Home Exam in Distrib'd Alg's

Due Date: Feb. 19

1) ~~Sps~~ Suppose that for a matching M, there are no augmenting paths of length ≤ 3.
Prove that $w(M) \geq \frac{2}{3} w(M^*)$, where M^* is the maximum cardinality matching.

2) Devise a construction of graphs w/ arbitrarily large girth and fractional chromatic number.

3) Given a torus $\{0,1,2\} \times \{0,1,\ldots,n-1\}$, w/ every vertex $(i,j) \in \{0,1,2\} \times \{0,1,\ldots,n-1\}$ connected to every other (i',j), $i \neq i'$, (w/ the same index j), and also every (i,j) connected to $(i, j+1 \pmod n)$)

and $(i, j-1 \pmod n)$.
Assume that every vx knows its i-index, but not its j-index.
Assume also that all vs have distinct Id numbers from the range $\{1, 2, \ldots, 3n\}$.

Devise a distributed 3-vx-coloring alg' for coloring the torus, ~~its~~ that runs in $\log^* n + O(1)$ time.

4)(a) Devise an $O(a \cdot \Delta)$-dg-coloring alg' that runs in $O(\log n)$ time, where a is the arboricity, and Δ is the max'm deg'.

(b) Devise an $O(\Delta^2)$-dg-coloring alg' that runs in $O(\log^* n)$ time, without using Linial's alg', (or any ~~an~~ vx-coloring analogue of Linial's alg').

5)(a) How large can be the maximum degree of a graph w/ neighborhood independence α and degeneracy d?
Prove your upper bound.

(b) Devise a variant of Luby's reduction for graphs w/ small neighborhood independence α.
Analyze the reduction.
What is the neighborhood independence of the graph that is obtained from the reduction?

6) Analyze an alg' for UPCAST in CONGEST(b) model, where b is an arbitrary parameter.
What is the running time of the alg'.

Good Luck!