Early Vision
(II)

Introduction to Computational and Biological Vision

CS 202-1-5261

Computer Science Department, BGU

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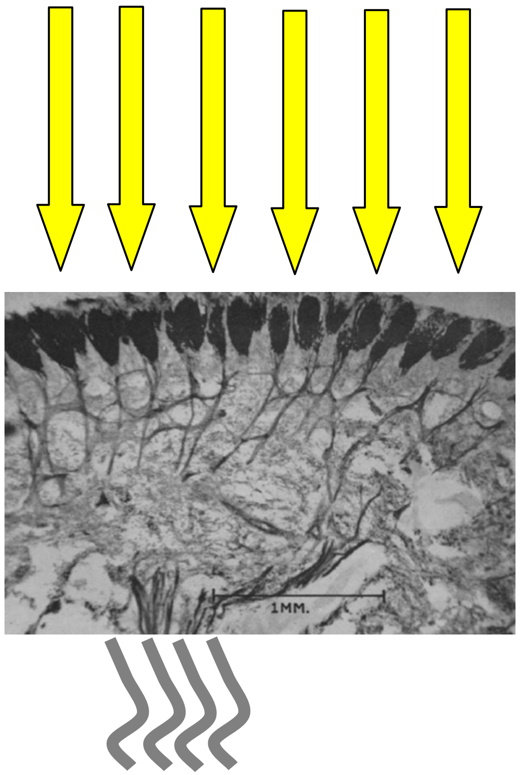
Linear systems and filtering

Input: Light



Output: Spike measurements

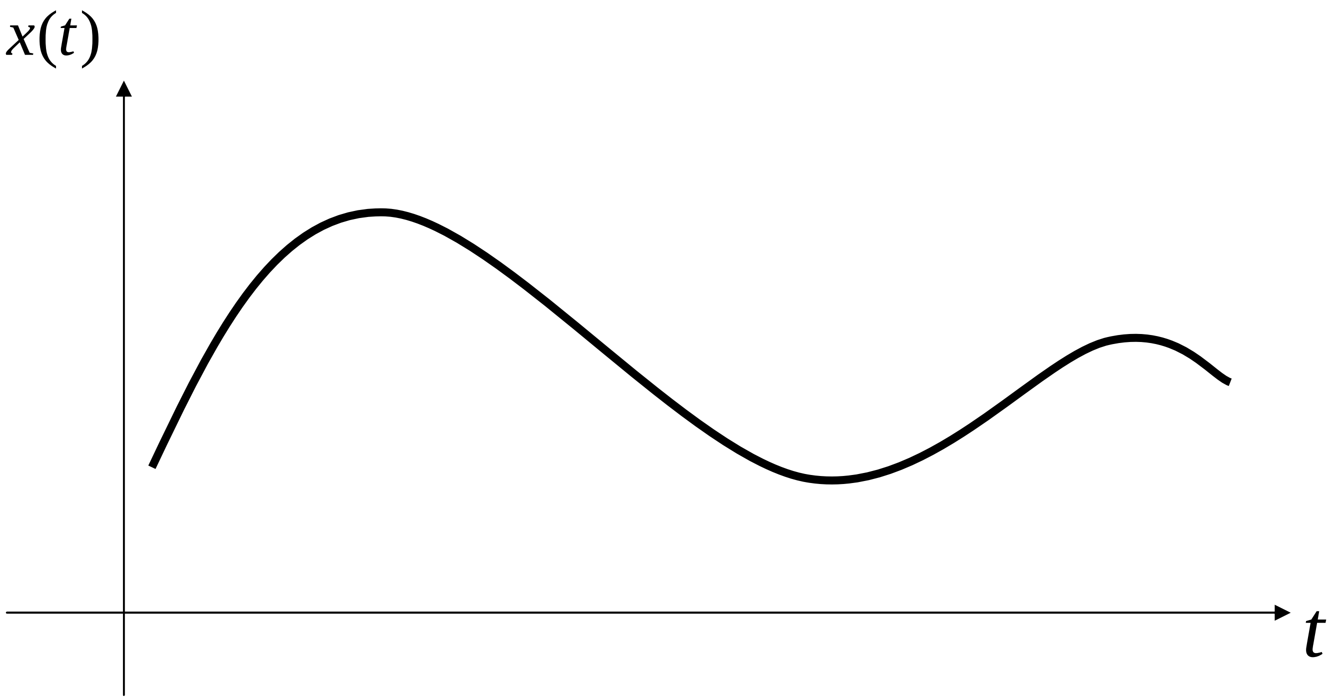
Lateral inhibition and linear systems

Where we left off:

- **What can be computed with lateral inhibition?**
- **What cannot be computed?**
- **What is an appropriate abstraction?**

