Sparse Graph Spanners
(2005; Elkin, Peleg)

Michael Elkin, Ben-Gurion University of the Negev, www.cs.bgu.ac.il/~elkinm

SYNONYMS: $(1+\epsilon,\beta)$-spanners; almost additive spanners.

1 PROBLEM DEFINITION

For a pair of numbers α, β, $\alpha \geq 1$, $\beta \geq 0$, a subgraph $G' = (V, H)$ of an unweighted undirected graph $G = (V, E)$, $H \subseteq E$, is an (α, β)-spanner of G if for every pair of vertices $u, w \in V$, $dist_{G'}(u, w) \leq \alpha \cdot dist_G(u, w) + \beta$, where $dist_G(u, w)$ stands for the distance between u and w in G. It is desirable to show that for every n-vertex graph there exists a sparse (α, β)-spanner with as small values of α and β as possible. The problem is to determine asymptotic tradeoffs between α and β on one hand, and the sparsity of the spanner on the other.

2 KEY RESULTS

The main result of Elkin and Peleg [4] establishes the existence and efficient constructibility of $(1+\epsilon, \beta)$-spanners of size $O(\beta n^{1+1/\kappa})$ for every n-vertex graph G, where $\beta = \beta(\epsilon, \kappa)$ is constant whenever κ and ϵ are. The specific dependence of β on κ and ϵ is $\beta(\kappa, \epsilon) = \kappa^{\log\log\kappa - \log\epsilon}$.

3 APPLICATIONS

The result of [4] was used in [3, 5, 6] for computing almost shortest paths in centralized, distributed, streaming, and dynamic centralized models of computations. The basic approach used in these results is to construct a sparse spanner, and then to compute exact shortest paths on the constructed spanner. The sparsity of the latter guarantees that the computation of shortest paths in the spanner is far more efficient than in the original graph.

4 OPEN PROBLEMS

The main open question is whether it is possible to achieve similar results with $\epsilon = 0$. More formally, the question is:
Is it true that for any $\kappa \geq 1$ and any n-vertex graph G there exists $(1, \beta(\kappa))$-spanner of G with $O(n^{1+1/\kappa})$ edges?
This question was answered in affirmitive for κ equal to 2 and 3 [1, 2, 4]. Some lower bounds were recently proved by Woodruff [8].

A less challenging question is to improve the dependence of β on ϵ and κ. Some progress in this direction was achieved by Thorup and Zwick [7].

5 CROSS REFERENCES

None is reported. Entry editors please feel free to add some.

6 RECOMMENDED READING

[1] Surender Baswana, Telikepalli Kavitha, Kurt Mehlhorn and Seth Pettie. New constructions of (alpha, beta)-spanners and purely additive spanners. In Proc. of Symp. on Discrete Algorithms (SODA), pp. 672-681, 2005.

[2] Dorit Dor, Shay Halperin and Uri Zwick. All Pairs Almost Shortest Paths In *SIAM Journal on Computing* 29, pp. 1740-1759, 2000.

[3] Michael Elkin. Computing Almost Shortest Paths. In *Transactions on Algorithms*, Vol. 1, No. 2, pp.283-323, 2005.

[4] Michael Elkin and David Peleg. $(1+\epsilon, \beta)$-spanner constructions for general graphs. In *SIAM J. Comput.*, Vol. 33, No. 3, pp. 608-631, 2004.

[5] Michael Elkin and Jian Zhang. Efficient Algorithms for Constructing $(1+\epsilon, \beta)$-spanners in the Distributed and Streaming Models. In *Distributed Computing*, 18(5), pages 375-385, 2006.

[6] Liam Roditty and Uri Zwick. Dynamic approximate all-pairs shortest paths in undirected graphs In *Proc. of Symp. on Foundations of Computer Science (FOCS)*, pp.499-508, 2004

[7] Mikkel Thorup and Uri Zwick. Spanners and Emulators with sublinear distance errors. In *Proc. of Symp. on Discrete Algorithms (SODA)*, pp. 802-809, 2006.

[8] David Woodruff. Lower Bounds for Additive Spanners, Emulators, and More. In *Proc. of Symp. on Foundations of Computer Science (FOCS)*, 2006.