Depth and Shape Inference

(III)

Introduction to Computational and Biological Vision

CS 202-1-5261

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Shape from Shading
Shape from Shading
Shape from Shading

Shading is more than contours
Shape from Shading

Shading is more than edge contours
Shape from Shading

Inverting the image formation process

Image formation = “Shading from shape” (and light sources)
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Image formation

**Image formation 1:** Where is a world/object point projected in the image plane?

**Image formation 2:** What is the amount of light that is reflected in the direction of the camera?
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represents the direction of light and reflectance.

\[ R(\phi_r, \theta_r) \]

\[ E(\phi_i, \theta_i) \]

\( \theta \) - Azimuth angle

\( \phi \) - Zenith angle
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The Bidirectional Reflectance Distribution Function (BRDF)

\[ f_\lambda(\phi_i, \theta_i; \phi_r, \theta_r) = \frac{R_\lambda(\phi_r, \theta_r)}{E_\lambda(\phi_i, \theta_i)} \]

Helmholtz’s reciprocity

\[ f(\phi_i, \theta_i; \phi_r, \theta_r) = f(\phi_r, \theta_r; \phi_i, \theta_i) \]

Isotropic materials:

\[ f(\phi_i, \theta_i; \phi_r, \theta_r) = f(\phi_i - \phi_r, \theta_i, \theta_r) \]
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Mirrored (perfectly secular) surfaces

\[ f_s(\phi_i, \theta_i; \phi_r, \theta_r) \propto \delta(\theta_r - \theta_i)\delta(\phi_r - \phi_i - \pi) \]
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Lambertian (perfectly diffused) surfaces

\[ f_L(\phi_i, \theta_i; \phi_r, \theta_r) = \text{const} \propto \rho \]

Albedo
Shape from Shading

Realistic (mixed) surfaces

\[ f(\phi_i, \theta_i; \phi_r, \theta_r) = \alpha \cdot f_L(\phi_i, \theta_i; \phi_r, \theta_r) + (1 - \alpha) f_S(\phi_i, \theta_i; \phi_r, \theta_r) \]
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What is the intensity reflected in the direction of the camera?

\[
f_\lambda(\phi_i, \theta_i; \phi_r, \theta_r) = \frac{R_\lambda(\phi_r, \theta_r)}{E_\lambda(\phi_i, \theta_i)} \quad \text{and} \quad R_\lambda(\phi_r, \theta_r) = f_\lambda(\phi_i, \theta_i; \phi_r, \theta_r)E_\lambda(\phi_i, \theta_i)
\]
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Total surface reflection towards the camera

\[ R(\phi_r, \theta_r) = \int_{\omega} f(\phi_i, \theta_i; \phi_r, \theta_r) E(\phi_i, \theta_i) \cos \theta_i \, d\omega \]

\[ R(\phi_r, \theta_r) \]

\[ \delta \omega = \sin \theta_i \delta \theta_i \delta \phi_i \]
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Total surface reflection towards the camera

\[ R(\phi_r, \theta_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} f(\phi_i, \theta_i; \phi_r, \theta_r) \cdot E(\phi_i, \theta_i) \cdot \sin \theta_i \cdot \cos \theta_i \, \delta \theta_i \, \delta \phi_i \]
**Shape from Shading**

Point light source from direction \((\phi_L, \theta_L)\)

\[
E(\phi_i, \theta_i) = E \cdot \frac{\delta(\theta_L - \theta_i) \cdot \delta(\phi_L - \phi_i)}{\sin \theta_L}
\]

\[
\int_{-\pi}^{\pi} \int_{0}^{\pi/2} E(\phi_i, \theta_i) \cdot \sin \theta_i \cdot \delta \theta_i \delta \phi_i = E
\]
Shape from Shading

Lambertian (perfectly diffused) surfaces

\[ f_L(\phi_i, \theta_i; \phi_r, \theta_i) = \text{const} = \bar{f} = \frac{1}{\pi} \rho \]

