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HTS Dynamo Flux Pump: The Impact of a Ferromagnetic Stator Substrate

Vladimir Sokolovsky and Leonid Prigozhin

***Abstract* — HTS dynamo magnetic flux pumps are perspective devices for contactless charging the superconducting magnets and coils. Using a thin shell model of a coated conductor with a ferromagnetic substrate and a Chebyshev spectral method for numerical solution, we show that such a substrate increases the pump-generated voltage if the superconducting layer is between the rotor and the substrate. For the opposite conductor orientation, the voltage decreases. We also demonstrate that the main substrate-depending pump characteristics can be extracted from solutions to problems with a given transport current and then used in a simple analytical description of charging a coil.**

Index Terms — HTS dynamo pump, coated conductor, ferromagnetic substrate, thin shell model, Chebyshev spectral method

I. INTRODUCTION

Electromagnetic induction and nonlinear resistivity of type-II superconductors enable HTS magnetic flux pumps to inject a high DC current into a closed-loop superconducting coil of a magnet and also to continuously compensate for the decay of this current [1-3]. Wireless magnet excitation eliminates excessive cryogenic losses and this is the reason for much recent interest in HTS pumps, see the reviews [4-7]. Dynamo-type pumps, first proposed in [8], have a simple structure: one or several permanent magnets are mounted on a rotating disk and, passing close to a superconducting tape (the stator) induce in it a traveling magnetic field wave generating an output voltage with a nonzero average. HTS dynamos have been intensively investigated experimentally; numerical simulations (see the comprehensive recent review [9]) helped to understand the physical mechanism of voltage generation and the impact on the dynamo pump efficiency of various geometrical factors and field-dependent current-voltage relation for the superconductor.

Although losses in coated conductors with magnetic substrates were studied experimentally and numerically in a number of works (see [10-12] and the references therein), our interest is the pump-generated DC voltage. Experimentally, the influence of a 0.1 mm thick slice of a strongly magnetic permalloy material attached to the superconducting stator tape has been studied in [13]: it was observed that if such hybrid superconductor/ferromagnet structure is oriented towards the rotor by its superconducting side, the generated voltage can be increased by 20-30%, whereas if the orientation is the opposite the voltage decreases. Technologically, it is more convenient to use coated conductors with ferromagnetic substrates. Here we investigate the influence of a magnetic stator substrate on the pump characteristics theoretically, using numerical simulations.

The authors of [13] have also simulated the operation of their pump numerically, using the H-formulation. This formulation is not convenient for modeling thin strip problems. For example, among the numerical methods for modeling the dynamo pump with a nonmagnetic stator substrate in [14], those using the H-formulation have had to artificially increase the superconducting strip thickness from 1 to 100 μm to match the strip finite element mesh with that of the surrounding space. The usage of an integral formulation in [15] resolved the high aspect ratio complication, decreased the problem dimension, and permitted solving this problem numerically tens of times faster. As is well known, the high aspect ratio is a serious obstacle to the efficient modeling of thin ferromagnetic layers too, and H-formulation is not the optimal one for this problem.

Incorporating the recently developed thin shell integral model of a coated conductor with a ferromagnetic substrate [11] into the pump model [3, 14, 15], we obtain a system of one-dimensional integro-differential equations. We solve this system using the fast and accurate spectral numerical method [11] and show that ferromagnetic substrates can increase the generated open-circuit DC voltage. Then, assuming a simplified lumped model of a coil, we simulate its charging during several thousands of rotor revolutions and demonstrate that such substrates accelerate this process. Finally, we show that this time-consuming simulation can be replaced by a simple analytical description of the charging process based on solutions to a few given transport current problems.

Our aim is a method for characterization of the impact of weakly and strongly magnetic substrates, and here we employ the simplest constitutive relations: a power current-voltage relation with a constant critical current density

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V. Sokolovsky is with Physics Department, Ben-Gurion University of the Negev, Beer-Sheva 84105 Israel (sokolovv@bgu.ac.il).

L. Prigozhin is with Blaustein Institutes for Desert Research, Ben-Gurion University of the Negev, Sde Boqer Campus 84990 Israel (leonid@bgu.ac.il).

