Logic-based Preference Modelling in Combinatorial Domains

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Talk Outline

- Why combinatorial domains?
- Logic-based preference representation with weighted formulas
- Results on expressive power, succinctness, complexity
- An application to combinatorial auctions
- Conclusion

Preferences in Combinatorial Domains

I'm interested in *collective decision making:* mapping the individual preference profiles of independent agents into a joint decision.

The alternatives often have a *combinatorial structure:* they are characterised by a tuple of variables ranging over a finite domain.

Examples:

- Allocate n indivisible goods to m agents: m^n alternatives
- Elect a committee of size k, from n candidates: $\binom{n}{k}$ alternatives

Weighted Propositional Formulas

Let PS be a set of propositional symbols (goods, candidates) and let $\mathcal{L}_{PS}$ be the propositional language over PS.

A *goal base* is a set $G = \{(\varphi_i, \alpha_i)\}_i$ of pairs, each consisting of a consistent propositional formula $\varphi_i \in \mathcal{L}_{PS}$ and a real number α_i.

The utility function u_G generated by G is defined by

$$u_G(M) \quad = \quad \sum \{\alpha_i \mid (\varphi_i, \alpha_i) \in G \text{ and } M \models \varphi_i\}$$

for all models $M \in 2^{PS}$. G is called the *generator* of u_G.

Example: $\{(p \vee q \vee r, 5), (p \wedge q, 2)\}$

Languages

Let $H \subseteq \mathcal{L}_{PS}$ be a syntactical restriction on *formulas* and let $H' \subseteq \mathbb{R}$ be a set of allowed weights *weights*.

Then $\mathcal{L}(H, H')$ is the language given by the class of goal bases conforming to restriction H and H'. Examples:

- $\mathcal{L}(pcubes, pos)$: the language of positive cubes (conjunctions of positive literals) with positive weights
- $\mathcal{L}(k\text{-}clauses, all)$: clauses of length $\leq k$ with arbitrary weights

Question: Are there simple restrictions on goal bases such that the utility functions they generate enjoy simple structural properties?

Some Expressivity Results

Formulas	**Weights**		**Utility Functions**
cubes/clauses/all	general	$=$	all
positive cubes/formulas	general	$=$	all
positive clauses	general	$=$	normalised
strictly positive formulas	general	$=$	normalised
k-cubes/clauses/formulas	general	$=$	k-additive
positive k-cubes/formulas	general	$=$	k-additive
positive k-clauses	general	$=$	normalised k-additive
literals	general	$=$	modular
atoms	general	$=$	normalised modular
cubes/formulas	positive	$=$	non-negative
clauses	positive	$\subset$	non-negative
strictly positive formulas	positive	$=$	normalised monotonic
positive formulas	positive	$=$	non-negative monotonic
positive clauses	positive	$\subset$	normalised concave monotonic

Comparative Succinctness

Let L and L' be two languages (classes of goal bases).

L is no more succinct than L' ($L \preceq L'$) iff there exist a mapping $f : L \to L'$ and a *polynomial* function p such that:

- $u_G \equiv u_{f(G)}$ for all $G \in L$ (they generate the same functions); and
- $size(f(G)) \leq p(size(G))$ for all $G \in L$ (polysize reduction).

Some Succinctness Results

$$\begin{aligned}
\mathcal{L}(pcubes, all) &\quad \perp \quad \mathcal{L}(complete\ cubes, all) \\
\mathcal{L}(pcubes, all) &\quad \prec \quad \mathcal{L}(cubes, all) \\
\mathcal{L}(pcubes, all) &\quad \prec \quad \mathcal{L}(positive, all) \\
\mathcal{L}(pclauses, all) &\quad \prec \quad \mathcal{L}(clauses, all) \\
\mathcal{L}(pcubes, all) &\quad \perp \quad \mathcal{L}(pclauses, all) \\
\mathcal{L}(cubes, all) &\quad \sim \quad \mathcal{L}(clauses, all)
\end{aligned}$$

Computational Complexity

Other interesting questions concern the complexity of reasoning about preferences. Consider the following decision problem:

> $\textsc{Max-Utility}(H, H')$
> **Given:** Goal base $G \in \mathcal{L}(H, H')$ and $K \in \mathbb{Z}$
> **Question:** Is there an $M \in 2^{PS}$ such that $u_G(M) \geq K$?

Some basic results are straightforward:

- $\textsc{Max-Utility}(H, H')$ is *in NP* for any choice of H and H', because we can always check $u_G(M) \geq K$ in polynomial time.
- $\textsc{Max-Utility}(all, all)$ is *NP-complete* (reduction from $\textsc{Sat}$).

More interesting questions would be whether there are either
(1) "large" sublanguages for which $\textsc{Max-Utility}$ is still polynomial,
or (2) "small" sublanguages for which it is already NP-hard.

Some Complexity Results

- $\textsc{Max-Utility}(\mathit{literals}, \mathit{all})$ is in P.
- $\textsc{Max-Utility}(\mathit{positive}, \mathit{positive})$ is in P.
- $\textsc{Max-Utility}(k\text{-}\mathit{clauses}, \mathit{positive})$ is NP-complete for $k \geq 2$.
- $\textsc{Max-Utility}(k\text{-}\mathit{cubes}, \mathit{positive})$ is NP-complete for $k \geq 2$.
- $\textsc{Max-Utility}(\mathit{positive}\ k\text{-}\mathit{clauses}, \mathit{all})$ is NP-complete for $k \geq 2$.
- $\textsc{Max-Utility}(\mathit{positive}\ k\text{-}\mathit{cubes}, \mathit{all})$ is NP-complete for $k \geq 2$.

