

Radial mixing and segregation of a binary mixture in a rotating drum: Model and experiment

Leonid Prigozhin^{1,*} and Haim Kalman²

¹*Department of Applied Mathematics and Computer Science, Weizmann Institute of Science, Rehovot, Israel*

²*Department of Mechanical Engineering, Ben-Gurion University of the Negev, P.O. Box 653, Beer-Sheva 84105, Israel*

(Received 10 April 1997; revised manuscript received 29 September 1997)

We studied mixing and size segregation of a binary mixture in a partially filled rotating drum. The proposed mathematical model of material transport accounts for the free-surface segregation and predicts the evolving nonuniform distribution of the mixture components. The theoretical distributions are in agreement with the experimental data obtained. [S1063-651X(98)14202-2]

PACS number(s): 81.05.Rm, 64.75.+g, 46.10.+z

I. INTRODUCTION

Industrial bulk solids are usually nonhomogeneous and it is essential to control their spatial distribution and to reduce the composition variability. The tendency of cohesionless polydisperse granular materials to segregate and, especially, the segregation of differing particles pouring down the free surface of a filling, plague the handling of bulk solids, although they are used sometimes for the separation of different fractions. Because of its practical importance, free-surface segregation has been studied by many researchers (see, e.g., [1–3]). Recently, a great interest in the properties of the granular state has led to many new publications on free-surface segregation in the physical literature ([4–6], see also review [7]).

Various aspects of segregation have been studied experimentally and also numerically (e.g., by means of molecular dynamics methods). Nevertheless, it is still difficult to relate quantitatively the characteristics of individual grains with the resulting distribution of species in a filling (see discussion in [2]).

The difference in the repose angles of mixture components has been chosen in [8] as the macroscopic parameter entirely determining segregation. However, such models are unable to describe size segregation: although the mixtures of particles differing only in size show the strongest tendency to segregate, their components have the same angle of repose [9]. The angle of repose of a mixture may reflect variations in the bulk composition but these variations seem in most cases a consequence rather than a cause of segregation.

A practicable macroscopic model of granular body formation that is able to account for free-surface segregation can be derived by combining a material-independent mathematical model of mass transfer with a phenomenological constitutive relation, or an *operator of segregation* [10]. The operator of segregation relates the granular material composition in surface flow to the composition of the layer generated at the surface of a growing pile or filling. This constitutive relation depends only on the mechanism of segregation that is specific for a given type of material, and can

be determined in a simple experiment.

By using the experimental data on size segregation of binary mixtures in a two-dimensional pile growing under a point source (Drahn and Bridgwater [2]), we identified the segregation operator for the binary mixtures in which particles differ only in size [10]. The first test of our model was based on comparison with another set of experiments in [2]. In the present study we use the same constitutive relation to predict the evolution of spatial distribution of particles in a rotating drum, a device used in numerous operations with bulk solids. Although both the configuration and the material of particles are different from those in [2], a good agreement of theoretical predictions and the experimental data has been achieved without any fitting of model parameters.

II. EXPERIMENT

The mixer used in the experiments was a cylindrical vessel, 141 mm in internal diameter and 36 mm in length. It is made of transparent plastic and is open at one end. Once the particles were introduced into the mixer, the open end was covered with a clinging nylon film (such as is often used for food sealing). The mixer was then placed vertically and rotated slowly. Experiments were carried out with binary mixtures that filled out various portions of the drum and contained various weight ratios of two sizes of spherical particles, $d_1 = 1.37$ and $d_2 = 2.51$ mm, with standard deviations not exceeding 5% in both cases. All particles were made of zirconium oxide with a material density of 4.14 g/cm^3 . The material dynamic angle of repose, measured after a typical avalanche, was about 27° – 28° ; the incline of the free surface before an avalanche was usually between 30° and 35° .

There was no sliding of granular material at the drum walls. The particles rotated with the drum until they arrived at the vicinity of the inclined filling surface, they then poured down this surface and were once more trapped into a rotating bulk. The surface flow took place in a thin boundary layer (2–4 particle diameters in thickness) and occurred either in avalanches or continuously, depending on the speed of rotation.

Some of the experiments were conducted for a qualitative comparison of the observed rotation-induced evolution of a given distribution of differently colored granules with the results of computer simulation of mixing and segregation. In

*Present address: CEEP, J. Blaustein Institute for Desert Research, Ben-Gurion University of the Negev, Sede Boqer Campus, 84990 Israel. Electronic address: leonid@bgumail.bgu.ac.il

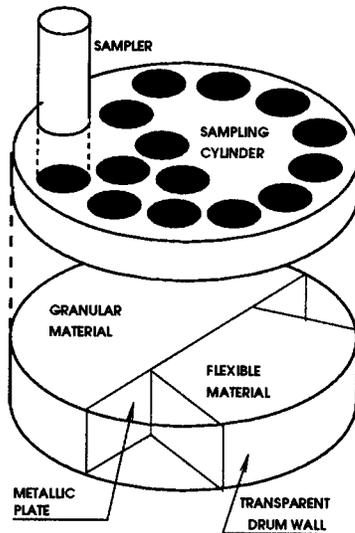


FIG. 1. Sampling procedure.

other experiments, which started from an approximately uniform initial distribution of species, the drum was rotated a number of times until the evolving nonuniform (due to segregation) radial distribution of large and small particles looked stable. Then the drum was stopped and samples were taken from several locations (the method is similar to the one used by Alonso *et al.* [3]).

