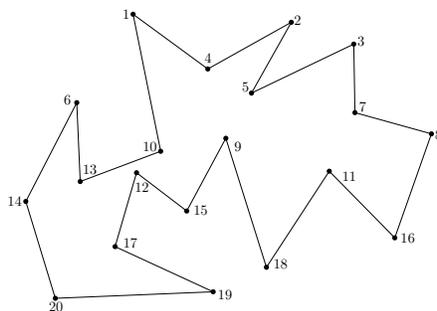


## Homework assignment no. 2

1. (a) Draw a polygon  $P$  and place guards in it, such that the guards cover the boundary of  $P$ , but there exists a point in the interior of  $P$  that is not seen by any of the guards.  
 (b) Define a family of polygons  $P_6, P_8, P_{10}, \dots$ , such that  $P_k$  has  $k$  vertices and there is a way to place  $k/2$  guards at every other vertex of  $P_k$  so that *not* every point in  $P_k$  is seen by a guard.
2. List the diagonals that will be added to the polygon below by the algorithm for partitioning a polygon into  $y$ -monotone pieces.



3. A simple polygon  $P$  is *star-shaped* if there exists a point  $c \in P$ , such that for every point  $p \in P$  the line segment  $\overline{cp}$  is contained in  $P$ . The point  $c$  is called a *center point* of  $P$ .  
 Let  $P$  be a star-shaped polygon with  $n$  vertices, and let  $c$  be a center point of  $P$ . Show that, after linear-time preprocessing, one can determine in  $O(\log n)$  time whether a query point  $q$  lies in  $P$ .
4. Prove that the query time of a three-dimensional kd-tree is  $O(n^{2/3} + k)$ .
5. Let  $P$  be a set of  $n$  points in the plane. Describe a compact data structure for queries of the following type: Given an axis-parallel rectangle  $R$ , return the highest point among the points of  $P$  that lie in  $R$ . (What is the size of the data structure and how fast can it handle a query?)
6. Let  $R$  be a set of  $n$  axis-parallel rectangles in the plane. We would like to be able to report all rectangles in  $R$  that are *fully* contained in a query axis-parallel rectangle. Describe a data structure of size  $O(n \log^{c_1} n)$  that supports such queries in time  $O(\log^{c_2} n + k)$ , where  $c_1, c_2$  are constants and  $k$  is the number of reported rectangles.

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