Linear systems and filtering

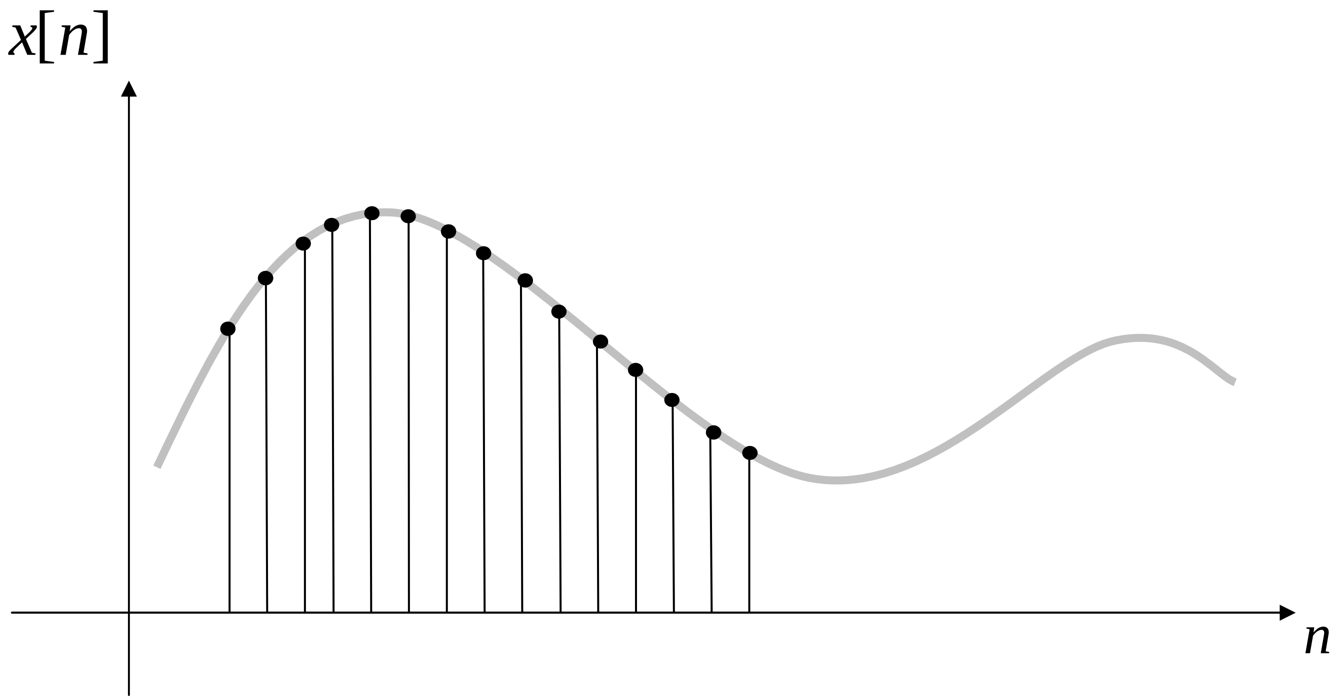
Continuous-time signals



Continuous signal – a time (or space) varying function x(t) of one (or more) independent variables