Albedo

\[ \int_{-\pi}^{\pi} \int_{0}^{\pi/2} \bar{f} \cdot \sin \theta_r \cdot \cos \theta_r \cdot \delta \theta_r \delta \phi_r = 1 \]

\[ \pi \bar{f} = 1 \]
Shape from Shading

Surface brightness – appearance in the Lambertian case and point light source

\[ f_L(\phi_i, \theta_i; \phi_r, \theta_r) = \rho \frac{1}{\pi} \]

\[ E(\phi_i, \theta_i) = \frac{\delta(\theta_L - \theta_i)\delta(\phi_L - \phi_i)}{\sin \theta_L} \]

\[
I(x, y) \propto R(\phi_r, \theta_r) = \int_{-\pi}^{\pi} \int_0^{\pi/2} f(\phi_i, \theta_i; \phi_r, \theta_r) \cdot E(\phi_i, \theta_i) \cdot \sin \theta_i \cdot \cos \theta_i \delta \theta_i \delta \phi_i
\]

\[ R = \rho \frac{1}{\pi} \cdot E \cdot \cos \theta_L \propto \rho(\hat{N} \cdot \hat{L}) \]
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So... How can we use all this for shape from shading?

\[ R \propto \rho(\hat{N} \cdot \hat{L}) \]
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Shape description via tangent plane and normal surface of the object at each point

\[ \mathbf{r}_x = \left( 1, 0, \frac{\partial H}{\partial x} \right) \quad \mathbf{r}_y = \left( 1, 0, \frac{\partial H}{\partial y} \right) \]

\[ \mathbf{N} = (N_x, N_y, N_z) = \frac{\mathbf{r}_x \times \mathbf{r}_y}{\|\mathbf{r}_x \times \mathbf{r}_y\|} = \frac{(-p, -q, 1)}{\sqrt{p^2 + q^2 + 1}} \]

\[ (x, y, H(x, y)) \]
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Different ways to represent directions (and function thereof)

\[ V = \frac{(v_x, v_y, v_z)}{\sqrt{v_x^2 + v_y^2 + v_z^2}} \]

\[ V = \frac{(-p_v, -q_v, 1)}{\sqrt{p_v^2 + q_v^2 + 1}} \]

\[ V \in S^2 \]

\[ f(V) = f(v_x, v_y, v_z) \]

\[ f(V) = f(p_v, q_v) \]

\[ f(V) \]
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Shading on Lambertian surface – General point source

\[ I = \rho (\hat{N} \cdot \hat{L}) = \rho \frac{-p \cdot L_x - q \cdot L_y + L_z}{\sqrt{p^2 + q^2 + 1} \sqrt{L_x^2 + L_y^2 + L_z^2}} = \rho \frac{p \cdot p_L + q \cdot q_L + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_L^2 + q_L^2 + 1}} \]
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Shading on Lambertian surface – Overhead point source

\[ I(x, y) = \rho(\hat{N} \cdot [0,0,1]) = \rho \frac{1}{\sqrt{p^2 + q^2 + 1}} = R(p, q) \]
**Shape from Shading**

The Reflectance Map – Lambertian surface from overhead source position

\[ R(p, q) = \frac{1}{\sqrt{p^2 + q^2 + 1}} \]
**Shape from Shading**

The Reflectance Map – Lambertian surface from general source position

\[ R(p, q) = \frac{p \cdot p_L + q \cdot q_L + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_L^2 + q_L^2 + 1}} \]

Gradient point of maximum brightness
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The Reflectance Map – typical real surfaces

\[ R(p, q) \]
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Surface orientation from shading

\[ I(x, y) = R(p, q) \]
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Photometric stereo
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Photometric stereo

\[ I_1(x, y) = R_1(p, q) \]
\[ I_2(x, y) = R_2(p, q) \]
The SFS problem (special case)

Given $I(x,y)$ of an (orthographic) projection of $H(x,y)$, and the reflectance map $R(p,q)$, find $H(x,y)$ everywhere.

$$I(x, y) = R(p, q) = R\left(\frac{\partial}{\partial x} H(x, y), \frac{\partial}{\partial y} H(x, y)\right)$$
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Shape recovery via characteristic strips

\[ I(x, y) = R(p(x, y), q(x, y)) \]
\[ p(x, y) = \frac{\partial}{\partial x} H(x, y) \]
\[ q(x, y) = \frac{\partial}{\partial y} H(x, y) \]

\[ H(x + \delta x, y + \delta y) \cong H(x, y) + p \delta x + q \delta y \]
\[ p(x + \delta x, y + \delta y) \cong p(x, y) + \frac{\partial p}{\partial x} \delta x + \frac{\partial p}{\partial y} \delta y \]
\[ q(x + \delta x, y + \delta y) \cong q(x, y) + \frac{\partial q}{\partial x} \delta x + \frac{\partial q}{\partial y} \delta y \]
Shape from Shading