The sampling procedure was as follows. When the drum was stopped, an incision was made in the nylon film wall, along and just above the line of the bed surface. It was done only after a rigid plate was fastened to the open side of the drum, its edge reaching the free surface line. A rectangular metallic piece was introduced through the incision into the mixer and flexible material was inserted between the rectangular piece and the drum wall in order to fix the piece in its place and to slightly compress the particles. All this was done to prevent the motion of particles when the drum is placed horizontally and samples are taken. After the drum was placed horizontally, the sampling circular plate (which had a diameter equal to the external diameter of the drum, a thickness 20 mm, and contained holes 22 mm in diameter at the specified distances from the center) was placed upon the drum (Fig. 1). This was done to control the exact location of the samplers which were made of a copper pipe with an external diameter 22 mm, internal diameter 20 mm, and a height 61 mm. The samplers were closed on one side and their walls sharpened on the other. The samplers were pushed down through the holes of the sampling cylinder and through the clinging film into the bed of particles until they reached the opposite drum wall. In this manner the particles were separated into the samplers and between them. After all the samplers were introduced, the drum was turned upside down and particles from each sample were screened.

III. SIMULATION OF MIXING AND SEGREGATION

Let a drum, partially filled with granular material, rotate around its axis which is horizontal. The particles in the drum participate in collective solid rotation and, reaching the filling surface inclined at about the angle of repose, pour down this surface and get mixed on the way. Then they are once

again trapped into the rotating bulk. At a low rotation speed, the pouring of particles down the free surface occurs in avalanches. As the speed increases, the time intervals between avalanches become shorter, and then a continuous surface flow regime is established (we do not consider the centrifugal regime at very high rotation speeds).

Using simple, purely geometrical argumentation, Metcalfe *et al.* [11] were able to analyze the avalanche mixing of equal but differently colored granules. The situation is more complicated if the particles differ in size. The fines easily find a niche at the lower part of the free surface where they are trapped. The larger particles have to roll down further. The resulting distribution has a higher concentration of fines in the central part of the filling. This radial segregation, which appears already at the beginning of rotation, is the main issue of our work. We do not consider axial segregation, which can also become significant after tens or hundreds of revolutions (see [5,6]), and, therefore, limit our consideration to a two-dimensional cross section of the drum.

Mixing and segregation occur in both avalanche and continuous regimes of surface flow and, in our opinion, the influence of avalanches on the distribution of particles is sometimes exaggerated. Mathematically, the model [11] of avalanche mixing in a rotating drum is a finite difference approximation of a simple continuous flow mixing model. These models yield similar results if the difference between surface incline angles before and after avalanches (a parameter in avalanche mixing model) is small; see also [12]. Similarly, although the surface flow in experiments [2] occurred by avalanches, the continuous model [10] was able to predict the resulting size distributions of particles in a pile. More important than the continuous or intermittent character of surface flow is the confinement of this flow to a thin boundary layer. This latter assumption is usually valid for avalanches and for continuous surface flows as well [7]. Only at higher rotation speeds does the layer of particles pouring down the free surface become thick so that the necessity for fines to percolate through this layer to the bulk surface can limit segregation [2]. Henceforth, we neglect this phenomenon. (A much more complicated model of transverse flow has been recently proposed and used in simulation of mixing of equal but differently colored granules [13]. Further evaluation and generalization of this model for simulating segregation in a thick surface flow layer would be interesting.)

Because of the stochastic nature of granular flow, Metcalfe *et al.* [11] simulated the surface transport of differently colored particles using a random map. However, the observed material distributions are stable and reproducible. A purely deterministic macroscopic model [10] allows us to calculate these distributions also for a polydisperse material.

A. Equations of mass transport

Let a mixture contain two types of approximately spherical particles, having the same mass density but different diameters d_1 and d_2 . We neglect the dependence of the bulk density and repose angle on material composition; the percolation of small particles through the bulk of material is also neglected. This is justified if the particle sizes are not too different. We suppose the free surface of a filling is initially flat and, as the rotation starts, becomes inclined approxi-

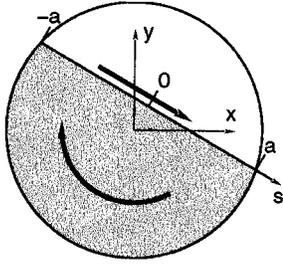


FIG. 2. Rotating drum.

mately at the angle of repose and remains stationary (fluctuations caused by avalanches are neglected). We also assume that all mixing and segregation occur in a thin surface layer.

Let $\kappa = \{\kappa^1(x, y, t), \kappa^2(x, y, t)\}$ be the time-dependent bulk composition of a filling in the neighborhood of a drum cross-section point (x, y) ,

$$\kappa^1 + \kappa^2 = 1, \quad (1)$$

and the initial concentrations $\kappa^i(x, y, 0)$ be given. We denote by $\kappa_-^i(s, t)$ the concentrations in the filling surface layer and by $\kappa_+^i(s, t)$ the concentrations in the flux of material pouring down the free surface (s is the coordinate of a point at this surface, see Fig. 2).