Color versions of the figures in this article are available online at http://ieeexplore.ieee.org for the superconductor and a constant magnetic susceptibility for the substrate. We also neglect currents in all layers of a coated conductor except the superconducting one.

II. THE MODEL

The HTS dynamo model [3, 14, 15] is developed for a rotating long permanent magnet passing close to a stationary long thin coated conductor strip (fig. 1). In this work we assume the same dynamo configuration and parameters as in [14, 15] (see table I) with one exception: now the coated conductor has a ferromagnetic substrate with the relative magnetic permeability μ_r and thickness δ; its width $2a$ is equal to that of the HTS layer.

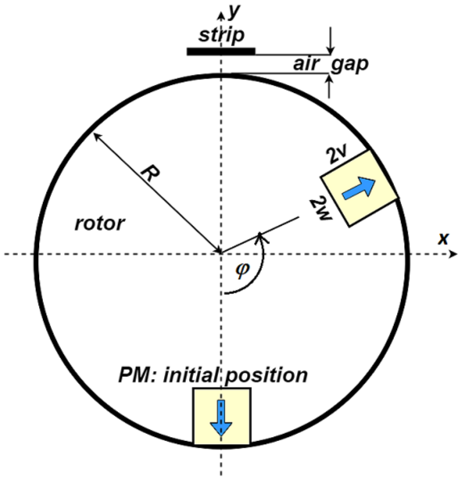

Fig. 1. A scheme of an HTS dynamo: the geometry of the problem.

We consider two possible orientations of the stator strip: the substrate is further from the rotor than the superconducting layer and vice versa. We consider only two layers of the coated conductor: the thin HTS layer and its substrate. Electrical current is allowed only in the HTS layer, for which we use the infinitely thin approximation, assume the sheet current density is directed along the strip and is the same in each cross-section. The power current-voltage relation, characterizing the HTS layer, is

$$e = e_0 \left| j / j_c \right|^{n-1} j / j_c .$$

Here $j(t,x)$ is the parallel to the z-axis sheet current density, $e(t,x)$ is the electric field, $e_0 = 10^{-4}$ V/m, and $j_c = I_c / 2a$ is the field-independent sheet critical current density.

Table I. HTS dynamo benchmark parameters.

permanent magnet	width, $2w$	6 mm
	height, $2v$	12 mm
	active length, l	12.7 mm
	Remanent flux density, B_r	1.25 T
HTS layer	width, $2a$	12 mm
	thickness	1 μm
	critical current, I_c	283 A
	power value, n	20
rotor external radius, R		35 mm
air gap, d		3.7 mm
frequency of the rotor rotation, f		4.25 Hz

If the substrate material is ferromagnetic, e.g. a Ni-W alloy, its magnetization influences the current distribution in the superconducting layer and should be taken into account.

The relative magnetic permeability of the substrate, μ_r, is of the order of tens for weak magnetic substrates and can be of the order of thousands and even more for the strong ones [16]. The thickness δ of a substrate layer, $30-100\,\mu\mathrm{m}$, is small comparing to its width, 4-12 mm. Hence, we can use the thin shell magnetization theory developed by Krasnov [17-19] in terms of “surface magnetization”, $\sigma(t,x)$, which is attributed to the mid-surface of the substrate. For a long strip σ can be regarded as scalar and is defined as

$$\sigma = \int_{-\delta/2}^{\delta/2} m_x(t,x,y)\,\mathrm{d}y ,$$

where m_x is the x-component of substrate magnetization.

This model is derived for the limit $\delta \to 0,\ \mu_r \to \infty$ while the product $\delta\mu_r$ remains constant.

As in [11], we use scaled dimensionless variables

$$\tilde{j} = \frac{j}{j_c}, \quad \tilde{\sigma} = \frac{\sigma}{a j_c}, \quad \tilde{\boldsymbol{h}} = \frac{\boldsymbol{h}}{j_c}, \quad \tilde{e} = \frac{e}{e_0},$$

$$(\tilde{x},\tilde{y}) = \frac{(x,y)}{a}, \quad \tilde{t} = \frac{t}{t_0}, \quad \tilde{I} = \frac{I}{I_c}, \quad \tilde{V} = \frac{V}{a e_0},$$

