7/3/2013  Exam on Distributed Algorithms
2nd term (Mond Bet)

Solve 3 questions out of 4.
Exam's duration: 2 and half hours.

1) a) Describe Luby's reduction from $(\Delta+1)$-coloring to the MIS problem. Prove its correctness.
   Analyze it. (Given an alg. that solves MIS in graphs with n vs and max' degree $\Delta$ within $T(n,\Delta)$ time, how much time will one need to solve $(\Delta+1)$-coloring in such graphs?)
   b) Suppose we have an MIS algorithm for unoriented trees that solves the problem in $n$-vertex trees with maximum degree $\Delta$ in $T(n,\Delta)$ time. Can it be used in conjunction with Luby's reduction to solve $(\Delta+1)$-coloring
in unoriented trees? Explain your answer.

2) Describe a construction of \( n \)-vertex graphs \( G \) with girth \((G) = \Omega(k)\) and with \( \Omega(n^{1+\frac{1}{b}}) \) edges, for an arbitrary positive integer parameter \( b \). Prove its correctness.

What are the implications of this construction to lower bounds for spanners? Prove these implications.

3) Given an \( n \)-vertex graph with arboricity \( a \), describe an algorithm that computes a \((2a + O(a^{2/3}))\)-coloring of this graph as efficiently as possible. Analyze your algorithm and its running time.

(The algorithm should be distributed.)
4) In a weighted graph \( G = (V, E) \), \( w: E \rightarrow \mathbb{R}^+ \) is a weight function, for a spanning tree \( T \), the bottleneck of \( T \) is the weight of its heaviest edge. The \textit{min-bottleneck-spanning tree} of \( G \) is the spanning tree with minimum bottleneck.

Describe a time- and communication-efficient distributed algorithm for computing a \textit{min-bottleneck-spanning tree} of \( G \). Prove its correctness and analyze its running time.

\textit{Good Luck!}