Homework assignment no. 4

1. Let P be a set of n points in the interior of an axis-parallel rectangle R. Assume that the points in P represent the houses in some living neighborhood R. One needs to determine the best location for a garbage dump in R. That is, one needs to find a point $g \in R$ that maximizes the expression $\min_{p \in P} \mathrm{dist}(p, g)$. Give an $O(n \log n)$-time algorithm for finding such a point g.

2. The Gabriel graph of a set $\mathcal{P}$ of points in the plane consists of all edges pq, $p, q \in \mathcal{P}$, such that the circle with diameter pq does not contain any point of $\mathcal{P}$ in its interior.

   (a) Prove that the Delaunay triangulation of $\mathcal{P}$ contains the Gabriel graph of $\mathcal{P}$.

   (b) Prove that pq is an edge of the Gabriel graph if and only if pq intersects the Voronoi edge between $\mathrm{Vor}\,(p)$ and $\mathrm{Vor}\,(q)$.

   (c) Show that the Gabriel graph can be computed in $O(n \log n)$ time.

3. Let S be a set of n (axis-aligned) squares in the plane. We wish to find a piercing set for S of minimum cardinality. That is, we wish to find a set of points P, such that (i) each square $s \in S$ is pierced by at least one point in P (i.e., $s \cap P \neq \emptyset$), and (ii) P is as small as possible (i.e., $|P| \leq |Q|$, for any other piercing set for S).

   Alma suggested the following algorithm: Let Q be an initially empty set. As long as S is non-empty do the following: Let $s_{\min} \in S$ be the smallest square in S, add the 4 corners of $s_{\min}$ to Q and remove $s_{\min}$ and all the squares in S that intersect $s_{\min}$ from S.

   Prove that (i) The set Q returned by Alma's algorithm is a piercing set for S.
   (ii) $|Q| \leq 4|P|$, where P is a minimum-cardinality piercing set for S.

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