If $J(s, t)$ is the total surface flux at point s , the surface flux of the i th type of particles is $\kappa_+^i J$, and the conservation law for each component may be written as

$$\partial_s(\kappa_+^i J) = \kappa_-^i V_n, \quad -a \leq s \leq a \quad (2)$$

where \vec{V} is the velocity in the rotating bulk and V_n is the value at the free surface of the component of \vec{V} normal to this surface.

Let ω be the angular velocity of rotation. Then $\vec{V} = \omega(y, -x)$ and it is easy to show that $V_n = -\omega s$. Adding Eqs. (2) for $i = 1, 2$ and taking into account that Eq. (1) holds also for both κ_- and κ_+ , we arrive at the equation for total surface flux: $\partial_s J = -\omega s$. Since at the point $s = -a$ the flux must be zero, the solution is $J = (\omega/2)(a^2 - s^2)$. The balance equation (2) can now be written as

$$\partial_s \{\kappa_+^i (a^2 - s^2)\} = -2s \kappa_-^i. \quad (3)$$

At the upper half of the free surface, κ_- is the composition of a bulk layer that has just arrived at this surface and is to be destroyed by pouring. This composition is known initially, since the initial composition of the bulk is given. The material of this layer feeds the surface flow, the composition of which determines (we will discuss this below) the material distribution in a new layer generated at the lower half of the free surface: κ_- for $0 \leq s \leq a$. This latter distribution provides the boundary condition for mass transport equations

$$\partial_t \kappa^i + \vec{V} \cdot \vec{\nabla} \kappa^i = 0 \quad (4)$$

describing the rigid solid rotation of the bulk. Equation (4) can be used to find the material composition κ and, in particular, κ_-^i at $-a \leq s \leq 0$ also for $t > 0$. This closes the model for mass transport.

Determining the composition of new layers (κ_-^i at $0 \leq s \leq a$) is generally the nontrivial part of this model. Using Eq. (3) we find first the concentrations of the surface flux at the point $s = 0$:

$$\kappa_0^i = \kappa_+^i(0, t) = -\frac{2}{a^2} \int_{-a}^0 s \kappa_-^i(s, t) ds. \quad (5)$$

If there is no segregation, these should also be the concentrations in a new layer: $\kappa_-^i \equiv \kappa_0^i$ in $(0, a)$. This is the case for differently colored particles with $d_1 = d_2$. Using a finite difference approximation, we solved numerically Eqs. (4) and (5) with an appropriate initial condition to simulate the mixing of nonuniformly distributed colored particles. The simulated evolution of material distributions is similar to the one observed experimentally (see Figs. 3 and 4). It should be noted, though, that according to the model if the drum is more than half full there exists a nonmixing rotating central core consisting of particles which never reach the free surface. In practice, however, there is some erosion of this core because of the finite thickness of the surface flow layer. The bed surface in these pictures is horizontal because, before making the photographs, we rotated the drum back at the angle of repose to stabilize the current distribution of particles.

In the general case it is necessary to know the operator of segregation which, provided the local composition of the surface flux is known, determines the local composition of a generated surface layer: $\kappa_-(s, t) = S(\kappa_+(s, t))$. This phenomenological constitutive relation characterizes the material tendency to segregate, a property specific to the granular state of matter. While it seems difficult to obtain this macroscopic relation theoretically by analyzing the chaotic surface flow of a mixture of particles, the segregation relation can be determined in an experiment. Below, we describe briefly the identification procedure used in [10] to find approximately the constitutive relation governing the free-surface size segregation of binary mixtures. Since composition of such a mixture is determined by the concentration of only one component, the operator S becomes a function:

$$\kappa_-^1 = S(\kappa_+^1), \quad \kappa_-^2 = 1 - S(\kappa_+^1).$$

Provided the segregation function is known, computing the material distribution is not difficult. The midpoint flux concentration κ_0^1 serves as the initial condition for ordinary differential equation (3) for κ_+^1 on the interval $(0, a)$ (with time being a parameter). We can write this equation as $\partial_s \{\kappa_+^1 (a^2 - s^2)\} = -2s S(\kappa_+^1)$ and solve it by dividing variables:

$$\int_{\kappa_0^1}^{\kappa_+^1(s)} \frac{du}{u - S(u)} = -\ln \left\{ 1 - \left(\frac{s}{a} \right)^2 \right\}, \quad 0 \leq s \leq a. \quad (6)$$

Each step of the numerical simulation should consist of (1) determining the midpoint flux concentration κ_0^1 from Eq. (5); (2) calculating the flux concentration $\kappa_+^1(s)$ for $0 \leq s \leq a$ using (6); (3) finding the new surface layer composition $\kappa_-^1 = S(\kappa_+^1)$ for $0 \leq s \leq a$; (4) rotation at a small angle [a step

