

Introduction

The atom magnetic trap is a device for trapping cold neutral or charged atoms by means of the magnetic field. The field is usually induced by a current in an electronic chip and the atoms, e.g. ⁸⁷Rb, are held at micro- or nano-kelvin temperatures in an isolated environment, such as vacuum, near the chip surface.

Magnetic traps are considered a powerful tool for study of the Bose-Einstein condensates, for coherent manipulation of the quantum state of the trapped atoms, and fundamental investigation of the atom-surface interaction. Such a device can be also used as a magnetic and an electric field probe, an element of a quantum computer, the atom SQUID, etc.

Increasing the life and coherence times of a magnetic trap is a high priority goal in the atom chip design. Superconductor nano-chips are able to generate stable magnetic traps having these times several orders of magnitude higher than those of the traps built upon conductors in the normal state.

Here we present a numerical method for simulating the 3D cold atom traps in which the magnetic field is induced by a current in a thin superconducting film in the mixed state. Our method is based on a variational formulation of thin film magnetization and transport current problems. The simulation results are presented for two magnetic traps: the self-sustained trap based on a thin square film, in which the super current is generated by two opposite pulses of the external magnetic field, and the trap built upon a Z-shaped thin film carrying a transport current. We compare our numerical results to the experiment data obtained for the magnetic trap based on a square superconducting film [1].

Numerical Approach

In the adiabatic approximation, the trapping of cold atoms occurs in the vicinity of a local minimum of the magnetic field modulus. The trap boundary is determined by a closed level surface surrounding this minimum and the trap depth is the difference between the maximal level of such closed surfaces and the minimum. The potential well of a stable trap, proportional to this depth, should be several times larger than the thermal energy of the atoms.

Computing magnetic field in the vicinity of a superconducting chip is based on the Biot-Savart law and needs an accurate calculation of the hysteretic sheet current density distribution, resulting from the history of variation of the external magnetic field or/and the applied transport current. The finite element method [1,2] allowed us to solve both the magnetization and transport current problems for thin type-II superconducting films of arbitrary shape and is applicable for both the power law and the critical-state current-voltage relations characterizing the superconductor.

Unlike the previous formulations of thin film magnetization problems written in terms of the magnetization function alone, our method is based upon a new (mixed) variational formulation and enables us to accurately compute not only the sheet current density but also the electric field. Numerical solution of the transport current problems for arbitrary shaped thin films is also new.

1. J. W. Barrett, L. Prigozhin, Electric field formulation for thin film magnetization problems. *Supercond. Science and Technology*, 2012, v. 25, 104002
2. J. W. Barrett, L. Prigozhin, V. Sokolovsky, Transport current and magnetization problems for thin type-II superconducting films. *Supercond. Science and Technology*, 2013, v. 26, 105009.

Traps generated by a superconducting square film

First we simulated the experiment [3]. The trap for ⁸⁷Rb atoms was based on a superconducting YBCO film 1 mm × 1 mm × 800 nm. One of the traps in [3] was created by the magnetic field induced by the film current resulting from two consequent pulses of the external magnetic field normal to the film: $0 \rightarrow 3\mu_0 J_c \rightarrow -\mu_0 J_c \rightarrow 0$. Qualitatively, our simulations (Fig. 1a) agree with the experiment (Fig. 1b); the numerical results are in dimensionless form: magnetic field is normalized by $\mu_0 J_c$ and all the sizes are given in the units of a half-side of the superconducting square. Magnetic traps created by the same pulses supplemented by a bias field normal to the film are shown in Fig. 2 (simulation).