Linear systems and filtering

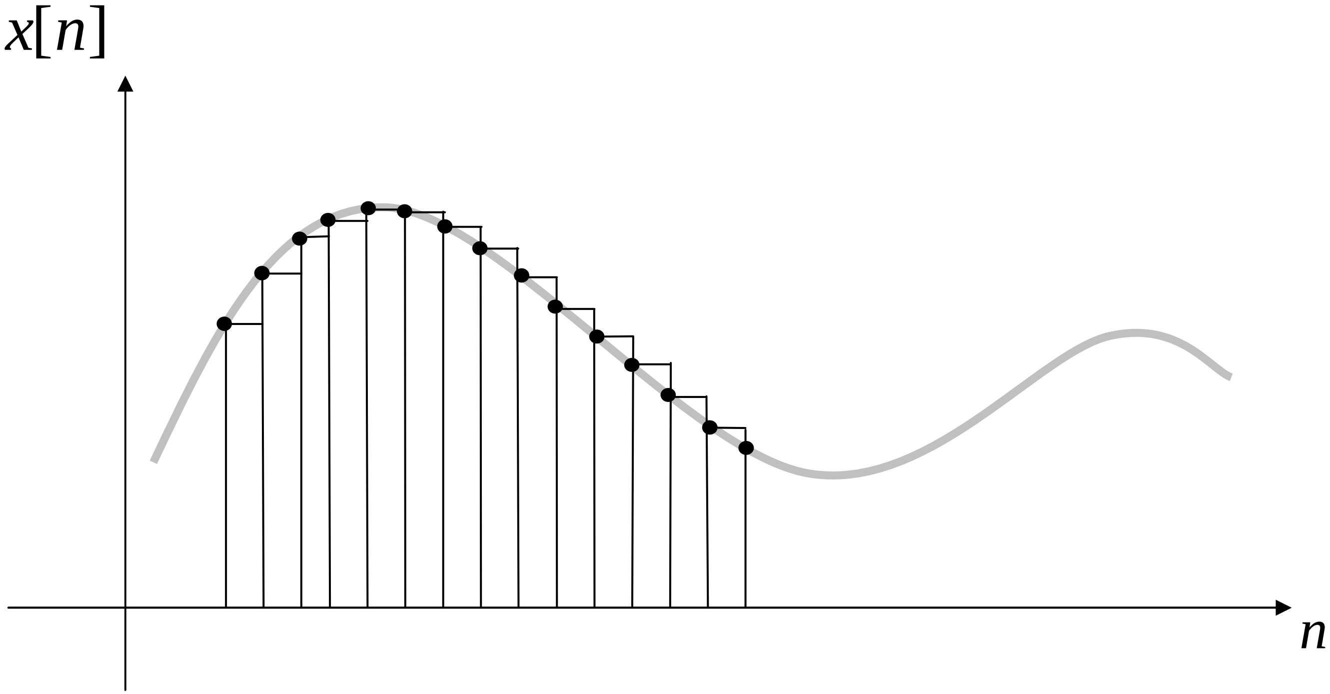
Discrete-time signals



Discrete signal – sequence x[n] of time (or space) ordered samples of a continuous signal

Linear systems and filtering

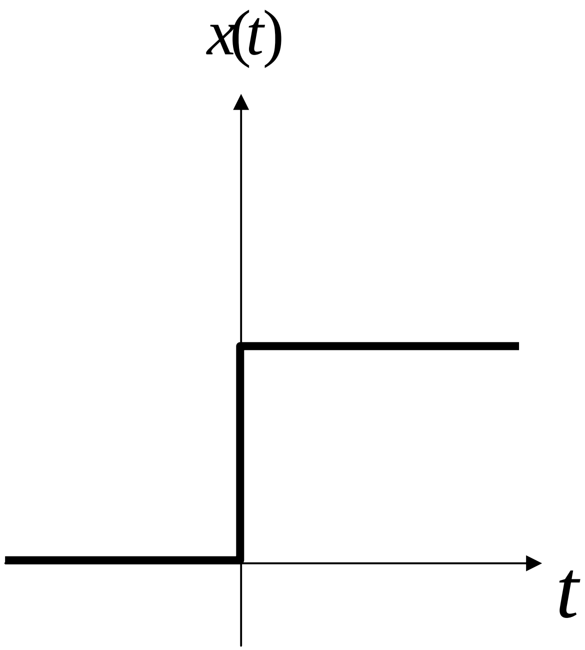
Discrete-time signals



Discrete signal – sequence x[n] of time (or space) ordered samples of a continuous signal