Shape recovery via characteristic strips

\[ I(x, y) = R(p(x, y), q(x, y)) \]

\[
\begin{align*}
\frac{\partial}{\partial x} I(x, y) &= \frac{\partial R(p, q)}{\partial p} \frac{\partial p(x, y)}{\partial x} + \frac{\partial R(p, q)}{\partial q} \frac{\partial q(x, y)}{\partial x} \\
\frac{\partial}{\partial y} I(x, y) &= \frac{\partial R(p, q)}{\partial p} \frac{\partial p(x, y)}{\partial y} + \frac{\partial R(p, q)}{\partial q} \frac{\partial q(x, y)}{\partial y}
\end{align*}
\]
Shape from Shading

Shape recovery via characteristic strips

\[ I(x, y) = R(p(x, y), q(x, y)) \]

\[ \frac{\partial}{\partial x} I(x, y) = \frac{\partial R(p, q)}{\partial p} \frac{\partial p(x, y)}{\partial x} + \frac{\partial R(p, q)}{\partial q} \frac{\partial p(x, y)}{\partial y} \]

\[ \frac{\partial}{\partial y} I(x, y) = \frac{\partial R(p, q)}{\partial p} \frac{\partial q(x, y)}{\partial x} + \frac{\partial R(p, q)}{\partial q} \frac{\partial q(x, y)}{\partial y} \]
Shape from Shading

Shape recovery via characteristic strips

\[ I(x, y) = R(p(x, y), q(x, y)) \]

\[ \frac{\partial}{\partial x} I(x, y) = \nabla R \cdot \nabla p \]

\[ \frac{\partial}{\partial y} I(x, y) = \nabla R \cdot \nabla q \]

\[ \delta H \approx p \delta x + q \delta y \]

\[ \delta p \approx \frac{\partial p}{\partial x} \delta x + \frac{\partial p}{\partial y} \delta y = \nabla p \cdot (\delta x, \delta y) \]

\[ \delta q \approx \frac{\partial q}{\partial x} \delta x + \frac{\partial q}{\partial y} \delta y = \nabla q \cdot (\delta x, \delta y) \]

A smart choice

\[ \delta x = \frac{\partial R(p, q)}{\partial p} \delta s \]

\[ \delta y = \frac{\partial R(p, q)}{\partial q} \delta s \]

\[ \delta p \approx \frac{\partial}{\partial x} I(x, y) \cdot \delta s \]

\[ \delta q \approx \frac{\partial}{\partial y} I(x, y) \cdot \delta s \]
Shape from Shading

Shape recovery via characteristic strips

\[ \delta x = R_p \delta s \]

\[ \delta y = R_q \delta s \]

\[ \delta H = (pR_p + qR_q) \delta s \]

\[ \delta p = I_x \delta s \]

\[ \delta q = I_y \delta s \]

\[ \dot{x} = R_p \]

\[ \dot{y} = R_q \]

\[ \dot{H} = pR_p + qR_q \]

\[ \dot{p} = I_x \]

\[ \dot{q} = I_y \]
Shape from Shading

Shape recovery via characteristic strips

Shape from Shading via Characteristic Curves

Given

- $I(x,y)$ of an (orthographic) projection of unknown $H(x,y)$
- The reflectance map $R(p,q)$
- Initial data $x_0, y_0, H(x_0, y_0), p(x_0, y_0), q(x_0, y_0)$

Develop a curve solution on $H(x,y)$ by taking small steps of size $\delta s$ via the system

\[
\begin{align*}
\delta x &= R_p \delta s \\
\delta y &= R_q \delta s \\
\delta H &= (pR_p + qR_q) \delta s \\
\delta p &= I_x \delta s \\
\delta q &= I_y \delta s
\end{align*}
\]
Shape from Shading

Shape recovery via characteristic strips
Shape from Shading

Shape recovery via characteristic strips