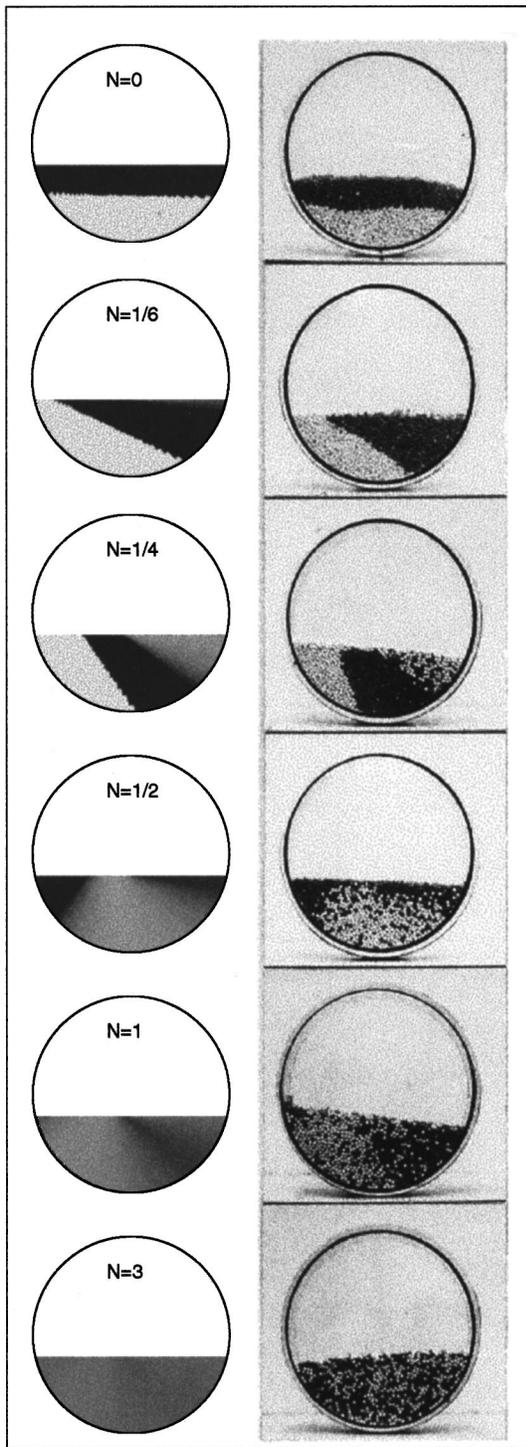


FIG. 3. Mixing of differently colored identical particles. Left—computer simulation, right—experiment. N is the number of revolutions. The drum is 36% full.

in the finite difference approximation of Eq. (4)] to obtain the new distribution of species in the drum.

B. Size segregation

The surface size segregation of a binary mixture has been studied experimentally by Drahn and Bridgwater [2] who measured the distribution of particles along the slope of a two-dimensional pile formed under a point source in a spe-

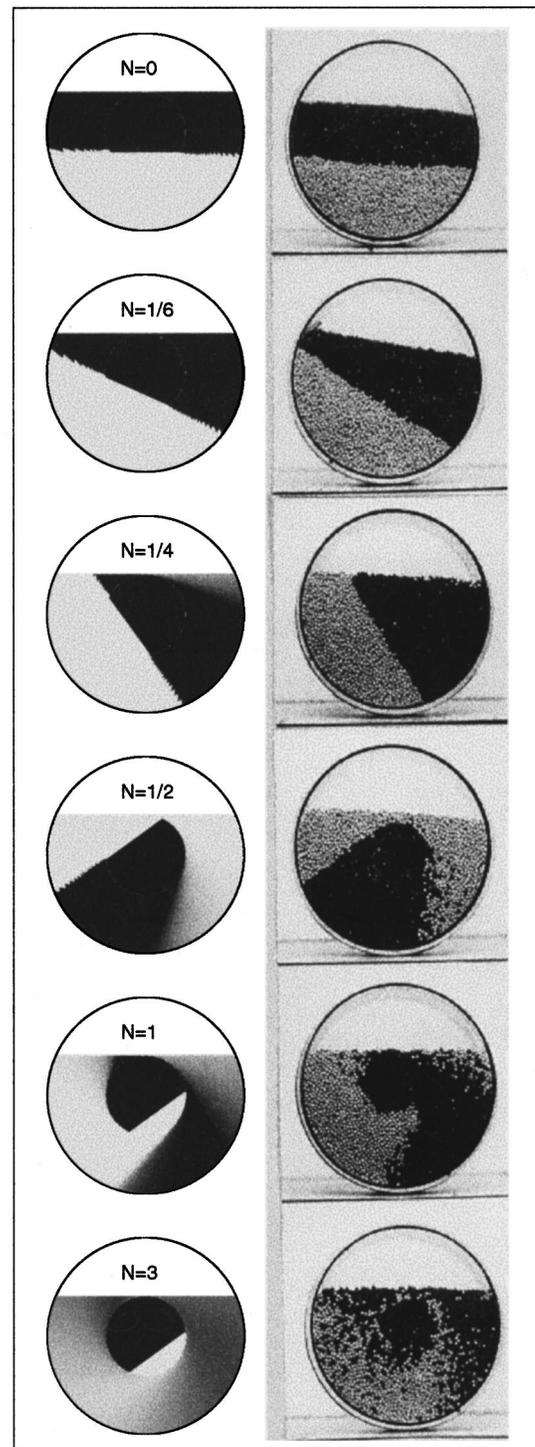


FIG. 4. As in Fig. 3; the drum is 75% full.

cially constructed apparatus (Fig. 5). Some of these experiments have been performed with a mixture in which one of the components, let us call it type 1, was present in small amounts (a tracer). It has been found that in such experiments the mean tracer position is not affected by the type of material and depends linearly on the diameter ratio for $0.4 < d_1/d_2 < 1.6$:

$$\frac{\langle s \rangle}{a} \approx 0.5 + 0.83 \left(\frac{d_1}{d_2} - 1 \right). \quad (7)$$