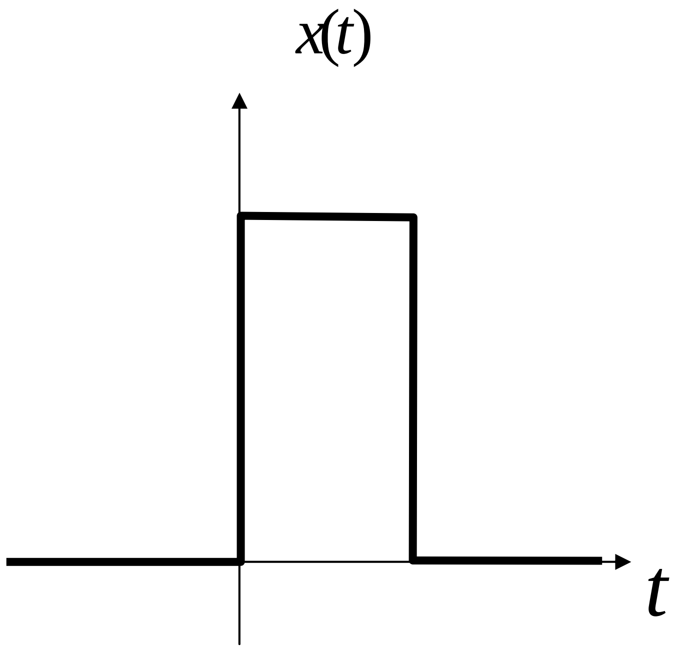
Linear systems and filtering

Some special signals



$$step(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Unit step function

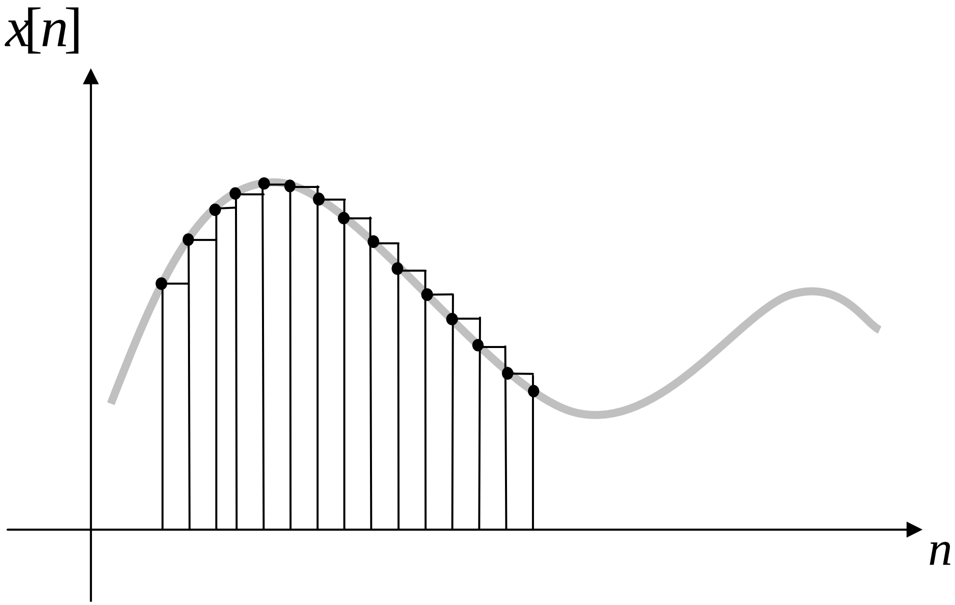


$$\delta_\Delta(t) = \begin{cases} \Delta^{-1} & 0 \leq t \leq \Delta \\ 0 & otherwise \end{cases}$$

Square functions

Linear systems and filtering

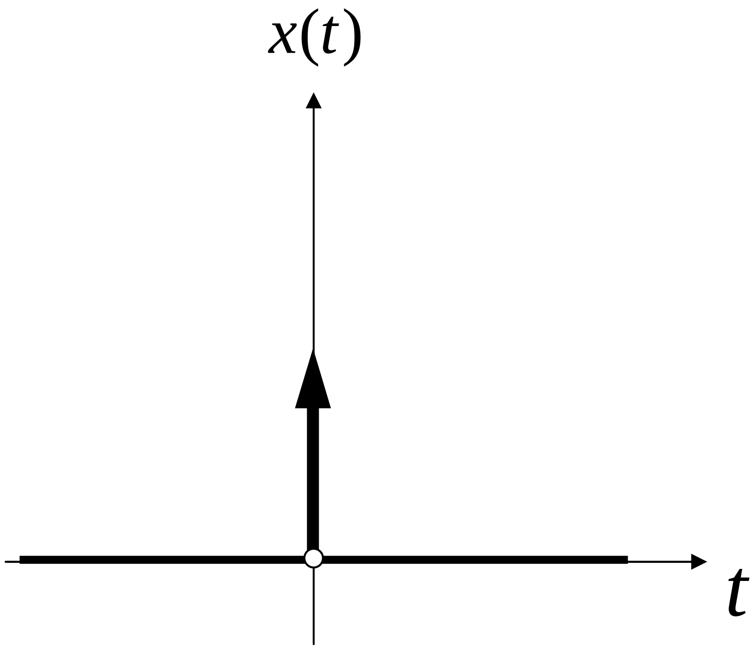
Approximation of continuous-time signals



$$x(t) \cong \tilde{x}_{\Delta}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta) \cdot \delta_{\Delta}(t - k\Delta) \cdot \Delta$$

Linear systems and filtering

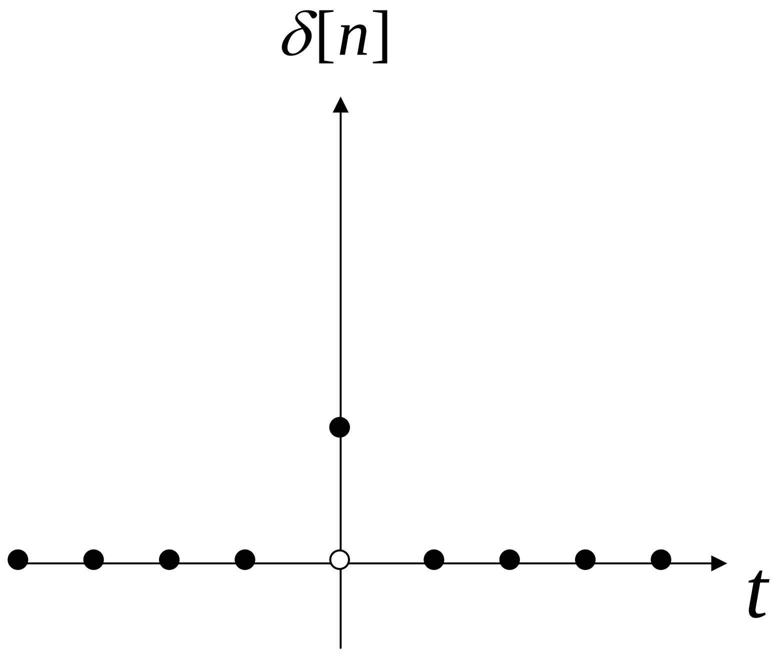
Some special signals



$$\delta(t) = 0 \quad \forall t \neq 0$$

$$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$

Dirac's Delta function (impulse)

