Homework assignment no. 1

1. Prove that the problem of computing the convex hull of a set of \( n \) points in the plane has an \( \Omega(n \log n) \) lower bound. Hint: Show that a set of \( n \) real numbers can be sorted in time \( O(n) \), plus the time needed for a single convex hull computation.

2. a. Let \( P_1 \) and \( P_2 \) be two disjoint convex polygons with \( n \) vertices in total. Describe an \( O(n) \) algorithm that computes the convex hull of \( P_1 \cup P_2 \).
   b. Develop an \( O(n \log n) \) algorithm for computing the convex hull of a set of \( n \) points in the plane that is based on the algorithm above.

3. Let \( C_1 \) and \( C_2 \) be two convex polygons with \( n_1 \) and \( n_2 \) vertices, respectively. (Each polygon is given by the sequence of its vertices in clockwise order.) Describe an \( O(n) \)-time algorithm for computing \( C_1 \cap C_2 \), where \( n = n_1 + n_2 \).

4. Let \( S_1 \) be a set of \( n \) disjoint horizontal segments, and let \( S_2 \) be a set of \( n \) disjoint vertical segments. Describe an \( O(n \log n) \)-time algorithm for counting the number of intersections in \( S_1 \cup S_2 \).

5. Let \( l_1, \ldots, l_n \) be \( n \) given lines in the plane, no two parallel and no three meeting at a common point. Let \( S \) be the set of their \( n(n-1)/2 \) intersection points. Give an algorithm for calculating the convex hull of \( S \) in time \( O(n \log n) \). (In particular, you cannot afford to calculate the entire set \( S \).) Prove your claims.