Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. In how many ways can we partition $A$ as $A_1 \cup A_2 \cup A_3$ with

- $1, 2 \in A_1$, $3, 4 \in A_2$, and $5, 6, 7 \in A_3$?
- $2 \in A_1$, $3, 4 \in A_2$, $5, 6 \in A_3$, and $|A_3| = 3$?

Let $A = \{(0, 0), (0, 2), (2, 0), (2, 2)\}$, and define $R$ on $A$ by $(x_1, y_1) R (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.

- Verify that $R$ is an equivalence relation on $A$.
- Determine the equivalence classes $[\{(0, 0)\}]$, $[(0, 1)]$, and $[(2, 2)]$.
- Determine the partition of $A$ induced by $R$.

Define the relation $R$ on $\mathbb{Z}^+$ by $x R y$ if $x/y = 2^n$ for some $n \in \mathbb{Z}$.

- Verify that $R$ is an equivalence relation on $\mathbb{Z}^+$.
- How many distinct equivalence classes do we find among $[i]$, $1 \leq i \leq 100$?
Let $A$ be a nonempty set and fix the set $B$, where $B \subseteq A$. Define the relation $\mathcal{R}$ on $\mathcal{P}(A)$ by $X \mathcal{R} Y$, for $X, Y \subseteq A$, if $B \cap X = B \cap Y$.

a) Verify that $\mathcal{R}$ is an equivalence relation on $\mathcal{P}(A)$.

b) If $A = \{a, b, c\}$ and $B = \{b, c\}$, find the partition of $\mathcal{P}(A)$ induced by $\mathcal{R}$.

c) If $A = \{a, b, c, d, e\}$ and $B = \{a, b, d\}$, find $[X]$ if $X = \{b, d, e\}$.

d) For $A = \{a, b, c, d, e\}$ and $B = \{a, b, d\}$, how many equivalence classes are in the partition induced by $\mathcal{R}$?

Define the relation $\mathcal{R}$ on $\mathbb{Z}^+$ by $a \mathcal{R} b$ if $\text{lcm}(a, 16) = \text{lcm}(b, 16)$.

a) Prove that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}^+$.

b) Determine each of the following equivalence classes: $[1], [2], [3], [10], [16], [25]$, $[32], [33], [48]$, and $[64]$.

How many of the equivalence relations on $A = \{a, b, c, d, e, f\}$ have (a) exactly three equivalence classes of size 2? (b) exactly one equivalence class of size 2? (c) one equivalence class of size 4? (d) at least one equivalence class with three or more elements?

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. For each of the following values of $r$, determine an equivalence relation $\mathcal{R}$ on $A$ with $|\mathcal{R}| = r$, or explain why no such relation exists. (a) $r = 6$; (b) $r = 7$; (c) $r = 8$; (d) $r = 9$; (e) $r = 11$; (f) $r = 22$; (g) $r = 23$; (h) $r = 30$; (i) $r = 31$. 