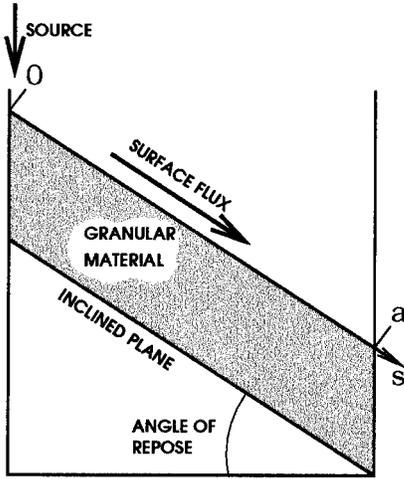


FIG. 5. Schematic diagram of Drahn and Bridgwater's apparatus.

Almost all tracers are concentrated under the source point or near the opposite side of a filling if the size ratio is less than 0.4 or more than 1.6, respectively.

Correlation (7) can be used to identify the segregation function. It is easy to see that for the configuration in Fig. 5 the surface flux J is proportional to $a-s$, and so, instead of Eq. (6), the theoretical distribution of particles in the apparatus of Drahn and Bridgwater is described by the equation

$$\int_{\kappa_0^1}^{\kappa_+^1(s)} \frac{du}{u-S(u)} = -\ln\left(1-\frac{s}{a}\right), \quad (8)$$

where κ_0^1 is the concentration of type-1 particles in the feed material. If these particles are tracers, their concentration is small and we assume $S(\kappa_+^1) \approx C\kappa_+^1$, where $C = C(d_1/d_2)$ is an unknown constant which depends on the size ratio. Substituting this relation into Eq. (8) and integrating, we find the density of spatial distribution of tracers

$$q = \frac{\kappa_-^1(s)}{\int_0^a \kappa_-^1(s) ds} = \frac{C}{a} \left(1 - \frac{s}{a}\right)^{C-1}$$

and the theoretical mean tracer position

$$\frac{\langle s \rangle}{a} = \frac{1}{a} \int_0^a sq(s) ds = \frac{1}{C+1}. \quad (9)$$

Equating expressions (7) and (9) yields the relation

$$C(d_1/d_2) = \begin{cases} \infty & \text{if } d_1/d_2 \leq 0.4 \\ \frac{1.6-d_1/d_2}{d_1/d_2-0.4} & \text{if } 0.4 < d_1/d_2 < 1.6 \\ 0 & \text{if } d_1/d_2 \geq 1.6, \end{cases} \quad (10)$$

in which particles of type 1 are assumed to be tracers. Using thus specified segregation operator, we obtained (see [10]), for various size ratios, theoretical distributions of tracers which were close to the experimental distributions [2].

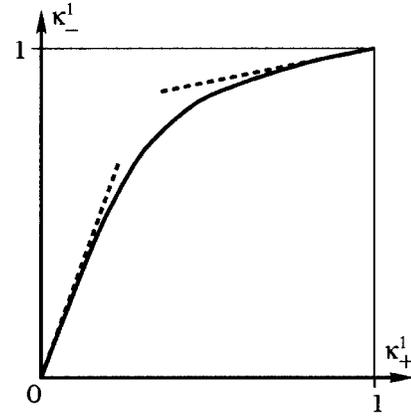


FIG. 6. Segregation function of binary mixture, $\kappa_-^1 = S(\kappa_+^1)$. Dashed lines—linear approximations, solid line—interpolation.

Therefore, if the source concentration of type-1 particles is low, a good approximation for S is $S(\kappa_+^1) = C_1\kappa_+^1$ with $C_1 = C(d_1/d_2)$ as determined by Eq. (10). On the other hand, if this concentration is high, the second component is a tracer and the same model yields $S(\kappa_+^1) = 1 - C_2(1 - \kappa_+^1)$, where $C_2 = C(d_2/d_1)$.

Furthermore, for a fixed size ratio, S should be a smooth monotone function of κ_+^1 . A smooth monotone interpolation that takes into account this function behavior in two limiting cases (Fig. 6) is given, e.g., by the parametric representation

$$\kappa_+^1 = A_+ \tau^2 + B_+ \tau, \quad \kappa_-^1 = A_- \tau^2 + B_- \tau, \quad (11)$$

where $\tau \in [0,1]$ is a parameter and

$$A_+ = \frac{C_1 + C_2 - 2}{C_1 - C_2}, \quad B_+ = \frac{2(1 - C_2)}{C_1 - C_2},$$

$$A_- = \frac{2C_1C_2 - C_1 - C_2}{C_1 - C_2}, \quad B_- = \frac{2C_1(1 - C_2)}{C_1 - C_2}.$$

Although this may be only a crude approximation of the segregation function, it takes into account all information at our disposal. The proposed parametric representation is convenient also because using Eq. (11) to change the variables in Eq. (8), one finds the integral analytically and obtains an algebraic equation in respect to $\tau(s)$,

$$\left(\frac{\tau}{\tau_0}\right)^{1/(C_1-1)} \left(\frac{1-\tau}{1-\tau_0}\right)^{1/(C_2-1)} = 1 - \frac{s}{a}, \quad (12)$$

where τ_0 is the only root of the second order equation $\kappa_+^1(\tau_0) = \kappa_0^1$ in the interval $(0,1)$. Numerical solution of Eq. (12) allowed us to calculate, using Eq. (11), distributions κ_-^1 and compare them with the distributions obtained by Drahn and Bridgwater in experiments where both types of particles were present in significant quantities. A good agreement of theoretical and experimental distributions has been obtained in all cases [10]. In the present work we use thus identified operator of segregation without any changes to predict the distribution of species in a rotating drum.

