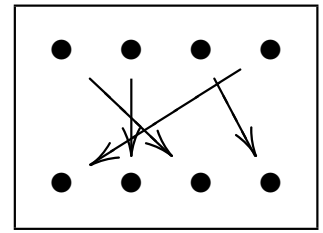

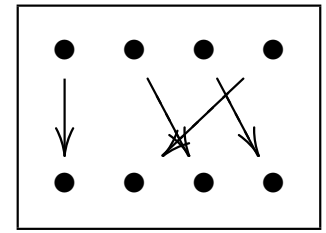

$\bullet \quad \bullet \quad \bullet$

At the Bottom Line

- Data-structure for large sets of size change graphs
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From Graphs to Sets: Union & Test

Union as disjunction —

$$\frac{\left\{\begin{array}{c} p(\bar{x}) \leftarrow \mu_1(\bar{x},\bar{y}),\ p(\bar{y}) \\ \vdots \\ p(\bar{x}) \leftarrow \mu_n(\bar{x},\bar{y}),\ p(\bar{y}) \end{array}\right\}}{p(\bar{x}) \leftarrow \bigvee_k \mu_k(\bar{x},\bar{y}),\ p(\bar{y})}$$

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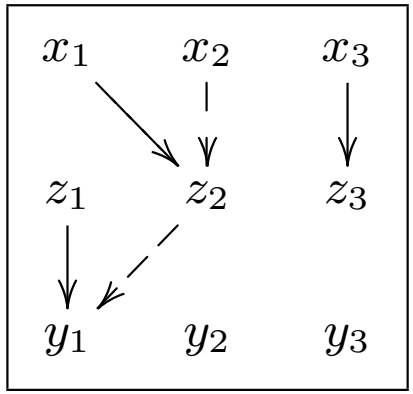
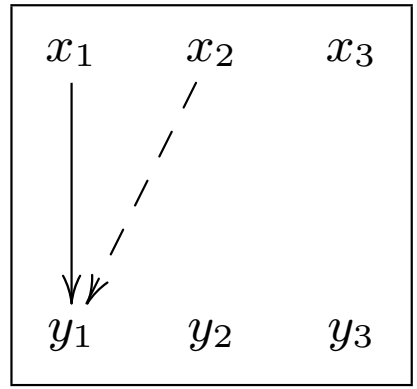
- What about pairwise compositions?

From Graphs to Sets: Composition

Composition as conjunction?? —





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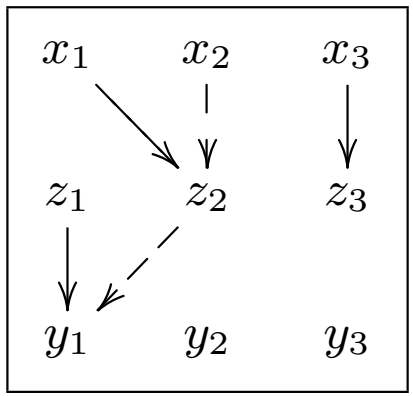

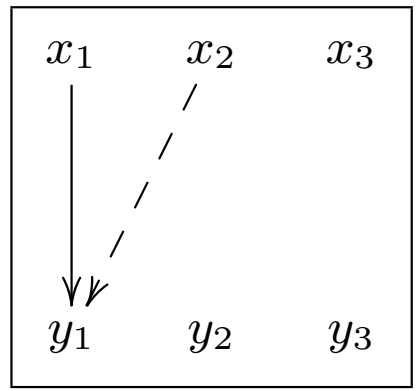

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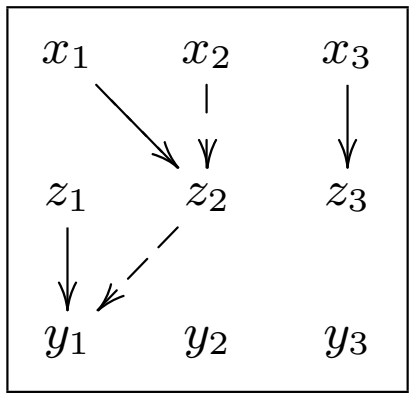