where $t_0 = a\mu_0 j_c / e_0$, μ_0 is the magnetic permeability of vacuum, I is the transport current, V is the voltage. Omitting the sign “~” to simplify our notations, we present the coated conductor model in dimensionless form (see [11]),

$$\begin{cases} \kappa^{-1}\sigma(t,x)+\partial_x\left(\dfrac{1}{2\pi}\displaystyle\int_{-1}^{1}\dfrac{\sigma(t,x')}{x-x'}\mathrm{d}x'\right)+\dfrac{s}{2}\,j(t,x) \\ \qquad = h_x^{\mathrm{e}}, \\ \partial_t\left(h_y^{\mathrm{e}}+\dfrac{1}{2\pi}\displaystyle\int_{-1}^{1}\dfrac{j(t,x')}{x-x'}\mathrm{d}x'+\dfrac{s}{2}\dfrac{\partial\sigma(t,x)}{\partial x}\right) \\ \qquad = \partial_x e(t,x), \\ \displaystyle\int_{-1}^{1} j(t,x)\mathrm{d}x = 2I(t), \qquad e = |\,j\,|^{n-1}\, j, \end{cases} \tag{1}$$

where $s=-1$ if the substrate is between the rotor and HTS layer and $s=+1$ if the HTS layer is between the rotor and substrate. This parameter accounts for the different directions of the tangential component of the superconducting-current-induced field and of the normal component of the substrate-magnetization-induced field above and below the corresponding layer.

The dimensionless parameter $\kappa = \chi\delta / a$, with $\chi = \mu_r - 1$ being the magnetic susceptibility of the substrate, solely characterizes the ferromagnetic substrate in this model. For a nonmagnetic substrate $\kappa = 0$. Numerical simulations in [11] and in this work showed that for $\kappa < 10^{-1}$ the impact of a substrate is small (a weakly magnetic substrate). It becomes significant for higher values of κ (a strongly magnetic substrate). As κ increases, the influence of the magnetic substrate saturates: the problem solutions tend to those for $\kappa = \infty$ and, typically, are almost the same for all $\kappa > 10$.

The field $\boldsymbol{h}^{\mathrm{e}} = (h_x^{\mathrm{e}}(t,x), h_y^{\mathrm{e}}(t,x))$ is the external magnetic field in the thin strip points. In our case it is the field induced by the rotating permanent magnet. The analytical expressions [20] were used to find this field at each moment in time; see [15] for details of this computation.

The instantaneous electromotive force in an electrical circuit is $\mathcal{E} = d\Phi / dt$, where the magnetic flux Φ depends on the circuit configuration. The electromotive force can be presented as the sum of the voltage V on the load and the stator voltage. The latter is estimated as $-V_r = l\langle e\rangle$, where the width-average electric field $\langle e\rangle = 0.5\int_{-1}^{1} e(t,x)\mathrm{d}x$ and l is the normalized "active length", usually taken equal to the length (in the z-axis direction) of the permanent magnet. The cycle-average value of $d\Phi / dt$ is zero in the open-circuit case, since the flux is induced by the rotating permanent magnet and is periodic. This value is negligible also in the case of load charging, since the transport current change during one rotor rotation is very small. Hence, the most important characteristic of an HTS dynamo, the DC voltage, can be calculated, similarly to [13-15, 21], as

$$\langle V\rangle = f\int_{t_0}^{t_0+1/f} V_r(t)\mathrm{d}t. \tag{2}$$

The one-dimensional integro-differential model of an HTS dynamo, (1)-(2), is simplified in many aspects. The main simplification is the long permanent magnet assumption, which makes possible to consider only a cross-section of the stator but needs the notion of an effective (active) length. This assumption, often employed in dynamo models with non-magnetic substrates (e.g., [3, 14, 15]), enables very fast numerical simulations but provides no realistic description of the closed current loops in the HTS layer, as do more accurate full-dimensional models [22-24]. Comparison of numerical solutions, obtained using the two types of models (see [23], fig. 7) showed, however, that the simplified model still produces a good approximation to the open circuit voltage curve if the strip width does not exceed the permanent magnet length. The thin shell magnetization model we employed for the substrate also makes numerical solution much easier. This simplification is, however, well justified [17-19].