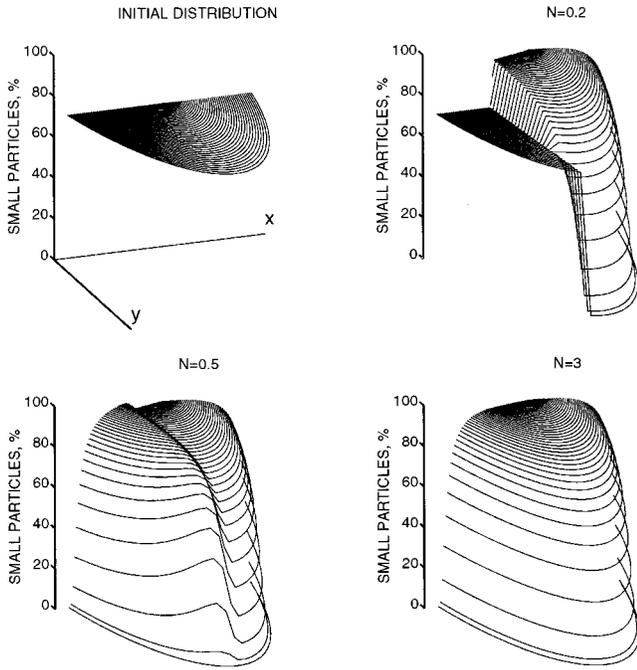


FIG. 7. Evolution of the material distribution. Binary mixture contains 70% of small particles, the drum is 20% full, particle diameter ratio is 0.64. N is the number of revolutions.

C. Segregation in a rotating drum

Comparing Eqs. (6) and (8), we see that only the right-hand side of Eq. (12) has to be modified because of a different geometry; the integral in the left side of Eq. (6) has been already calculated using the substitution of the parametric representation (11). We obtain an algebraic equation in respect to parameter τ ,

$$\left(\frac{\tau}{\tau_0}\right)^{1/(C_1-1)} \left(\frac{1-\tau}{1-\tau_0}\right)^{1/(C_2-1)} = 1 - \left(\frac{s}{a}\right)^2, \quad (13)$$

and, solving it numerically for $0 < s < a$, find the spatial distribution $\tau = \tau(s)$. As in Eq. (12), the value of τ_0 is determined by the relation $\kappa_+^1(\tau_0) = \kappa_0^1$. In this case, however, the “feed concentration” κ_0^1 is not given *a priori* and should be calculated from Eq. (5). Each value of parameter τ determines a unique point on the curve $\kappa_-^1 = S(\kappa_+^1)$. Using Eq. (11), we calculate the new surface layer concentration $\kappa_-^1(s)$ for $0 < s < a$ and are then able to simulate the rotation step of the numerical algorithm described in Sec. III A. [In our realization of this algorithm we used a parametric representation for function S , therefore it was convenient to solve Eq. (6) in terms of parameter τ . If the dependence $\kappa_-^1 = S(\kappa_+^1)$ were given explicitly, Eq. (6) could be equivalently solved for κ_+^1 .]

As an example, we present the results of a computer simulation which began with a uniform initial distribution of particles (Fig. 7). Time oscillations of κ_0^1 , the surface flux concentration of fines at $s=0$, are shown in Fig. 8. The period of these oscillations depends on the filled drum portion. The amplitude is determined by the initial material distribution and decreases exponentially as the stationary particle distribution is established. Note that the κ_0^1 values are greater

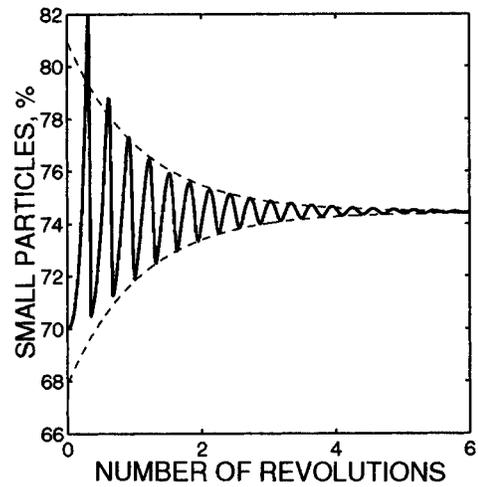


FIG. 8. Concentration of fines in surface flow at the midpoint (solid line). The oscillations decrease exponentially as the stationary material distribution is established (dashed lines—exponential approximation).

than the proportion of fines in the drum: small particles, concentrating in the central part of the filling, appear at the free surface more frequently than the large ones.

In the stationary state, the concentration κ_-^1 depends only on the distance from the drum axis and is equal to κ_-^1 at the corresponding surface point. Let h be the distance between the drum axis and filling surface, $r(s) = \sqrt{h^2 + s^2}$, and $F(s)$ be the intersection of the filling cross section and a circle having its center at the drum axis and radius $r(s)$ (see Fig. 9). By $F(s)$ we denote also the area of this region; its calculation is elementary.

