Object Identification and Recognition (II)

Introduction to Computational and Biological Vision

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2D Regions and Shape Analysis

2D Shape representation

\[ X = (x_1, x_2, \ldots, x_n) \approx \]

\[ Y^1 = (y_1^1, y_2^1, \ldots, y_n^1) \leftrightarrow \]

\[ Y^2 = (y_1^2, y_2^2, \ldots, y_n^2) \leftrightarrow \]

\[ \vdots \]

\[ Y^m = (y_1^m, y_2^m, \ldots, y_n^m) \leftrightarrow \]
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2D Shape representation: What can the features describe?

\[ X = (x_1, x_2, \ldots, x_n) \]

**Region** (more explicit)

<table>
<thead>
<tr>
<th>Representation Scope</th>
<th>Contour (more compact)</th>
<th>Region (more explicit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global Features</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local Features</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2D Regions and Shape Analysis

Contour representation

- Parametric curve (continuous)
- Point list (continuous and discrete)
- Chain code (discrete)
- Polygonal approximation (continuous and discrete)
- Piecewise polynomial approximation

\[ \alpha(t) \]

4-neighbourhood

8-neighbourhood

\[
\begin{align*}
\text{NW} & \quad \text{N} & \quad \text{NE} \\
\text{W} & \quad \text{E} & \quad \text{SW} & \quad \text{S} & \quad \text{SE} \\
\text{FL} & \quad \text{F} & \quad \text{FR} \\
\text{L} & \quad \text{R} \\
\text{BL} & \quad \text{B} & \quad \text{BR}
\end{align*}
\]
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Area representation

- Explicit (discrete)
- Sweeping lines (constant/varying orientation)
- Standard regions
- Hierarchical representations (e.g., quad tree)
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Skeleton representation

- 1D curve (or graph) representation for 2D regions
- Captures explicitly
  - shape geometry
  - general structure of the shape in terms of parts
  - general “strokes”
  - local symmetry
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Skeleton representation via the medial axis

\[ d(p, \partial S) = \min_{q \in \partial S} (p, q) \]

\[ B(p, S) = \{ q \in \partial S \mid d(p, q) = d(p, S) \} \]

\[ S^*(S) = \{ p \in S \mid \|B(p, S)\| > 1 \} \]

**Medial axis:** the set of all points having more than one closest point on the object’s boundary.

**Medial axis Transform:** \[ S \leftrightarrow MA(S) = \{ (p, d(p, \partial S)) \mid p \in S^*(S) \} \]
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Skeleton representation via the medial axis
2D Regions and Shape Analysis

Skeleton representation via the medial axis


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Shape invariants

\[ x_i = 1 \quad \Rightarrow \quad S \text{ is } \langle \text{your shape type of interest}\rangle \]

Example (…perhaps the only obvious one)

\[ x_1 = \text{Area}(S) \]
\[ x_2 = \text{Perimeter}(S) \]
\[ x_3 = SF(S) = 4\pi \frac{x_1}{x_2} \]

\[ S = \begin{cases} 
\text{Circle} & x_3 = 1 \\
\text{Non circle} & x_3 < 1 
\end{cases} \]
Shape Moments (from region representation)

\[ h_s(x, y) = \begin{cases} 
0 & (x, y) \not\in S \\
1 & (x, y) \in S 
\end{cases} \]

\[ M_{pq} = \iint_I h_s(x, y) x^p y^q \, dx \, dy \]

\[ M_{pq} = \sum_i \sum_j h_s(i, j) i^p j^q \]
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Shape Moments (from contour representation)

\[ \oint_{\partial S} \left( \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right) dx dy = \int_{\partial D} L dx + M dy \]

\[ L(x, y) = 0 \]

\[ M(x, y) = \frac{1}{p + 1} x^{p+1} y^q \]

\[ M_{pq} = \int_S x^p y^q dx dy = \int_S \left( \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right) dx dy = \oint_{\partial S} \frac{1}{p + 1} x^{p+1} y^q dy \]
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Shape Moments

\[
M_{pq} = \iint_{S} h_{S}(x, y)x^{p}y^{q}dx\,dy
\]

\[
M_{00} = \iint_{S} h_{S}(x, y)dx\,dy
\]

\[
M_{10} = \iint_{S} x \cdot h_{S}(x, y)dx\,dy
\]

\[
M_{01} = \iint_{S} y \cdot h_{S}(x, y)dx\,dy
\]
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First order moments and central moments

\[ C = \left( \frac{M_{10}}{M_{00}}, \frac{M_{01}}{M_{00}} \right) \]

\[ M_{pq} = \iint_{I} h_S(x, y) x^p y^q \, dx \, dy \]