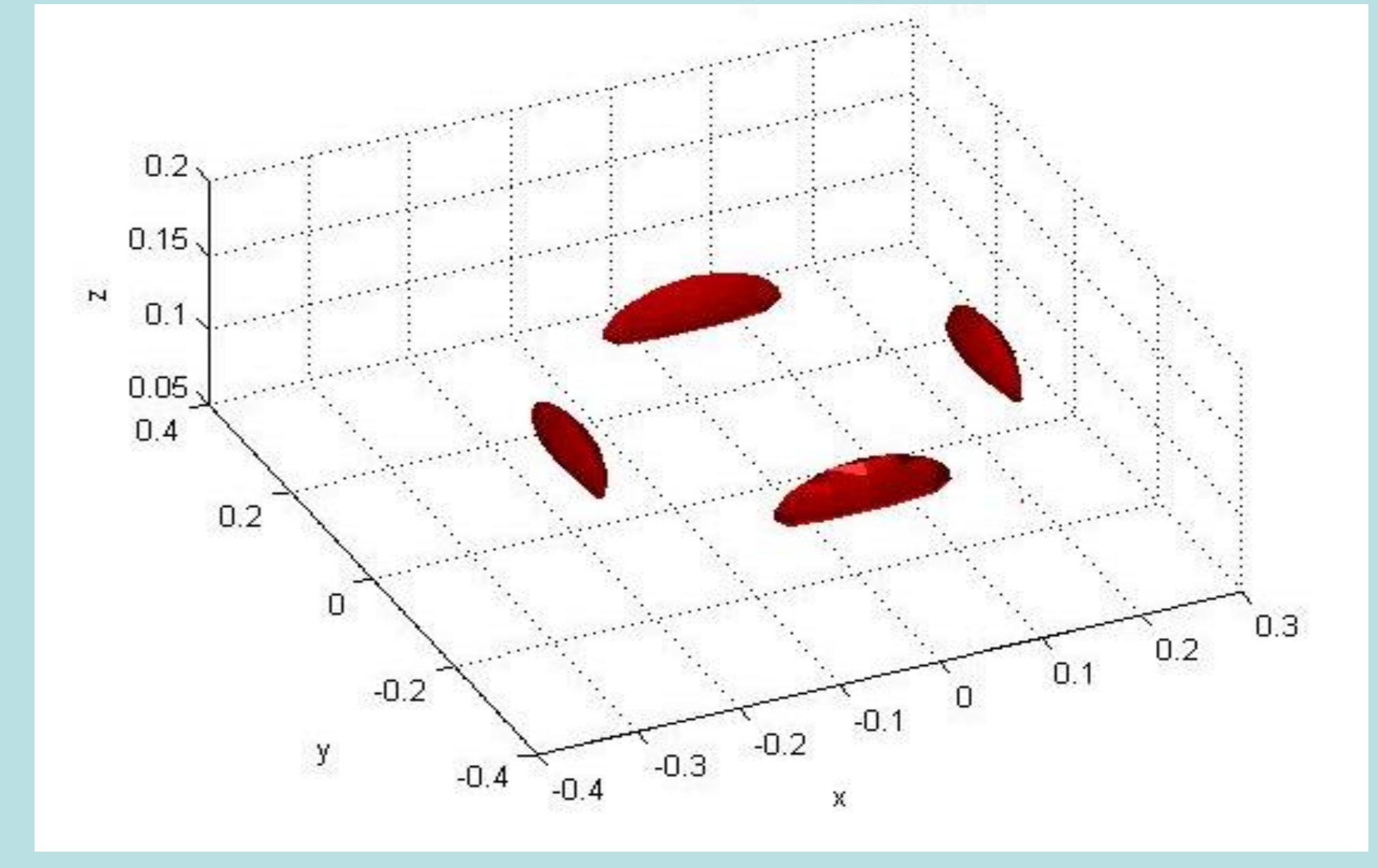