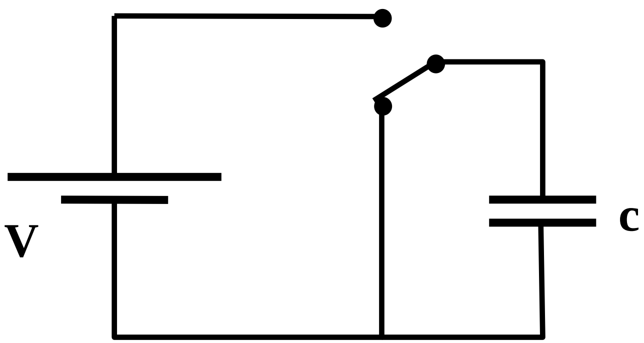


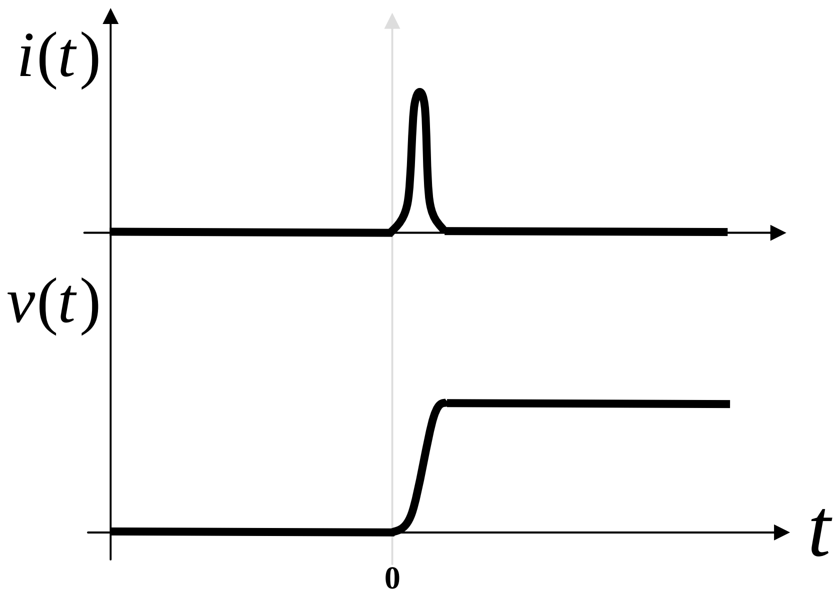
$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

Kronecker's Delta function

Linear systems and filtering

Where this Delta comes from?





$$\int_{-\infty}^{+\infty} i(t)dt = Q = c \cdot V$$

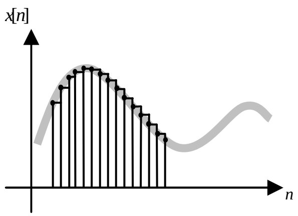
$$i(t) \overset{?}{=} \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad \Longrightarrow \quad i(t) = c \cdot V \cdot \delta(t)$$

Linear systems and filtering

Delta function properties

$$\delta(t) = \lim_{\Delta \to 0} \delta_{\Delta}(t)$$

$$step(t) = \int_{-\infty}^{t} \delta(s)ds$$



$$x(t) = \lim_{\Delta \to 0} \tilde{x}_{\Delta}(t) = \int_{-\infty}^{\infty} x(s) \cdot \delta(t-s)ds$$

Linear systems and filtering

Shift invariant linear systems

Homogeneity: $L[\alpha \cdot x] = \alpha \cdot L[x]$

Additivity: $L[x_1 + x_2] = L[x_1] + L[x_2]$

Superposition: $L[\alpha \cdot x_1 + \beta \cdot x_2] = \alpha \cdot L[x_1] + \beta \cdot L[x_2]$

Shift invariance: $y(t) = L[x(t)] \quad \Leftrightarrow \quad y(t-s) = L[x(t-s)]$

Linear systems and filtering

Predicting output of linear systems

$$x(t) \cong \tilde{x}_\Delta(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\cdot\delta_\Delta(t-k\Delta)\cdot\Delta$$

$$y(t) = L[\lim_{\Delta\to 0}\tilde{x}_\Delta(t)] = L\left[\lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty} x(k\Delta)\cdot\delta_\Delta(t-k\Delta)\cdot\Delta\right]$$

Additivity

Linear systems and filtering

Predicting output of linear systems

$$y(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} L\left[x(k\Delta) \cdot \delta_{\Delta}(t - k\Delta) \cdot \Delta\right]$$

$$y(t) = \int_{-\infty}^{+\infty} L\left[x(s) \cdot \delta(t-s) ds\right]$$

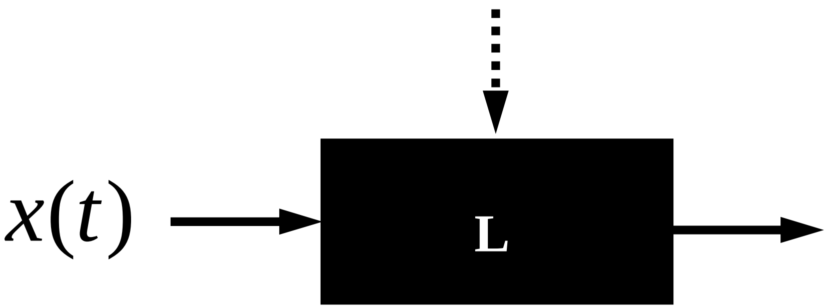
Homogeneity

$$y(t) = \int_{-\infty}^{+\infty} x(s) \cdot L[\delta(t-s)] ds$$

Linear systems and filtering

Predicting output of linear systems

$$h(t) = L[\delta(t)]$$ **Impulse response / Point spread function**



$$y(t) = \int_{-\infty}^{+\infty} x(s) \cdot h(t-s)\,ds$$

Convolution :