\[ \overline{M}_{pq} = \iint_{I} h_S(x, y) (x - \overline{x})^p (y - \overline{y})^q \, dx \, dy \]
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Second order central moments

\[ \overline{M}_{20} = \iint_S (x - \bar{x})^2 \, dx \, dy \]

\[ \overline{M}_{20} = \iint_S x^2 \, dx \, dy \quad \bar{x} = \frac{M_{10}}{M_{00}} - \bar{x}^2 \iint_S \, dx \, dy \]

\[ \overline{M}_{20} = M_{20} - 2\bar{x}M_{10} + \bar{x}^2 M_{00} \]

\[ \overline{M}_{20} = M_{20} - \bar{x}^2 M_{00} \]
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Second order moments

\[
\begin{align*}
\bar{M}_{20} &= M_{20} - \bar{x}^2 M_{00} \\
\bar{M}_{11} &= M_{11} - \bar{x}\bar{y} M_{00} \\
\bar{M}_{02} &= M_{02} - \bar{y}^2 M_{00}
\end{align*}
\]

\[
\begin{align*}
\mu'_{20} &= \bar{M}_{20} / \bar{M}_{00} = M_{20} / M_{00} - \bar{x}^2 \\
\mu'_{11} &= \bar{M}_{11} / \bar{M}_{00} = M_{11} / M_{00} - \bar{x}\bar{y} \\
\mu'_{02} &= \bar{M}_{02} / \bar{M}_{00} = M_{02} / M_{00} - \bar{y}^2
\end{align*}
\]

\[
R = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\]

\[
\tilde{C} = \text{COV}[S] = \begin{bmatrix}
\mu'_{20} & \mu'_{11} \\
\mu'_{11} & \mu'_{02}
\end{bmatrix} = R \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_1
\end{bmatrix} R^T
\]
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Second order moments

\[
\begin{bmatrix}
\mu'_2 & \mu'_1 \\
\mu'_1 & \mu'_0
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{bmatrix}
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\]

Shape orientation:
\[
\theta = \frac{1}{2} \arctan \frac{2\mu'_1}{\mu'_2 - \mu'_0}
\]

\[
\lambda_i = \frac{1}{2} \left( \mu'_2 + \mu'_0 \pm \sqrt{4 \mu'^2_1 + (\mu'_2 - \mu'_0)^2} \right)
\]

Shape eccentricity:
\[
e = \sqrt{1 - \frac{\lambda_2}{\lambda_1}}
\]
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Aspect ratio

\[ AR = \frac{\Delta y}{\Delta x} \]

\[ AR(\theta) = \frac{\Delta y(\theta)}{\Delta x(\theta)} \]

\[ \overline{AR}(\theta) = \max \{ AR(\theta) \mid \theta \in [0..2\pi] \} \]

\[ \overline{AR}(\theta) = \lambda_1 / \lambda_2 \]
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Total boundary curvature

\[ \alpha(s) \Rightarrow \kappa(s) \]

\[ E = \int_0^l \kappa^2(s) ds \]

\[ E[\text{circle of radius } R] = \int_0^{2\pi R} \frac{1}{R^2} ds = \frac{2\pi}{R} \]

\[ E = \frac{l}{4\pi^2} \cdot \int_0^l \kappa^2(s) ds \]

Scale invariant
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Computing skeletons

The naïve approach:

- For each interior point \( p \) of \( S \)
  - Find the set \( B(p, S) \) of closest boundary points
  - If \( ||B(p, S)|| > 1 \) mark as a skeleton point

Blum’s Grassfire Transform:

- Set the boundary on fire
- Mark points where fire fronts meet
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Skeletonization via Distance Transform

\[
D(p) \leftarrow 0 \quad \forall p \in \text{Image} \\
\text{Front} \leftarrow \partial S \\
D(p) \leftarrow 1 \quad \forall p \in \text{Front} \\
\text{While Front} \neq \emptyset \\
\text{Pick one pixel } p \in \text{Front} \\
\text{Front} \leftarrow \text{Fron} \setminus \{p\} \\
\text{For all } q \in \text{Neighborhood}(p) \\
D'(q) = D(p) + d(p, q) \\
\text{if } D'(q) < D(q) \\
D(p) \leftarrow D'(q) \\
\text{Front} \leftarrow \text{Front} \cup \{q\} \\
\text{end}
\]

end
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Medial Axis Transform and Skeletonization via Distance Transform

Image/Shape

Distance Transform

Medial Axis Transform

Ridges and singularities
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Voronoi Diagrams for skeletonization

• Sample the boundary of shape $S$ densely enough.
• Compute the Voronoi diagram $V$.
• Take the subset of $V$ that is fully contained in $S$.

[Tagliasacchi 2014]