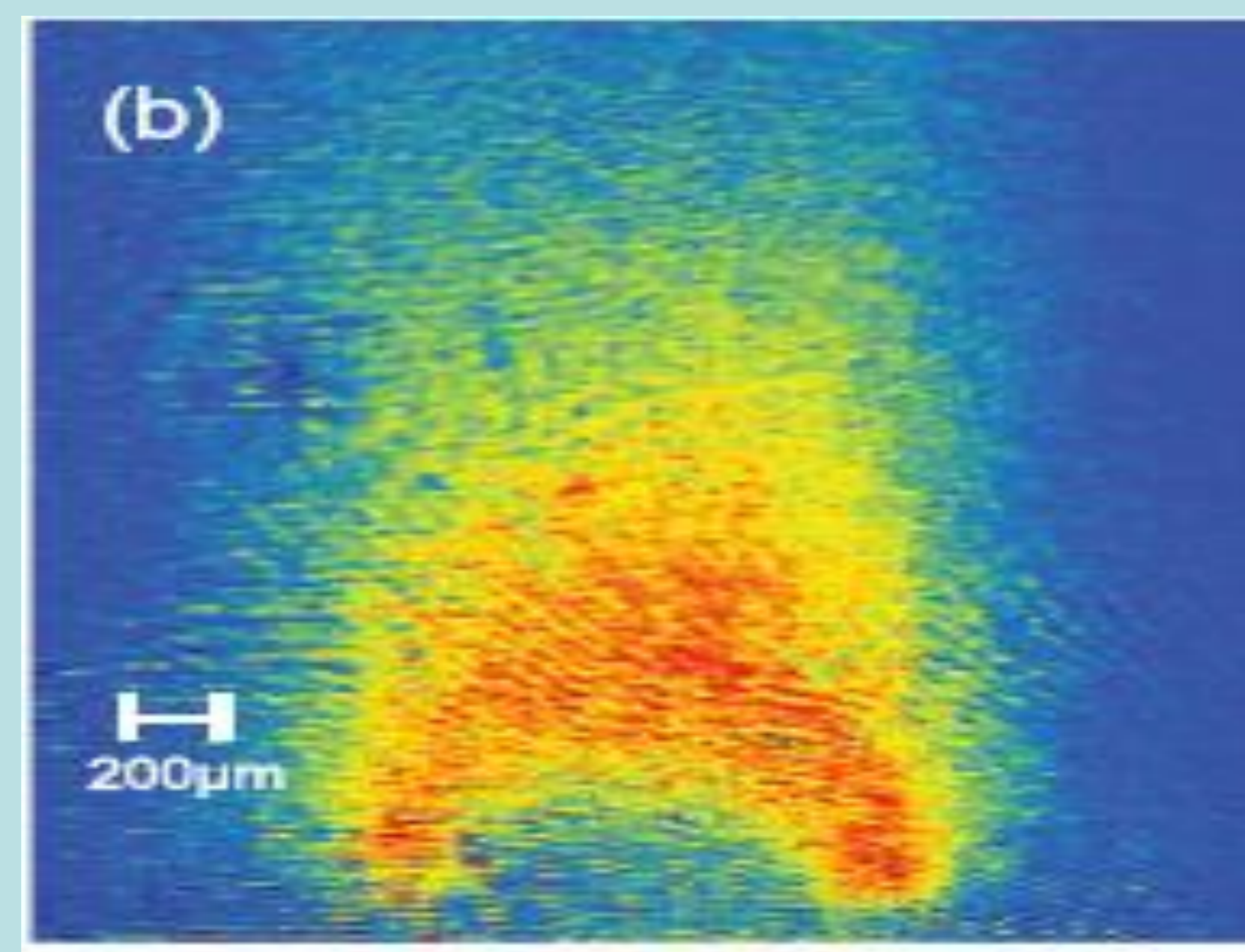