Impulse linear filtering :

$$y(t) = x(t) * h(t)$$

Linear systems and filtering

Properties of convolutions

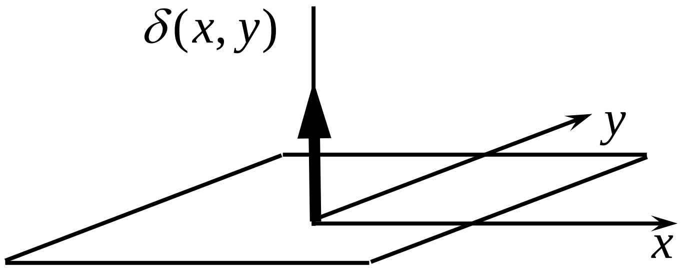
Commutative : $$x * h = h * x$$

Associative : $$(x * h_1) * h_2 = x * (h_1 * h_2)$$

Distributive : $$(x * h_1) + (x * h_2) = x * (h_1 + h_2)$$

Linear systems and filtering

2D convolutions



2D impulse (delta)

$$h(x, y) = L[\delta(x, y)]$$

2D impulse response

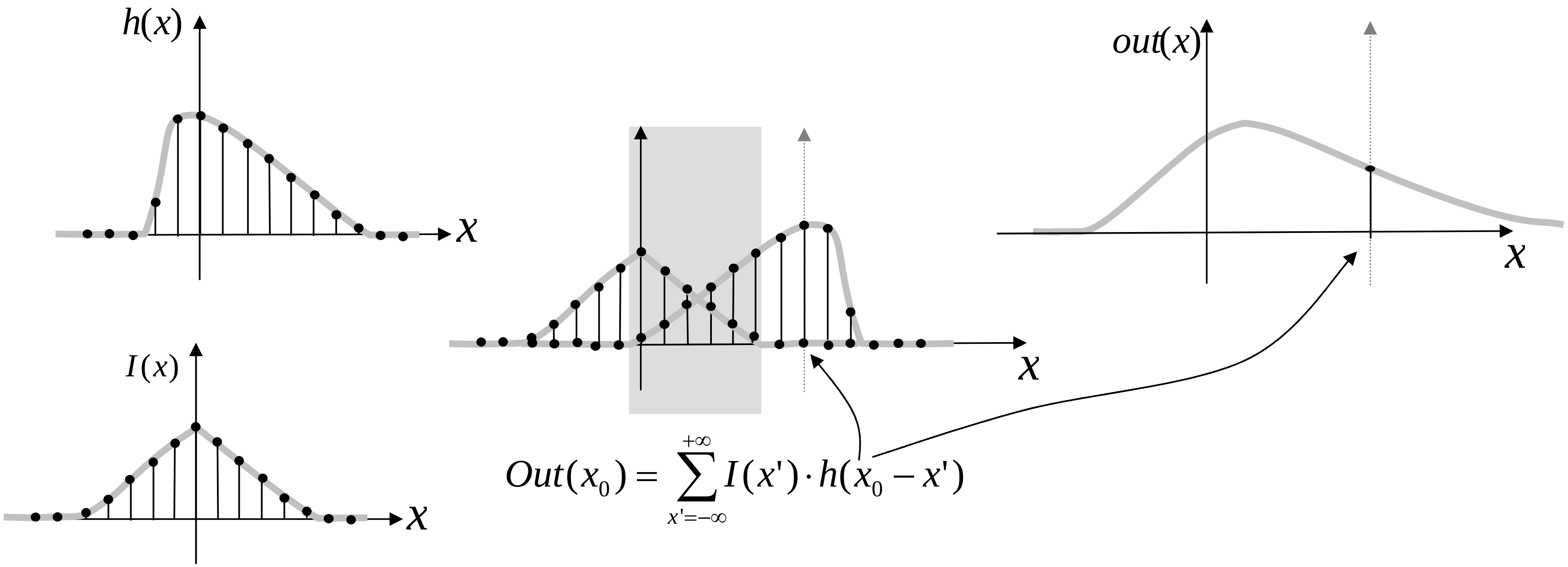
$$f(x, y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} I(x', y') \cdot h(x - x', y - y') dx' dy'$$