Combinatorial Auctions

In a *combinatorial auction*, the auctioneer puts several goods on sale and the other agents submit bids for entire bundles of goods.

Weighted formulas can be used as *bidding languages* in CAs. We are working on *winner determination algorithms* for this setting.

- Integer Programming.
- Heuristic-guided search using *branch-and-bound* algorithms.
  - Nodes in the search tree are partial allocations.
  - Moves: allocating one more item.
  - Use heuristic to get upper bound on expected social welfare for a given branch and prune hopeless branches.
  - Need to develop heuristic for each language.

Experiments: $\mathcal{L}(pcubes, positive)$

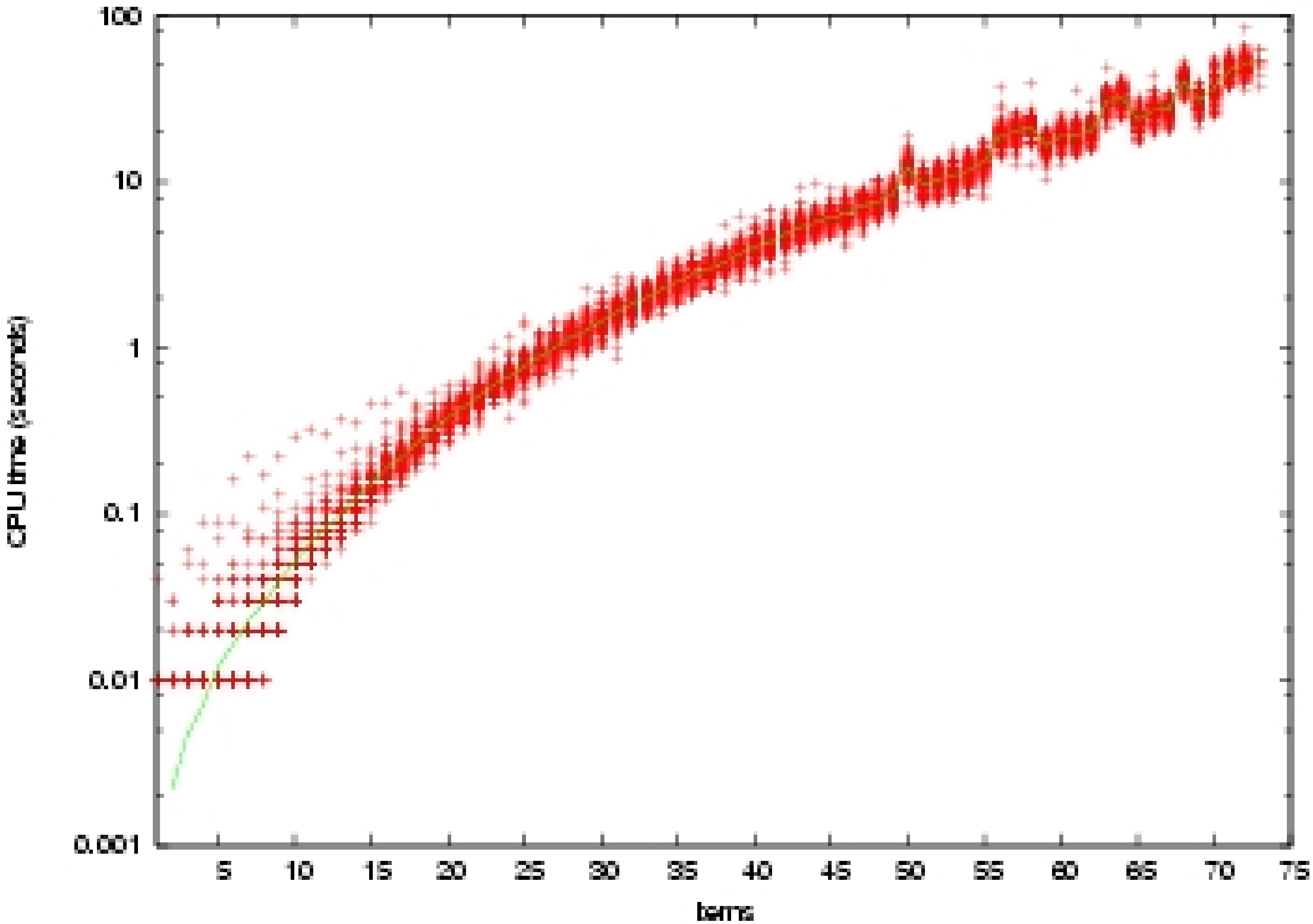

Figure: 20 bidders (around 1400 goals for 70 bidders)

Conclusion

Compact preference representation in combinatorial domains is relevant to a number of applications, and weighted goals are an interesting class of languages for doing this. Ongoing work:

- Fill in missing technical results on expressivity, succinctness and complexity to get global picture
- Aggregation operators other than $\sum$ (particularly $\max$)
- Applications: negotiation, auctions, voting

Y. Chevaleyre, U. Endriss, and J. Lang. *Expressive Power of Weighted Propositional Formulas for Cardinal Preference Modelling*. Proc. KR-2006.

J. Uckelman and U. Endriss. *Preference Representation with Weighted Goals: Expressivity, Succinctness, Complexity*. Proc. AiPref-2007.

J. Uckelman and U. Endriss. *Winner Determination in Combinatorial Auctions with Logic-based Bidding Languages*. Under review.