Problem 4
Due: 30/11/17

Consider the following linear program, where $G = (V, E)$ is a directed graph with positive edge weights $w : E \rightarrow \mathbb{R}_+$ in which every node is reachable from a given node $s \in V$:

$$
\begin{align*}
\text{max} & \quad \sum_{v \in V} d_v \\
\text{subject to} & \quad d_v - d_u \leq w(u, v) && \forall (u, v) \in E \\
& \quad d_s = 0
\end{align*}
$$

• What problem is solved by the above LP?

• Write the dual of the above LP. Describe (and prove) the general structure of an optimal solution to the dual which is a vertex of the feasible region.\(^1\)

\(^1\)You may have noticed that we have seen this constraint matrix before, and it is totally unimodular.