Let f_1 be the total concentration of type-1 material in the drum. Then

$$f_1 = \frac{\int_0^a \kappa_-^1(s) dF(s)}{\int_0^a dF(s)} = \frac{\int_0^a \kappa_-^1(s) dF(s)}{F(a) - F(0)}. \quad (14)$$

If the drum is less than half full, $F(0) = 0$. Otherwise, $F(0)$ is the area of the non-mixing rotating core. Strictly speaking, f_1 should be defined as the concentration of type-1 material outside this core. However, this makes no difference for our examples below where the material is initially uniformly distributed. Parameter τ_0 completely determines $\tau(s)$, the solution to Eq. (13). Using Eq. (11), we can rewrite Eq. (14) in terms of τ and regard it as a nonlinear equation for τ_0 . Solving this equation numerically, we obtain τ_0 , $\tau(s)$ and, using Eq. (11) once more, we get $\kappa_-^1(s)$. This determines the sta-

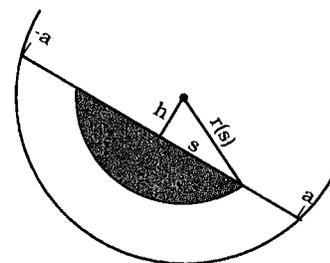


FIG. 9. Cross section. $F(s)$ is represented by the shaded region.

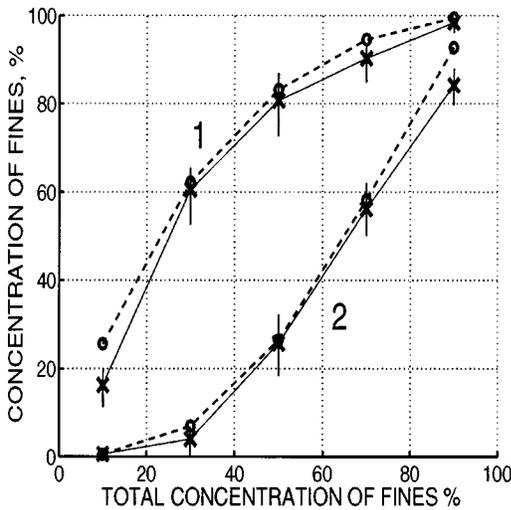


FIG. 10. Stationary concentration of fine particles as a function of the total concentration of fines in the drum: 1—at the distance $r=0.42R$, 2— $0.83R$ from the drum axis. Notations: \circ —theoretical, \times —mean experimental concentration of fines in the samples. Short vertical lines connect the minimal and maximal experimental values of sample concentrations. The drum is 62% full, particle diameter ratio is 0.55.

tionary concentration κ^1 everywhere (everywhere outside the nonmixing core if it exists). Note that the stationary radial distribution of material in a, say, 20%-filled drum is the same as in an 80%-filled drum outside the unmixing core.

Numerically calculated theoretical stationary radial distributions have been compared with the experimental data obtained by means of the sampling procedure described in Sec. II. To make the comparison, we first calculated the radial distributions of particles and then, integrating over the sampler cross section, also the theoretical sample concentrations. In the first series of experiments, the drum was $(62 \pm 2)\%$ filled, and the sampling plate was such that several samples could be taken simultaneously at each of the two distances from the center, $r_1=0.42R$ and $r_2=0.83R$, where R is the drum radius. The total weight portion of fines varied from 10% to 90%. The calculated and experimentally determined sampler concentrations are very close (Fig. 10).

In the second series of experiments, the stationary spatial distributions of fines were measured using the sampling plates which allowed us to take samples from locations at various distances from the center. The total weight concentration of fines in all these experiments was 50%; the drum was either 39% or 56% filled. The correspondence of theoretical and experimental concentrations is also quite satisfactory, see Fig. 11.

These results show that our mixing and segregation model is able to predict the material distributions for various drum fillings and mixture compositions. Note that the size segregation operator of a binary mixture in this model has been identified using the data obtained for an entirely different geometric configuration and different materials. Therefore the agreement obtained confirms that this operator depends only on the particle diameter ratio.

IV. DISCUSSION

A theoretical model for mixing and radial size segregation of a binary mixture in a rotating drum was derived and tested

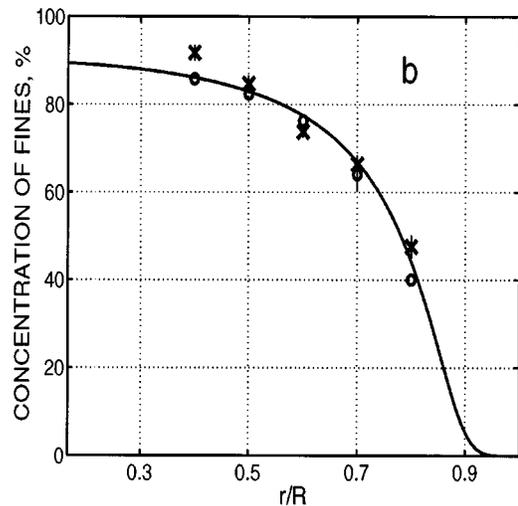
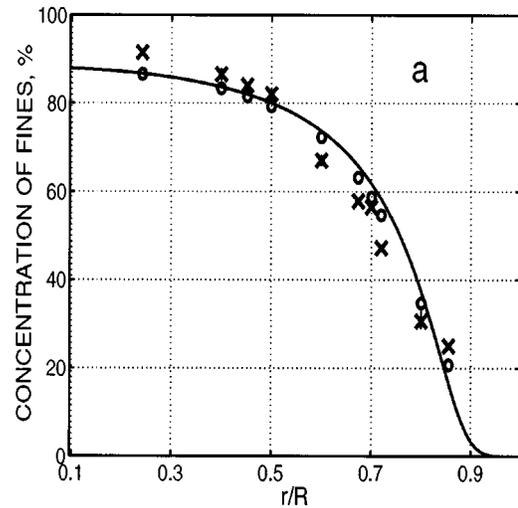