For numerical solution of equations (1) we employed the method presented in [11] and based on the method of lines for integration in time and Chebyshev spectral approximation for discretization in space. The values of unknowns j and σ are found in $N+1$ nodes of the Chebyshev mesh $x_k = -\cos(\pi k / N),\ k = 0,...,N.$ This is an extension of the spectral method [15], developed there for HTS dynamos with a non-magnetic stator substrate and shown to be more efficient than all numerical methods considered in [14]. Our numerical simulations were performed in Matlab R2020b on a PC with the Intel® Core™ i7-9700 CPU 3.00 GHz.

III. OPEN CIRCUIT VOLTAGE

To model the open circuit conditions, we set $I(t) = 0$ and compare voltages computed for different ferromagnetic substrates characterized by their values of parameter κ. To avoid the transient effects, we present our simulation results for the second rotor rotation cycle. The computations were performed using the Chebyshev mesh with $N = 200$ and the computation time was about 20 seconds per cycle. For $N = 100$ this time was less than 5 seconds and the difference in computed DC voltages was less than 1%.

First, let the HTS layer of the coated conductor faces the rotor, so in the model (1) we set $s = 1.$ For small κ the influence of the ferromagnetic substrate is negligible, the voltage curve and the DC voltage are similar to those for a non-magnetic substrate. As this parameter increases, the voltage curve changes and the two negative peaks become deeper (fig. 2).

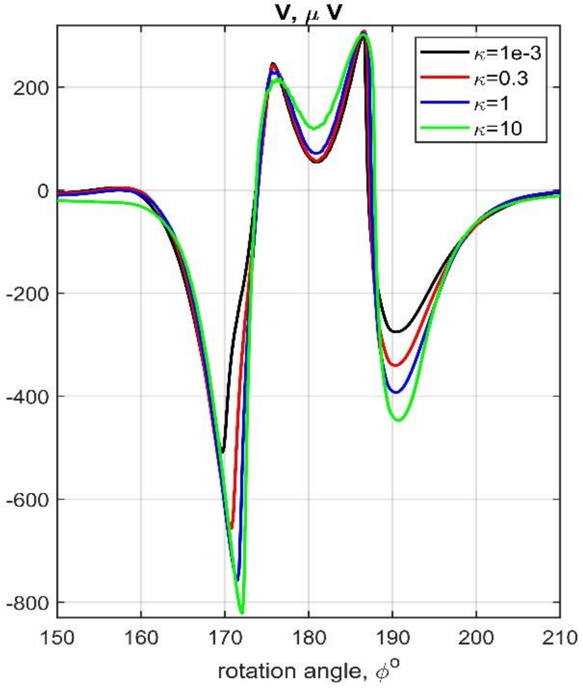

Fig. 2. The open-circuit voltage V_r during the second rotation cycle: the influence of ferromagnetic substrate; the HTS layer is between the rotor and substrate. The deeper negative peaks are for the larger κ vlues.

The DC voltage changes from -10.1 to -16 μV (fig. 3, top) and saturates: the results for $\kappa = 10$ and $\kappa = 100$ are close. We conclude that, if the HTS layer is between the rotor and the substrate, the ferromagnetic substrate can significantly increase the DC voltage (its absolute value, the sign is unimportant).

In case of the opposite orientation (the ferromagnetic substrate is between the rotor and HTS layer, $s = -1$) the substrate shields the magnetic field and the DC voltage decreases; it becomes negligible for large κ (fig. 3, bottom).

A similar influence of the ferromagnetic slice inserted before or behind the coated conductor dynamo stator has been observed in [13].