$$= I(x, y) * h(x, y)$$

Linear systems and filtering

Discrete convolutions on regular grids

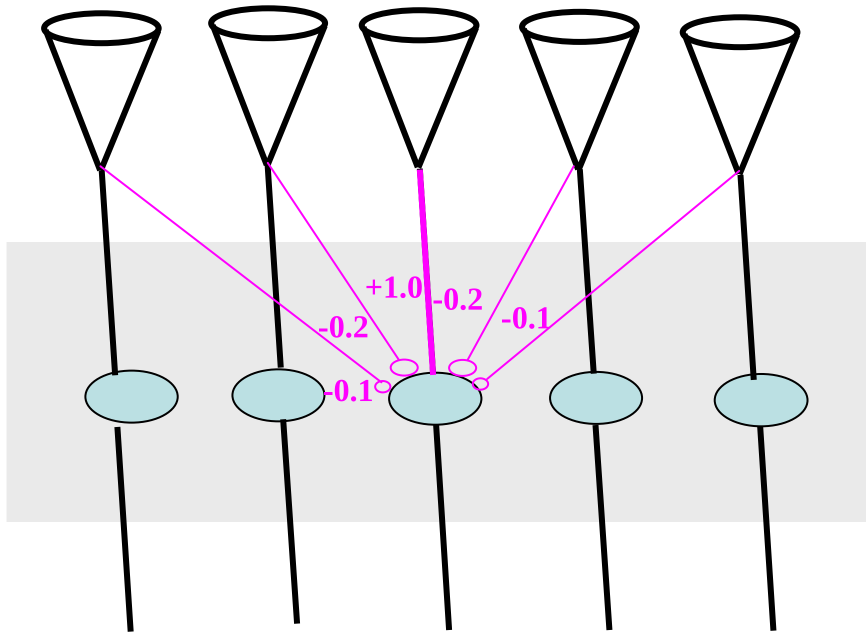
$$Out(x,y) = I(x,y) * h(x,y) = \sum_{x'=-\infty}^{+\infty} \sum_{y'=-\infty}^{+\infty} I(x',y') \cdot h(x-x', y-y')$$



A system is shift invariant linear iff the response is a weighted sum on the inputs

Linear systems and filtering

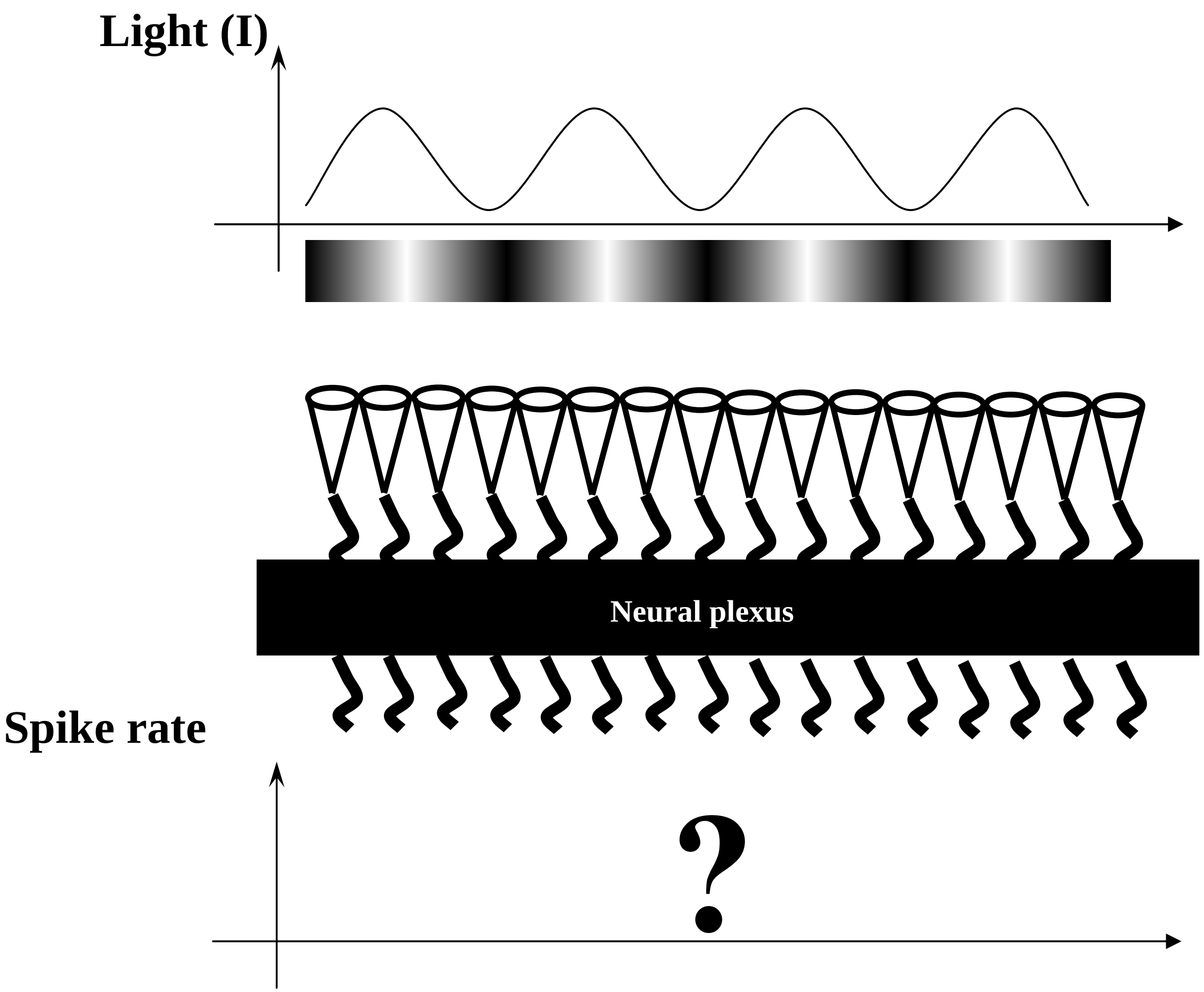
Back to lateral inhibition



Lateral inhibition = FIR (Finite Impulse Filter)

