Assign't 3, Distr' Alg's

Due Date: Jan. 22, 2017.

1) Given a metric $\mathcal{M} = (V, d)$ "$d_\mathcal{M}$" on ~~n~~ n vertices, an MST of $\mathcal{M}$ is the ~~min'm~~ spanning tree of $\mathcal{M}$ w/ min'm wt.

A Steiner minimum tree (SMT) is a tree $\mathcal{T}$ on a superset $X \supseteq V$, s.t. for every pair $u, v \in V$,

$d_\mathcal{T}(u,v) \geq d_\mathcal{M}(u,v)$.

Prove that $w(\mathcal{T}) \geq \frac{1}{2} w(MST)$, i.e., SMT cannot be more than twice lighter than the MST.

2) Given a matching M, an odd length path $P = (e_1, e_2, e_3, \dots, e_{2q+1})$, for some $q = 0, 1, 2, \dots$, in which $e_2, e_4, \dots, e_{2q} \in M$, and such that its two endpoints are not M-matched, is called an <u>augmenting path</u> for M.

Suppose that for a matching M, there are no augmenting paths of length ≤ 3.

Prove that $w(M) \geq \frac{2}{3} w(M^*)$, where M^* is the max'm cardinality matching.

3) Devise a construction of triangle-free graphs G w/ arbitrarily large degeneracy $degen(G)$ and such that $\chi(G) = degen(G) + 1$.

4) The <u>fractional chromatic number</u> $\chi_f(G)$ of a graph G is the smallest number t such that there exists a probability distribution $\mathcal{D}$ over independent sets of G s.t. for every vx v, ~~given~~ $\mathbb{P}_{S \sim \mathcal{D}}(v \in S) \geq \frac{1}{t}$.

Devise a construction of graphs w/ arbitrarily large girth and chromatic number.

5) Generalize the greedy alg' for constructing $(2t-1)$-spanners of size $O(n^{1+1/t})$ that we saw in class to weighted graphs. Analyze the tradeoff betw' the stretch and the number of edges in resulting spanners.

6) Show that any tree metric d_T satisfies the four point condition, i.e., for every x, y, u, v

$$d_T(x,y) + d_T(u,v) \leq$$

$$\leq \max\{d_T(x,u) + d_T(y,v), \; d_T(x,v) + d_T(y,u)\}.$$

Enjoy!