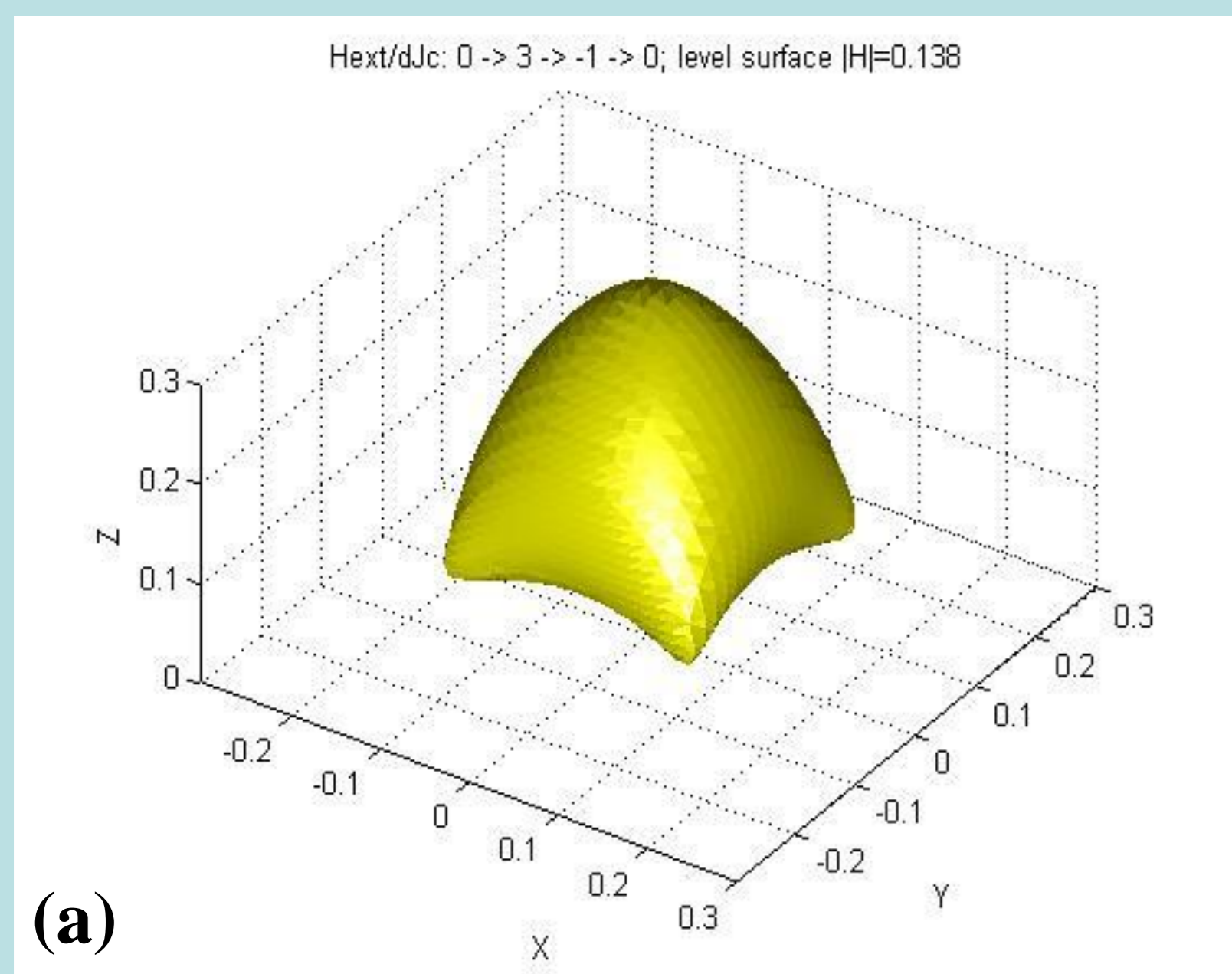