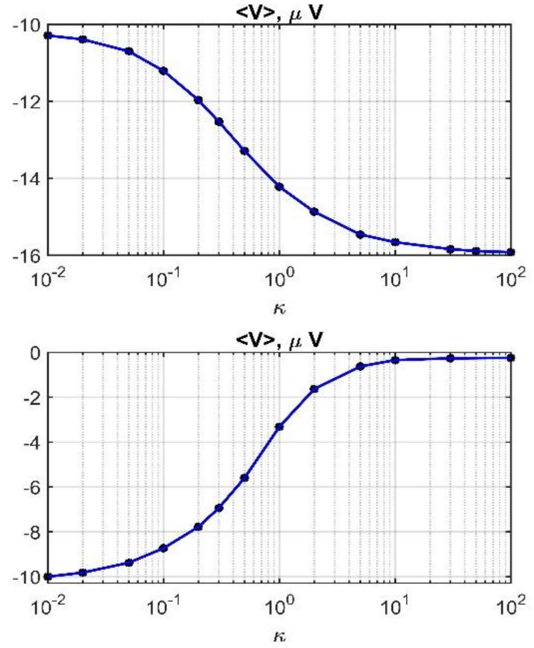

Fig. 3. Influence of a ferromagnetic substrate on the generated DC voltage $<V>$. Top: HTS layer is between the rotor and the substrate. Bottom: the substrate is between the rotor and HTS layer.

To illustrate the effect of shielding, we consider the magnetic field distribution in the absence of the superconducting layer. The field can be written (see [11]) as

$$\boldsymbol{h} = \boldsymbol{h}^e - \frac{1}{2\pi}\begin{Bmatrix} \displaystyle\int_{-1}^{1} \frac{\partial_{x'}\sigma(t,x')(x-x')}{(x-x')^2+y^2}\mathrm{d}x' \\ \displaystyle\int_{-1}^{1} \frac{\partial_{x'}\sigma(t,x')y}{(x-x')^2+y^2}\mathrm{d}x' \end{Bmatrix}. \qquad (3)$$

Noting that with $s = 0$ the model (1) describes separate independent magnetizations of the superconducting and ferromagnetic layers in the field of a rotating permanent magnet, we computed the field (3) for $\kappa = 10$ (strongly magnetic substrate) and several rotor positions (fig. 4). Clearly, the ferromagnetic substrate shields the superconducting layer placed on the opposite to the rotor side of the coated conductor and amplifies the magnetic field on the side that faces the rotor.

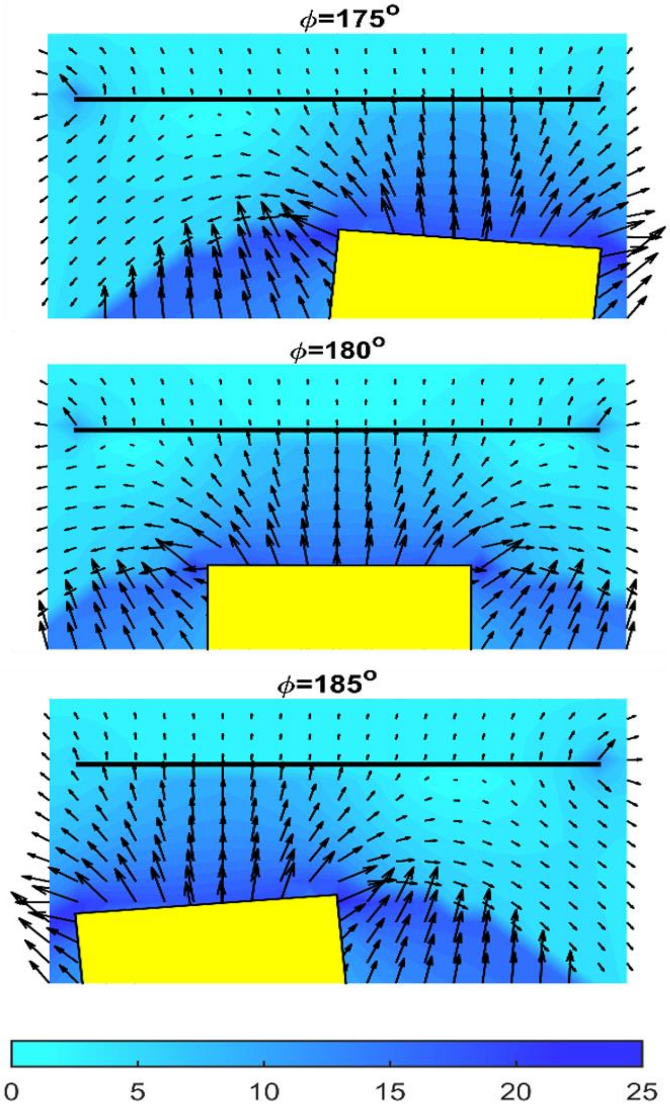

Fig. 4. Shielding by the substrate layer. Magnetic field $\boldsymbol{h}/j_c$ for three rotor positions characterized by the rotor rotation angle ϕ. Black line is the ferromagnetic substrate, the partly seen yellow rectangle is the permanent magnet.