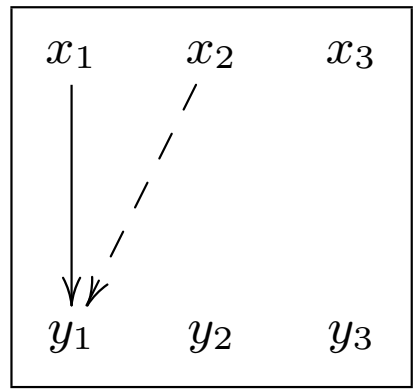

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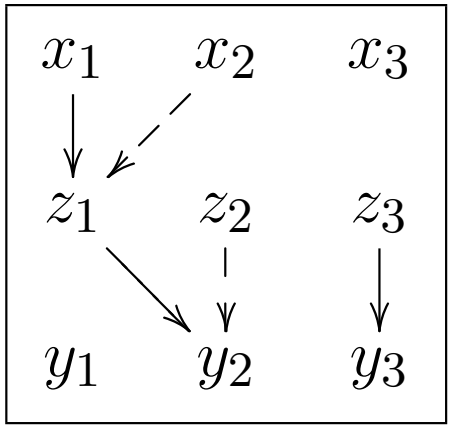

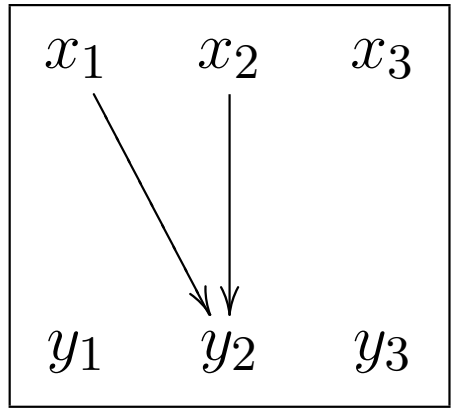

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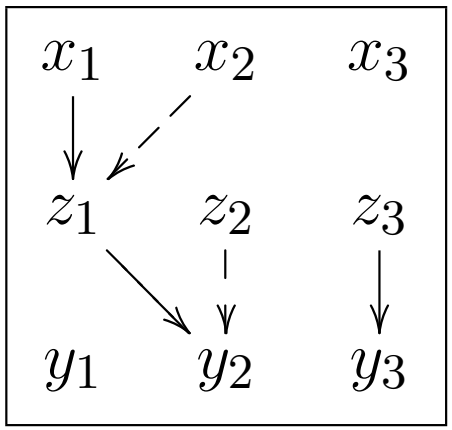

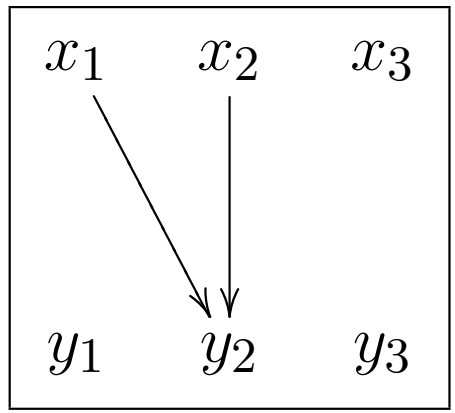

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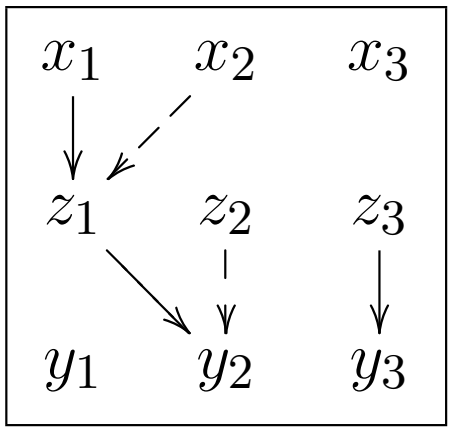

