Homework assignment no. 2

1. (a) Draw a polygon $P$ and place guards in it, such that the guards cover the boundary of $P$, but there exists a point in the interior of $P$ that is not seen by any of the guards.

(b) Define a family of polygons $P_6, P_8, P_{10}, \ldots$, such that $P_k$ has $k$ vertices and there is a way to place $k/2$ guards at every other vertex of $P_k$ so that not every point in $P_k$ is seen by a guard.

2. Give an efficient algorithm to determine whether a polygon $P$ with $n$ vertices is monotone with respect to some line, not necessarily a horizontal or vertical one. [dBcVko]

3. A simple polygon $P$ is star-shaped if there exists a point $c \in P$, such that for every point $p \in P$ the line segment $cp$ is contained in $P$. The point $c$ is called a center point of $P$.

Let $P$ be a star-shaped polygon with $n$ vertices, and let $c$ be a center point of $P$. Show that, after linear-time preprocessing, one can determine in $O(\log n)$ time whether a query point $q$ lies in $P$.

4. Prove that the query time of a 3-dimensional kd-tree is $O(n^{2/3} + k)$.

5. Let $R = \{R_1, \ldots, R_n\}$ be a set of $n$ axis-parallel rectangles in the plane. We would like to build a football stadium somewhere in $A = R_1 \cup \cdots \cup R_n$. For this purpose, we need an efficient algorithm for finding the largest-area axis-parallel rectangle that is contained in $A$. Describe such an algorithm.

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