IV. CHARGING A SUPERCONDUCTING COIL

We now assume the superconducting layer is between the rotor and substrate (the DC voltage is increased due to the ferromagnetic substrate) and consider first the problem with a given transport current I. Let the current be changed linearly from zero to a prescribed value I during the first cycle and then remain constant. The DC voltage was now computed for the third cycle. Our numerical simulations showed that for each κ the voltage $<V>$ depends almost linearly on the transport current I (fig. 5).

Denoting by $V_0(\kappa)$ the open-circuit DC voltage, we approximate this dependance as

$$<V>=V_0(\kappa)-R_{\text{eff}}(\kappa)I\ , \tag{4}$$

where R_{eff} is a constant playing the role of an effective stator resistance. The voltage becomes zero at almost the same transport current, $I_0(\kappa)$, for all κ. We find $R_{\text{eff}}(\kappa)=V_0(\kappa)/I_0(\kappa)$ (table II).

To simulate charging a superconducting magnet by an HTS dynamo we consider, as in [25] for the case of a nonmagnetic substrate, the dynamo in a closed circuit with a simplified lumped load having the resistance R and inductivity L. We use the same values of these parameters as in [25]: $R=0.88\,\mu\Omega,\ L=0.24\,\text{mH}$.

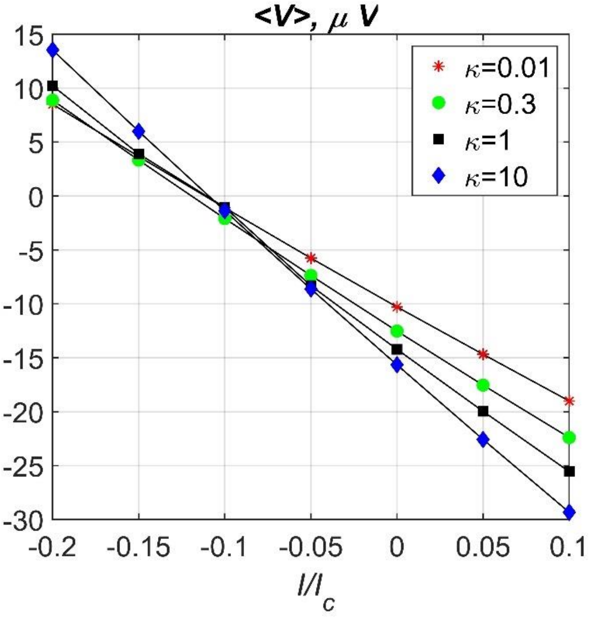

Fig. 5. Pump-generated DC voltage as a function of transport current for different ferromagnetic substrates.

Table II. Dependance of the main pump characteristics on the ferromagnetic substrate.

κ	0.01	0.3	1	10
V_0 (μV)	-10.3	-12.5	-14.2	-15.7
I_0 (A)	-31.1	-34.0	-34.0	-31.1
R_{eff} (μΩ)	0.330	0.369	0.418	0.503
$I_{\text{sat}}\ (\text{A})$	8.5	10	10.9	11.4
$\tau\ (s)$	198	192	185	174

The circuit equation

$$V_0(\kappa)-R_{\text{eff}}(\kappa)I=RI+LdI/dt$$

yields

$$I=I_{\text{sat}}\left[1-\exp(-t/\tau)\right], \tag{5}$$

where the saturation current and the characteristic charging time are, respectively,

$$I_{\mathrm{sat}} = V_0/(R_{\mathrm{eff}} + R), \quad \tau = L/(R_{\mathrm{eff}} + R).$$

Previously, this electrical circuit model was used in [1, 26]. In our case the model parameters depend on κ , see table II.

Solution (5) completely ignores oscillations of the pump voltage during each cycle. On the other hand, not using the linear approximation (4), we can supplement our model (1) by the differential equation

$$RI + LdI/dt = V_r(t), \qquad (6)$$

in which the right hand side takes into account only the part of the pump voltage ripples related to the resistance of the stator but, anyway, has the cycle-averaged value equal to that of the total voltage. It is easy to incorporate the evolutionary equation (6) into our numerical scheme that uses the method of lines for integration in time. The employed numerical method is fast and we were able to model several thousands of cycles and compare the analytical solution (5) with the numerical solution of the system (1),(6); see fig. 6.

