Homework assignment no. 1

1. Prove that the problem of computing the convex hull of a set of n points in the plane has an $\Omega(n \log n)$ lower bound. Hint: Show that a set of n real numbers can be sorted in time $O(n)$, plus the time needed for a single convex hull computation.

2. A *slab* is a region of the plane that is defined by two parallel lines. Its *width* is the distance between the lines defining it. Let P be a set of n points in the plane. The *width* of P is the width of a minimum-width slab that contains P. Describe an $O(n \log n)$-time algorithm for computing the width of P. (Hint: Show that the width of P is determined by a pair of parallel lines supporting the convex hull of P, where at least one of them contains an edge of the convex hull.)

3. Let P be a set of n points in the plane. A point $p \in P$ is *h-maximal*, for $0 \leq h \leq n-1$, if the number of points in P which are both above and to the right of p is at most h. p is *maximal* if it is 0-maximal.
   (i) Give a four-point example P that has both a maximal point in the interior of $CH(P)$, and a vertex of $CH(P)$ that is not maximal.
   (ii) Describe an $O(n \log n)$-time algorithm to compute the h-maximal points of P, where $|P| = n$ and h is a non-negative integer.

4. Let C_1 and C_2 be two convex polygons, each with n vertices. (Each polygon is given by the sequence of its vertices in clockwise order.)
   (i) Show that the number of intersection points between edges of C_1 and edges of C_2 is $\Theta(n)$.
   (ii) Describe an $O(n)$-time algorithm for computing all these intersection points.

5. Let S_1 be a set of n disjoint horizontal segments, and let S_2 be a set of n disjoint vertical segments. Describe an $O(n \log n)$-time algorithm for *counting* the number of intersections in $S_1 \cup S_2$.

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