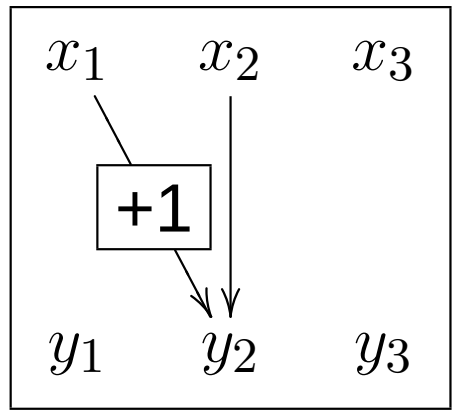

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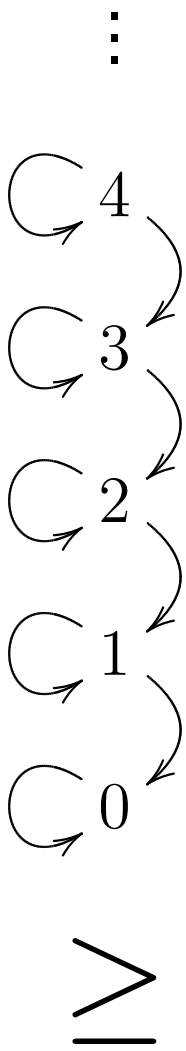

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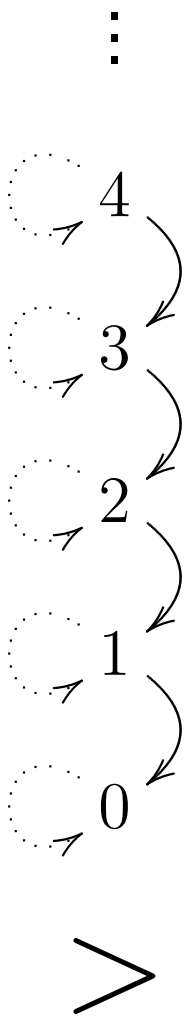

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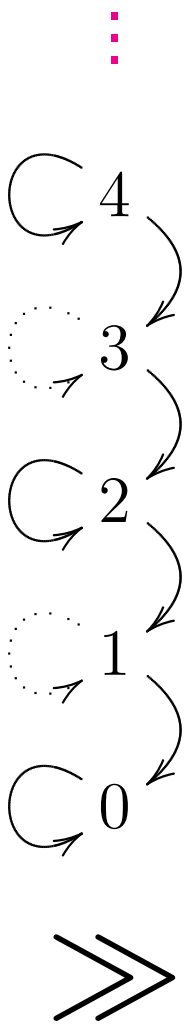

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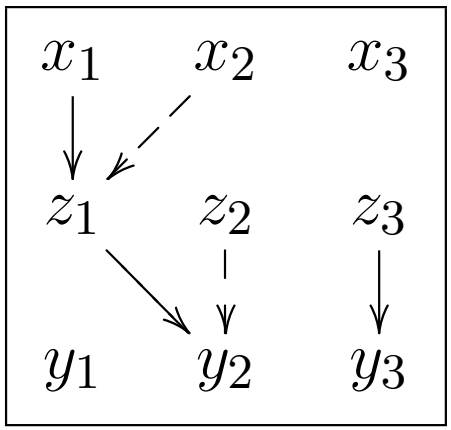

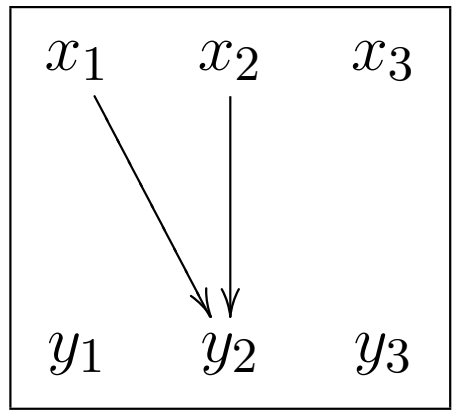

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Set-based Operations

We now have all of the required set-based operations

- Let G be a set of scg's of the form $p(\bar{x}) \leftarrow c_i(\bar{x}, \bar{y}),\ p(\bar{y})$ and $C(\bar{x}, \bar{y}) = \vee_i c_i(\bar{x}, \bar{y})$
- To compute the closure G^* as the fixed point of $G \cup (G \circ G)$
- Compute instead the fixed point of $C(\bar{x}, \bar{y}) \vee \exists \bar{z}.C(\bar{x}, \bar{z}) \wedge C(\bar{z}, \bar{y})$
- Apply test for termination

