An advanced greedy square jigsaw puzzle solver

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The problem

From this

To this
Current best known solver

- By Cho et al.
- 432 parts, 28 x 28.
- Uses clues
- The solver is based on a probabilistic approach
Problem properties

- Know to be NP-Complete
- Using square parts
  - Simple to code
  - Hard to solve
- A brute force algorithm will take $O(n!)$ time, where $n$ is the number of parts
- If we had an accurate parts compatibility function we could have solve in polynomial time using a greedy algorithm
Compatibility metrics
Finding out the likelihood that two given parts are neighbours
Compatibility metrics

\[ D_{LR}(x_i, x_j) = \sum_{k=1}^{K} \sum_{l=1}^{3} (x_i(k, K, l) - x_j(k, 1, l))^2 \]
New compatibility metrics

- Backward difference-based compatibility
- Central difference-based compatibility
- Square absolute dissimilarity-based metric (SAD)

$$SAD_{LR}(x_i, x_j) = \sum_{k=1}^{K} \sum_{l=1}^{3} \sqrt{|x_i(k, K, l) - x_j(k, 1, l)|}$$
New compatibility metrics

Compatibility accuracy test

- Dissimilarity
- Backward difference
- Central difference
- Square abs dissimilarity (SAD)

<table>
<thead>
<tr>
<th>Part Size</th>
<th>Metric accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>84.0 81.4 76.9</td>
</tr>
<tr>
<td>25</td>
<td>77.7 76.5 68.7</td>
</tr>
<tr>
<td>20</td>
<td>67.5 67.3 58.3</td>
</tr>
<tr>
<td>16</td>
<td>48.2 48.1 41.0</td>
</tr>
</tbody>
</table>

% Metric accuracy
Performance metrics
Measuring the quality of a given solution
Performance metrics

- Original Image
  
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

- Solution Image
  
<table>
<thead>
<tr>
<th>3</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

- Direct comparison metric – No cell is placed correctly

- Neighbour comparison metric – Many neighbours are placed correctly
Estimation metrics
Using performance metrics to imply the convergence of an algorithm is equivalent to using clues.
(Best buddies metric)
The shifting problem
Regions are often placed shifted to their original location
The shifting problem
The shifting problem – A simple example

(a) Greedy solver
basic solution

(b) The basic solution
segmentation map

(c) Best shifted
solution
Greedy algorithm with modules approach

Chains of modules, each target a different problem
Greedy algorithm with modules approach

<table>
<thead>
<tr>
<th>Calculate compatibility</th>
<th>Dissimilarity metric</th>
<th>SAD metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improve compatibility results</td>
<td>Relaxation labeling</td>
<td>Loopy belief propagation</td>
</tr>
<tr>
<td>Greedy algorithm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shifting algorithm</td>
<td>Segmentation methods</td>
<td>Shifting methods</td>
</tr>
</tbody>
</table>
Improve compatibility results
Solver test results

- Sample of 10 images
- Size of 400 × 400
Direct comparison test

<table>
<thead>
<tr>
<th>Number of parts</th>
<th>No improvements</th>
<th>SAD Compatibility</th>
<th>SAD + Shift via segmentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.21</td>
<td>1.36</td>
<td>64.77</td>
</tr>
<tr>
<td>256</td>
<td>0.40</td>
<td>0.27</td>
<td>78.10</td>
</tr>
<tr>
<td>400</td>
<td>0.21</td>
<td>0.19</td>
<td>14.04</td>
</tr>
</tbody>
</table>
Neighbor comparison test

- No improvements
- SAD Compatibility
- SAD + Shift via segmentation

Number of parts:
- 100: 41.51, 55.52, 89.01
- 256: 30.14, 54.93, 91.07
- 400: 19.28, 34.55, 59.18

% Reconstruction accuracy
Solver test results - Example 1

Greedy with no added improvements

(a) 0% direct, 22.168% neighbour, 18 seconds.

With SAD metric

(b) 0% direct, 53.2227% neighbour, 18 seconds

With SAD metric + Shift

(c) 100% direct, 100% neighbour, 23 seconds

256 parts
Solver test results - Example 2

Greedy with no added improvements

With SAD metric

With SAD metric + Shift

Simple object, difficult background

256 parts
Solver test results - Example 3

Greedy with no added improvements

With SAD metric

With SAD metric + Shift

Computer graphics, with repetitive patterns

256 parts
Q & A

Future research:

• Improve compatibility results algorithms (RL)
• GA to improve compatibility functions
• Smart shifting methods

Thank you!