Fig. 1 Self-sustained magnetic trap based on a superconducting square film: a) the simulated magnetic field level surface; b) experiment: the atom cloud (reproduced from [1]).

Fig. 2. Simulated traps created by the pulses $0 \rightarrow 3\mu_0 J_c \rightarrow -\mu_0 J_c \rightarrow 0$ and the $0.5\mu_0 J_c$ bias field: the $0.1\mu_0 J_c$ magnetic field level surface is shown.

3. M. Siercke, K. S. Chan, B. Zhang, M. Beiang, M. J. Lim, R. Dumke, Reconfigurable self-sufficient traps for ultracold atoms based on a superconducting square, *Phys. Rev. A*, 85, 041403 (R), 2012

A trap based on a z-shaped superconducting film

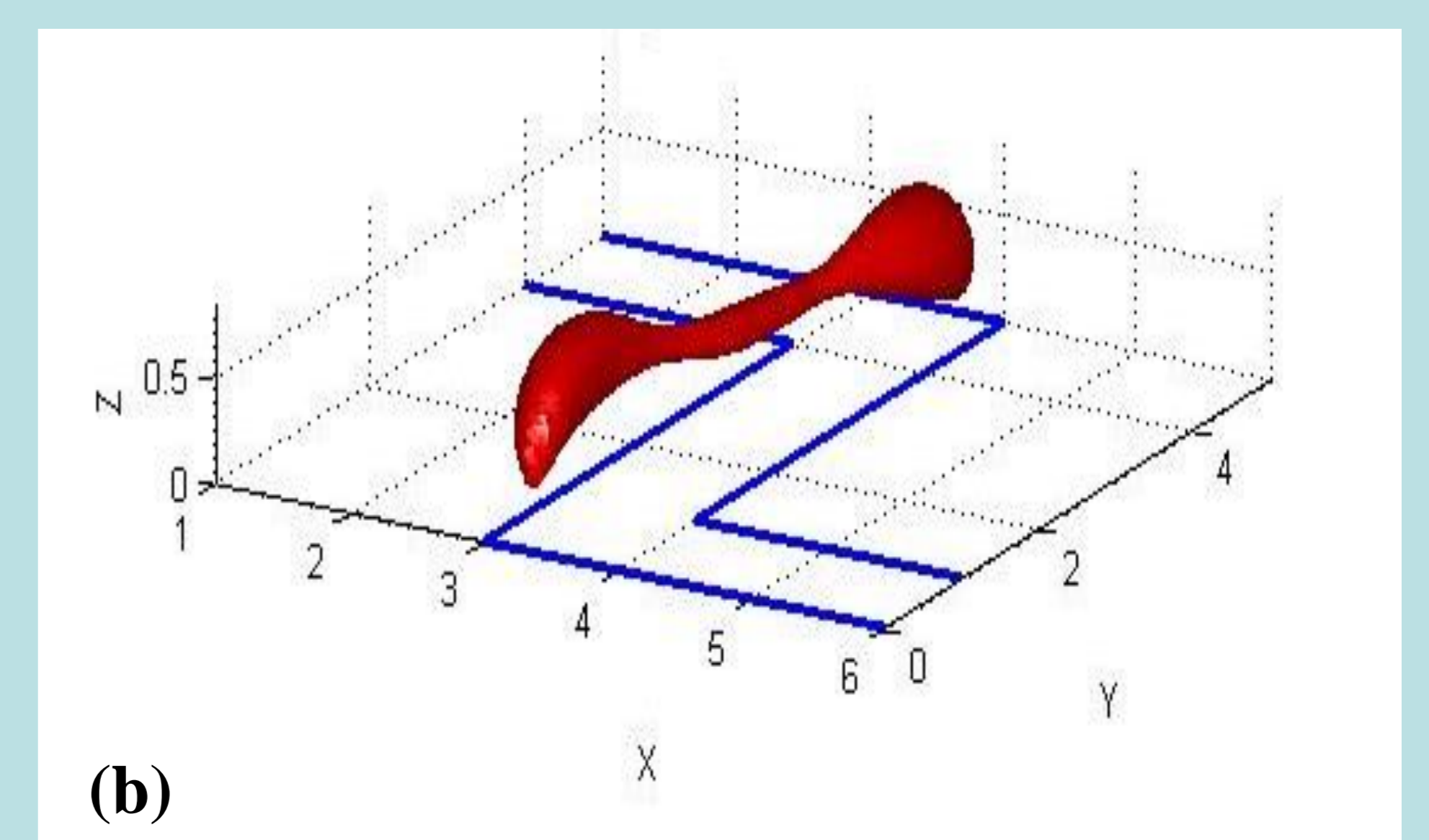
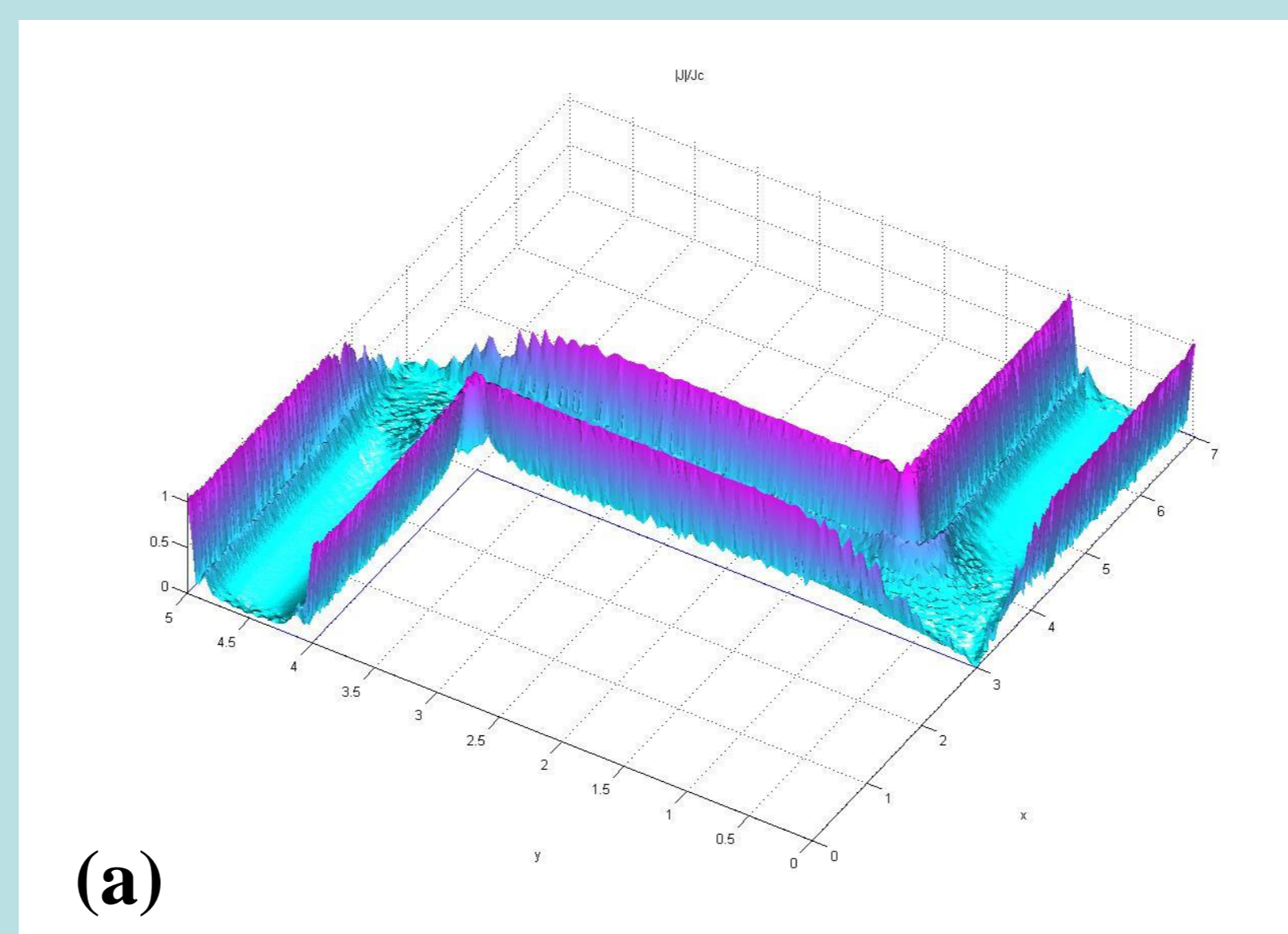
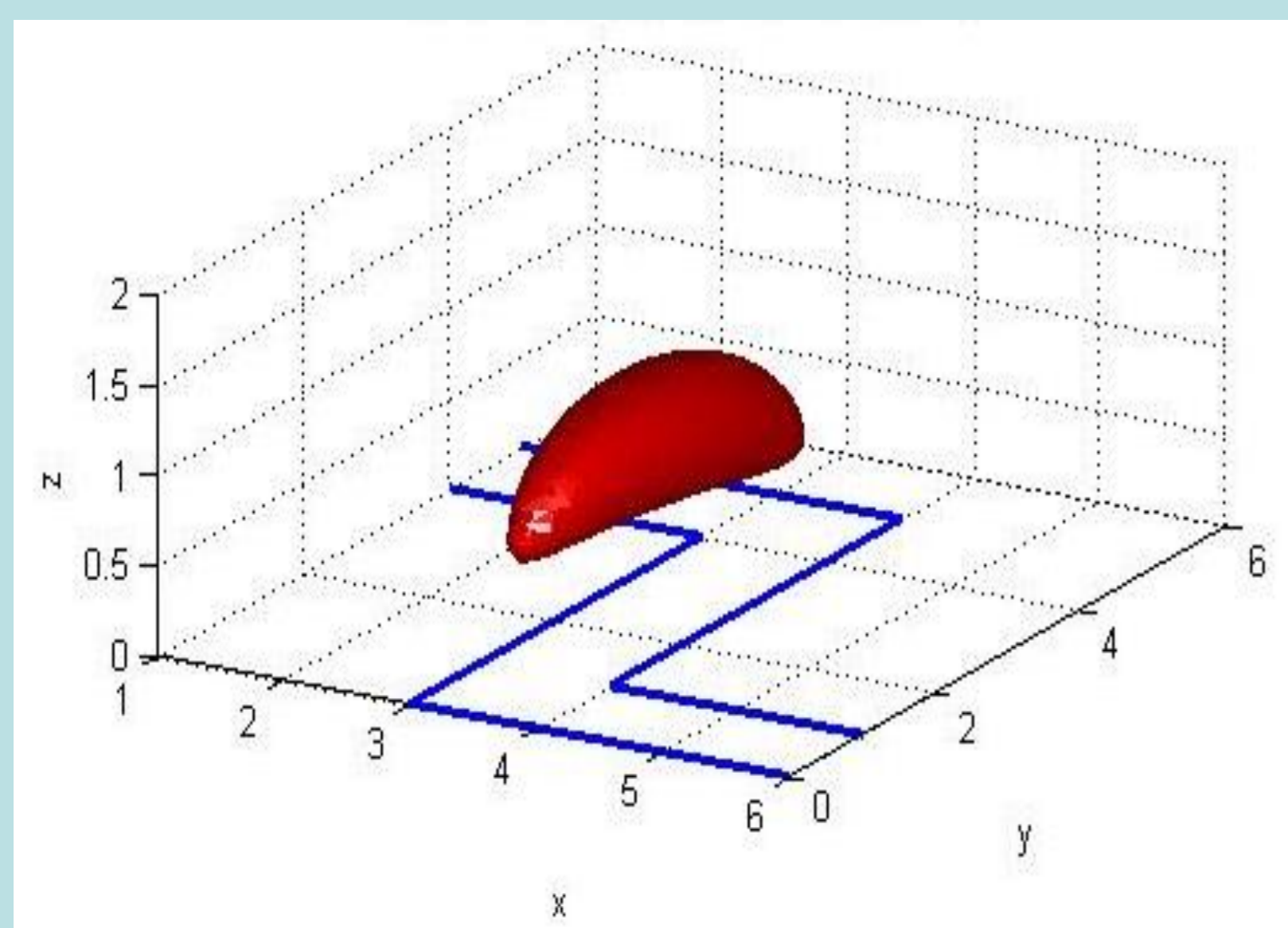


Fig. 3 Simulation: the transport current $0.7I_c$ and x -directed bias field $0.1\mu_0 J_c$ are applied. Magnetic field level surface $|H| = 0.045\mu_0 J_c$ is shown.

Fig. 4 The simulated transport current pulse $0 \rightarrow 0.7I_c \rightarrow 0$: a) the modulus of sheet current density; b) the magnetic trap at the bias field of $0.007\mu_0 J_c$; the magnetic field level is $0.004\mu_0 J_c$.

Magnetic field is normalized by $\mu_0 J_c$ and all the sizes are given in the units of a width of the superconducting strip.

Conclusion

Our method, based on a new variational formulation of thin film magnetization and transport current problems, allows one to simulate magnetic traps based upon a superconducting film of arbitrary shape, carrying a transport current or/and magnetized by the applied magnetic and current pulses. The method can be employed for designing the superconducting chips.