FIG. 11. Stationary distribution of fines. Drum filling: (a) 56%, (b) 39%. Total concentration 50%, particle diameter ratio 0.55. Thick solid lines—theoretical distributions, \circ —theoretical, \times —mean experimental concentrations in the samples. Short vertical lines connect the minimal and maximal experimental values of sample concentrations.

experimentally. This model consists of material transport equations, determined by the problem's geometry, and an operator of surface segregation, a phenomenological relation characterizing the segregation properties of granular material. The operator of size segregation for a binary mixture has been identified in [10]. In this work we employed this relation, which is supposedly independent of geometrical configuration, to simulate the size segregation of binary mixture in a rotating drum. The calculated and experimental material distributions agree well without any fitting of model parameters.

Obviously, our simplified model does not account for many factors that influence mixing and segregation. Thus we assumed that the surface flow is confined to a very thin boundary layer. This limits the applicability of our model to slow rotation. (Similarly, the model of granular body formation [10] is limited to cases where the source discharge rate is low.) Provided this condition is satisfied, it is less important whether the surface flow occurs continuously or by avalanches.

In this study we ignored the percolation of fines in the bulk of material, which can also lead to segregation. Generally, such percolation is significant only if the fine particles are several times smaller than the large ones, or in the presence of vibration. Since the diameter ratio was 0.55 in our experiments, no significant percolation in a motionless bulk should be expected. It is, however, possible that repeated revolutions of the bulk enhance interparticle percolation.

The axial segregation in our experiments was not entirely negligible: the bands containing a higher proportion of large particles could be observed near the drum ends after only 15–20 revolutions, when the rotation was stopped and the samples taken. The appearance of the first two axially segregated bands at the drum ends has also been reported in [6], where the interaction of axial and radial segregation was

studied experimentally. As far as we know, no explanation of the wall effect has been presented.

Despite these and other simplifications, our model succeeded in predicting the material distribution in a slowly rotating drum. This confirms the conjecture [10] that the material distribution inside granular bodies formed under various conditions can be predicted by a model in which the transport equations account for all “external” conditions (such as the geometrical configuration or the distribution of material sources). The segregation properties of polydisperse granular material in such a model are characterized by a constitutive relation (the operator of segregation), which does not depend on these external conditions and can be identified experimentally.

-
- [1] B. Roseman and M. B. Donald, *Br. Chem. Eng.* **7**, 823 (1962); R. L. Brown and J. C. Richards, *Principles of Powder Mechanics* (Pergamon Press, Oxford, 1970); J. C. Williams, *Powder Technol.* **15**, 245 (1976); J. Bridgwater, in *Granular Matter. An Interdisciplinary Approach*, edited by A. Mehta (Springer, New York, 1994), pp. 161–193.
- [2] J. A. Drahn and J. Bridgwater, *Powder Technol.* **36**, 39 (1983).
- [3] M. Alonso, M. Satoh, and K. Miyanami, *Powder Technol.* **68**, 145 (1991).
- [4] F. Cantelaube and D. Bideau, *Europhys. Lett.* **30**, 133 (1995); E. Clement, J. Rajchenbach, and J. Duran, *ibid.* **30**, 7 (1995); G. H. Ristow, *ibid.* **28**, 97 (1994).
- [5] O. Zik, D. Levin, S. G. Lipson, S. Shtrikman, and J. Stavans, *Phys. Rev. Lett.* **73**, 644 (1994); G. Metcalfe and M. Shattuck, *Physica A* **233**, 709 (1996).
- [6] K. M. Hill and J. Kakalios, *Phys. Rev. E* **52**, 4393 (1995); K. M. Hill, A. Caprihan, and J. Kakalios, *Phys. Rev. Lett.* **78**, 50 (1997).
- [7] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, *Rev. Mod. Phys.* **68**, 1259 (1996).
- [8] T. Elperin and A. Vikhansky, *Phys. Rev. E* **53**, 4536 (1996); T. Boutreux and P. G. de Gennes, *J. Phys. I* **6**, 1295 (1996).
- [9] M. A. Carson, *Earth Surf. Proc.* **2**, 363 (1977).
- [10] L. Prigozhin, *Chem. Eng. Sci.* **48**, 3647 (1993).
- [11] G. Metcalfe, T. Shinbrot, J. J. McCarthy, and J. M. Ottino, *Nature (London)* **374**, 39 (1995).
- [12] B. A. Peratt and J. A. Yorke, *Europhys. Lett.* **35**, 31 (1996).
- [13] D. V. Khakhar, J. J. McCarthy, T. Shinbrot, and J. M. Ottino, *Phys. Fluids* **9**, 31 (1997).