Towards a Constraint Solver

- Using general purpose constraints solvers such as CLP(R): maintain the set of disjuncts

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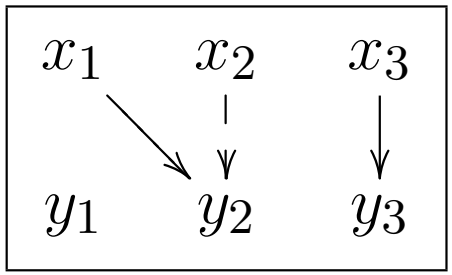

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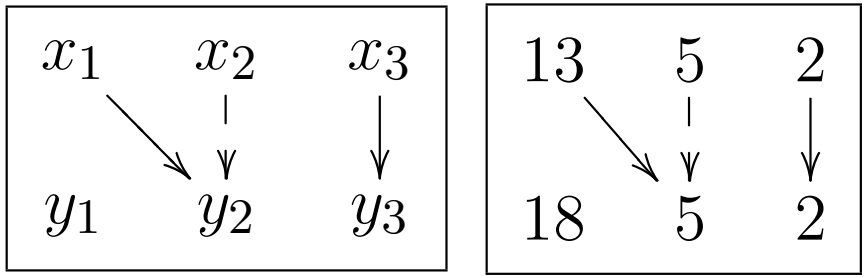

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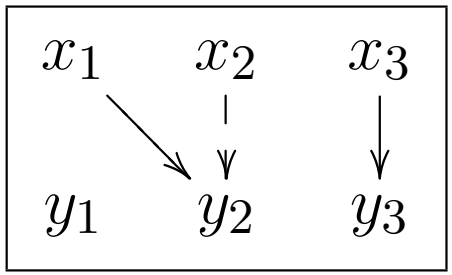

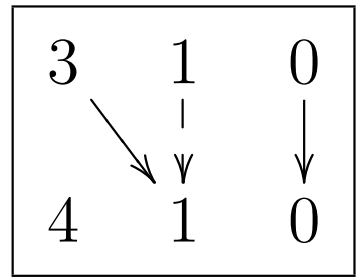

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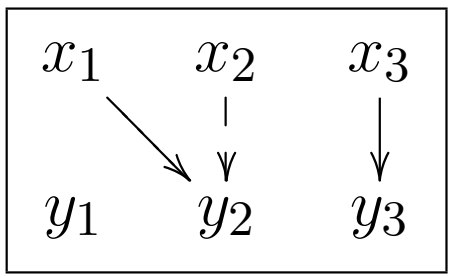

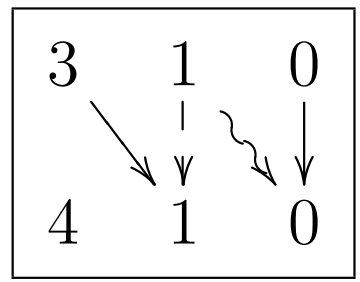

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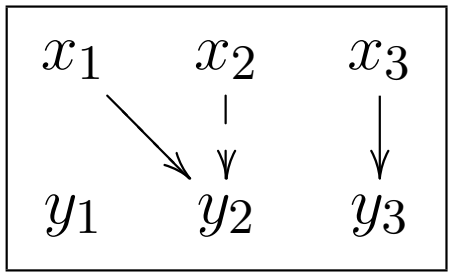