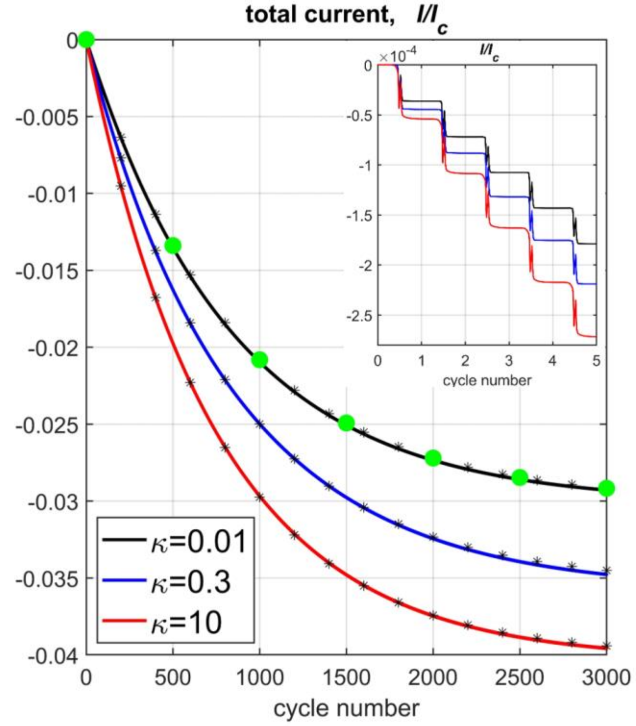

Fig. 6. Numerical simulation of load charging by dynamos with different stator substrates. Solid lines – solutions to the model (1),(6). Black stars indicate the corresponding analytical solutions (5). Green circles – solution [25] for a nonmagnetic substrate. The inset illustrates the current ripples, accompanying the process but not seen in the current curves for 3000 cycles. The lines from the upper to the lower are, respectively, for κ = 0.01, 0.3, 10.

The solutions to (1),(6) practically coincide with the corresponding analytical solutions (5), which suggests that in modeling charging the current ripples can be ignored. This statement is further confirmed by comparison of our solution for $\kappa = 0.01$ with the solution [25] for a nonmagnetic substrate (green circles in fig. 6). Although the authors of [25] employed a different approach to voltage ripple simulation, resulting in much larger ripples, their ripples have the same cycle-average values and our charging current curves coincide. The presence of ripples can, however, increase the AC loss.

VI. CONCLUSION

The benchmark HTS dynamo pump problem [14] was in this work extended to the case of the pump stator made of a coated conductor with a ferromagnetic substrate. Such a substrate changes the superconducting current density distribution. To our knowledge, the impact of a ferromagnetic substrate on the dynamo pump performance has not been studied yet, although a similar problem for a ferromagnetic slice attached to the stator was previously considered in [13].

In our work [11], using the thin shell magnetization theory [17-19], we presented a new model of a coated conductor with a ferromagnetic substrate and an efficient Chebyshev spectral numerical method. The model makes use of the high width-to-thickness ratio of the substrate and superconducting layers and is much simpler than the previously proposed two-dimensional models (for which the high aspect ratio presents a difficulty). In this work we extended this approach to modeling the HTS dynamo pumps and showed that magnetic stator substrate can accelerate contactless charging of a coil and increase the current if the superconducting layer is oriented towards the rotated permanent magnet (fig. 6). In the setup of the magnetic pump benchmark problem [14], the open circuit voltage changes from 10 μV for a nonmagnetic substrate to 16 μV for a strongly ferromagnetic one (fig. 3). For the opposite stator orientation, the magnetic field is shielded by the substrate and the pump-generated voltage decreases.

For a given transport current, determining the pump-generated voltage using our model and numerical scheme takes less than a minute on a PC. Results of such simulations can be used to determine also the effective lumped model parameters of the pump and replace simulation of charging a coil during thousands rotor rotations by a simple analytical formula.

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