Homework assignment no. 2

1. (a) Draw a polygon P and place guards in it, such that the guards cover the boundary of P, but there exists a point in the interior of P that is not seen by any of the guards.

   (b) A *rectilinear polygon* is a simple polygon whose edges are either horizontal or vertical. Let P be a rectilinear polygon with n vertices. Show that $\lfloor n/4 \rfloor$ guards are sometimes necessary to guard P.

2. A simple polygon P is *star-shaped* if there exists a point $c \in P$, such that for any point $p \in P$ the line segment $\overline{cp}$ is contained in P. The point c is called a *center point* of P.

   (a) Let P be a star-shaped polygon, and let c be a center point of P. Show that, given a query point q, one can determine in $O(\log n)$ time whether q lies in P. Assume that P is given as an array of its n vertices in sorted order along the boundary.

   (b) Give an expected linear-time algorithm to decide whether a simple polygon is star-shaped.

3. Given n inequalities $a_i x + b_i y \geq 1$, for $i = 1, \ldots, n$, describe an expected linear-time algorithm that finds a point (x, y) (if exists) that (i) satisfies all these inequalities, and (ii) is nearest to the origin under the L_1 distance (where $d_1(p, q) = |q_x - p_x| + |q_y - p_y|$).

4. Let R be a set of n axis-parallel rectangles in the plane. We would like to be able to report all rectangles in R that are *fully* contained in a query axis-parallel rectangle. Describe a data structure of size $O(n \log^{c_1} n)$ that supports such queries in time $O(\log^{c_2} n + k)$, where c_1, c_2 are constants and k is the number of reported rectangles.

5. **Bonus:** Let S be a set of n line segments on the x-axis. For a segment $s \in S$, consider the segments in S whose left endpoint lies in s and whose right endpoint lies to the right of s. Let $p(s)$ be the rightmost endpoint among the right endpoints of these segments. Describe an algorithm that finds the point $p(s)$, for each $s \in S$, in total time $O(n \log n)$.

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