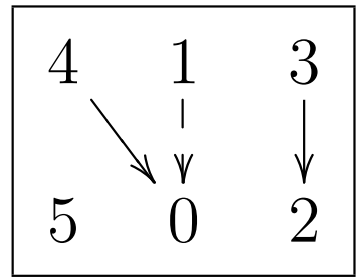

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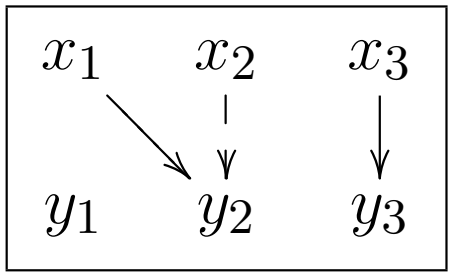
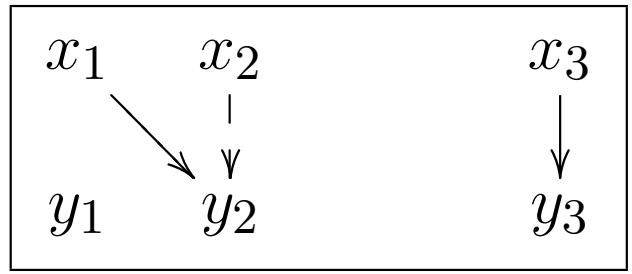


- In practice often less



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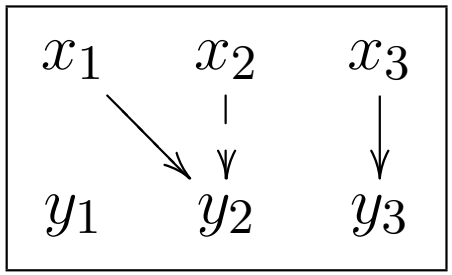

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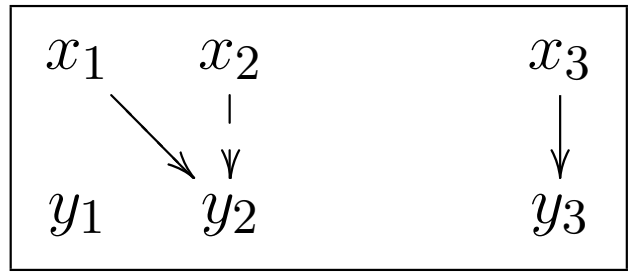

Number of nodes in largest partition

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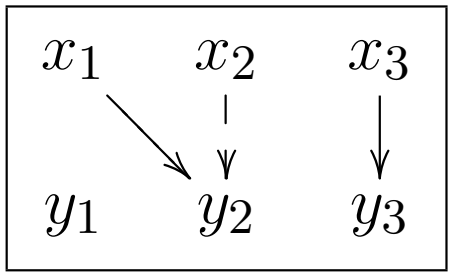

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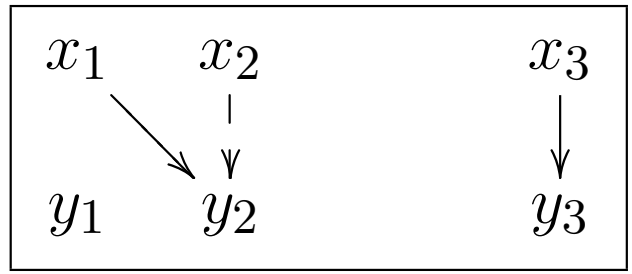

Number of nodes in largest partition

- There are subtleties (remember $\gg$) — But it works.

And in k Bits?

- Each variable x and y is represented as $x = x_k x_{k-1} \cdots x_0$ and $y = y_k y_{k-1} \cdots y_0$;
- Each constraint $x \geq y$ and $x \gg y$ is encoded for k bits.
- (example for $k = 2$):
  $x_1 x_0 \geq y_1 y_0 \equiv (x_1 \wedge \neg y_1) \vee ((x_1 \leftrightarrow y_1) \wedge (x_0 \wedge \neg y_0))$
- Similar for $\gg$

From k Bits to BDD's

All components for size change termination analysis are represented as Boolean formula and standard Boolean operations. This facilitates an implementation based on well-studied data structures and well-engineered tools for representing and manipulating Boolean formulae.

At the Bottom Line

- Data-structure for large sets of size change graphs
- Set-based operations: union, composition, and test (for termination condition)
- Prototype is implemented using standard BDD package

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Related Work: Size change termination in Polynomial time; Ben-Amram and Lee. They show conditions which enable to avoid performing the expensive closure operation.

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Future Work: Apply to richer domains: δ-SCG's and others.

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- Set-based operations: union, composition, and test (for termination condition)
- Prototype is implemented using standard BDD package