# ASSIGNMENT 3 - DISTRIBUTED ALGORITHMS

Due Date - 14/01/07

(1) (a) Prove that the analysis of the message complexity of $Dijkstra$ algorithm that was studied in class is tight in the following sense: for arbitrary integers $n$ and $D$, $1 \leq D \leq n1$, there exists an n-vertex graph $G = (V, E)$ of diameter $D$, for which there is an execution of $Dijkstra$ algorithm that requires $\Omega(n \cdot D + |E|)$ messages.

(b) Repeat the previous section with the additional requirement that the graph $G$ has $|E| = O(n)$ edges.

(2) Give an example for an execution of the $Bellman - Ford$ algorithm requiring $\Omega(n^3)$ messages.

(3)

**Definition 0.1.** Given a graph $G = (V, E)$ and a set of trees $\mathcal{T} = \{T_1, T_2, \ldots T_n\}$, where $\forall i \in \{1, 2, \ldots, n\} : T_i = (V(T_i), E(T_i)), V(T_i) \subseteq V$ and $E(T_i) \subseteq E$, $\mathcal{T}$ is a called a *Cover* of G if the following conditions hold:

(a) $\bigcup_{i=1}^{n} E(T_i) = E$ and $\bigcup_{i=1}^{n} V(T_i) = V$.

(b) For each $e = (u, v) \in E$, there exists a tree $T_i \in \mathcal{T}$ so that $u, v \in V(T_i)$.

**Definition 0.2.** The *Overlap* of a Cover $\mathcal{T}$ in a vertex $v \in V$ is the number of trees $T \in \mathcal{T}$ that contain $v$:
$Overlap_v(\mathcal{T}) = |\{T | T \in \mathcal{T}, v \in V(T)\}|$.

**Definition 0.3.** The *Overlap* of a *Cover* $\mathcal{T}$ is the maximal *Overlap* of $\mathcal{T}$ in some vertex $v \in V$:
$Overlap(\mathcal{T}) = max_{v \in V}\{Overlap_v(\mathcal{T})\}$.

**Definition 0.4.** The *Diameter* of a *Cover* $\mathcal{T}$ is the maximal Diameter of a tree within this Coverage:
$Diam(\mathcal{T}) = max_{T \in \mathcal{T}}\{Diam(T)\}$.

Given a graph $G$ and a Cover $\mathcal{T}$ of $G$. Forthermore, each vertex $v \in V(G)$ knows to which trees $T \in \mathcal{T}$ it belongs, and the identity of his neighbours in each of these trees. Let $d = Diam(\mathcal{T})$ and $l = Overlap(\mathcal{T})$.

Describe a synchronizer that uses the Cover $\mathcal{T}$, and analyze its complexity. (Your analysis can rely on $d$ and $l$). Of course, your synchronizer should be as effective as possible.

(4)

**Definition 0.5.** Given a graph $G = (V, E)$, a sub-graph $G' = (V, H)$, $H \subseteq E$, is considered a *k-spanning* graph of $G$ if for each edge $e = (u, v) \in E$ there exist a path $p$ in G' for which $|p| \leq k$. $(k \geq 1)$

Given a graph $G = (V, E)$ and a k-spanning sub-graph $G'$, where $k \geq 1$. Each vertex $v \in V$ knows the identity of its neighbours in $G'$, and it also knows that value of the constant $k$. Let $h = |E(G')|$.

Describe a synchronizer that uses $G'$ and analyze its complexity. (Your analysis can rely on $h$). Of course, your synchronizer should be as effective as possible.

(5) Consider a 15-processor asyncronous network with the processors $\{P_0, P_1, \cdots, P_14\}$. The processors constantly run a synchronizer. Let $v$ and $v'$ be two processors in the network, and suppose that at a certain moment, the pulse counter at $v$ shows $p = 27$. What is the range of possible pulse numbers in at $v'$ in each of the following cases:

(a) The network is a ring (with the processors arranged according to their numbers), $v$ is processor number 11, $v'$ is processor number 2 and the synchronizer used is $\alpha$.

(b) The network is a full balanced binary tree (4 levels), $v$ is the root, $v'$ is one of the leaves and the synchronizer used is $\beta$.

(c) The same as in $(b)$, except both $v$ and $v'$ are leaves.