Examin (Mod Aleph) in Distributed Algorithms

Solve 3 questions out of four.

1. Def: A set $S \subseteq V$ is called a $k$-ruling set, for a pos int param $k$,
if

(1) for every $v \in V$ there exists a "ruling" $u \in U$ with
$\text{dist}_G(u, v) \leq k$.

(2) Every two $u, u' \in U$ are at distance at least 2 one from another.

Example: The set $\{a, b\}$ is a 1-ruling set for this graph.

The problem: We are given an $L$-coloring, for a positive integer $L$. Devise an algorithm that builds an $O(\log L)$-ruling set for a given graph $G$ (for which we have an $L$-coloring). Analyze its running time and prove its correctness.
2. Def: A coloring \( \varphi \) of a graph \( G = (V, E) \) is called an \( \textit{\( L \)-coloring \( B \)-defective} \) (for a pair of integers \( L \) and \( B \)) if \( \varphi \) assigns every \( v \in V \) a color from \([L]\) and for every \( v \in V \), it has at most \( B \) neighbors of the same color.

Problem: Given an \( \textit{\( L \)-coloring \( B \)-defective} \) \( \varphi \) of a graph \( G = (V, E) \) w/ \( \Delta(G) = \Delta \) and \( \Delta \leq L, B = \Delta^2 + O(1) \). Devise an efficient \( (\Delta - 1) \)-coloring \( \text{alg}' \) (that might employ this defective coloring).
3) Invoke Mycielski's construction on $C_7$. Describe the resulting graph. What are its properties?

$\begin{array}{c}
\text{C}_7 \\
\text{Diagram of } C_7
\end{array}$

(What is its girth, chromatic number, and number of vertices?)

4) Define the notion of $k$-spanner. Describe the notion of $\delta$ synchronizer based on $k$-spanner, for a fixed $k$. Prove its correctness.

What are $\text{Timepulse}(S)$, $\text{Commulse}(S)$?

How do these compare with $\text{Timepulse}(S)$, $\text{Commulse}(\infty)$, and with $\text{Timepulse}(L)$, $\text{Commulse}(L)$, and with $\text{Timepulse}(B)$, $\text{Commulse}(B)$?

Good luck!