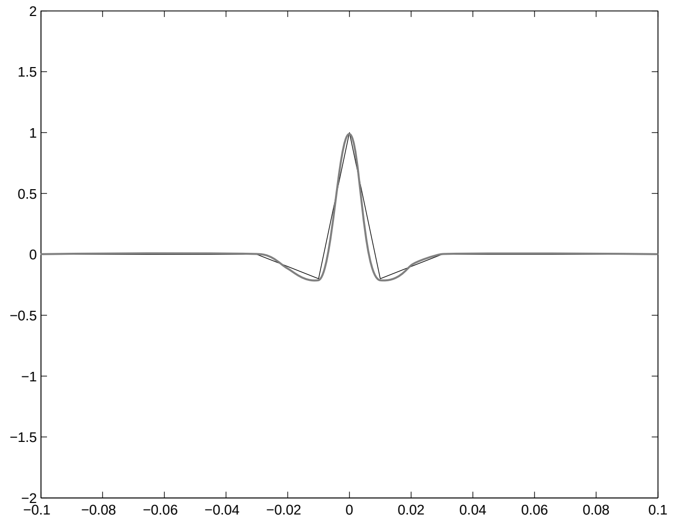
Linear systems and filtering

Linear systems and periodic signals

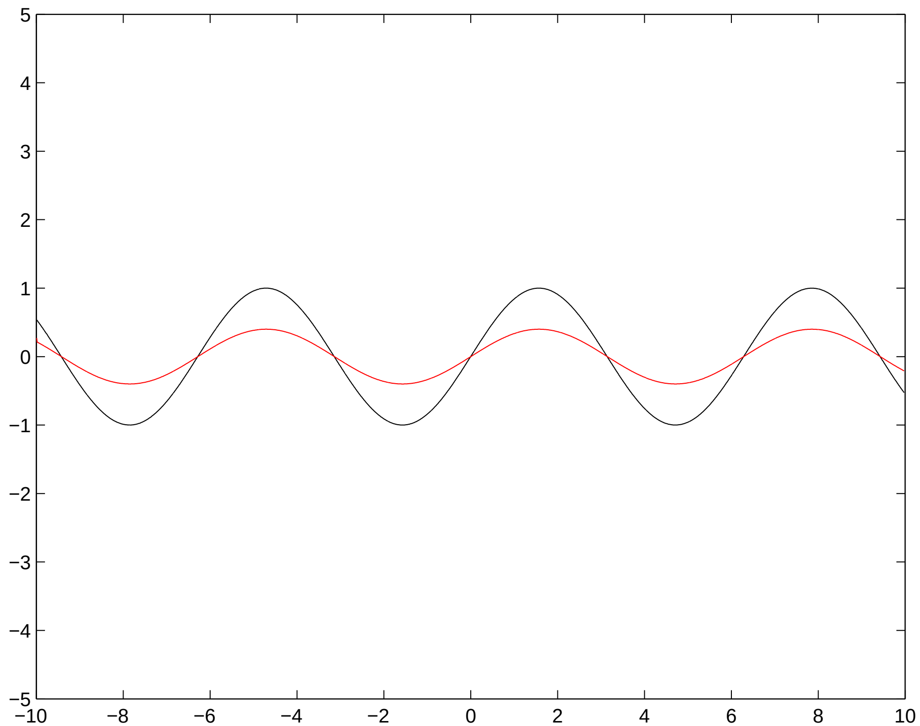


Linear systems and filtering

Linear systems and periodic signals

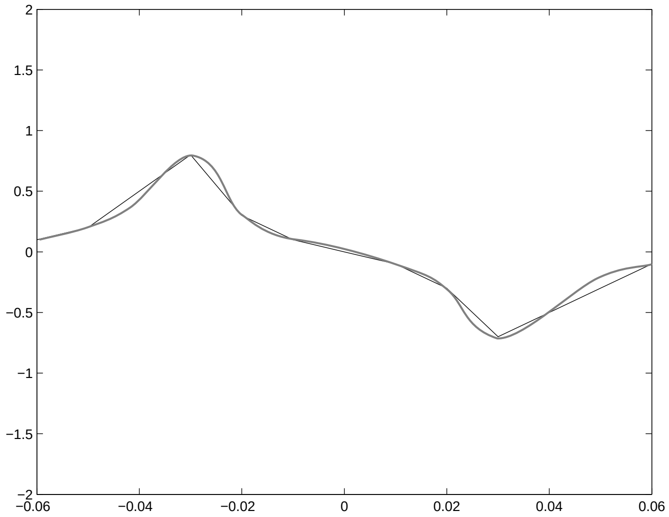


$$h(t) = [-0.1 \quad -0.2 \quad 1.0 \quad -0.2 \quad -0.1]$$

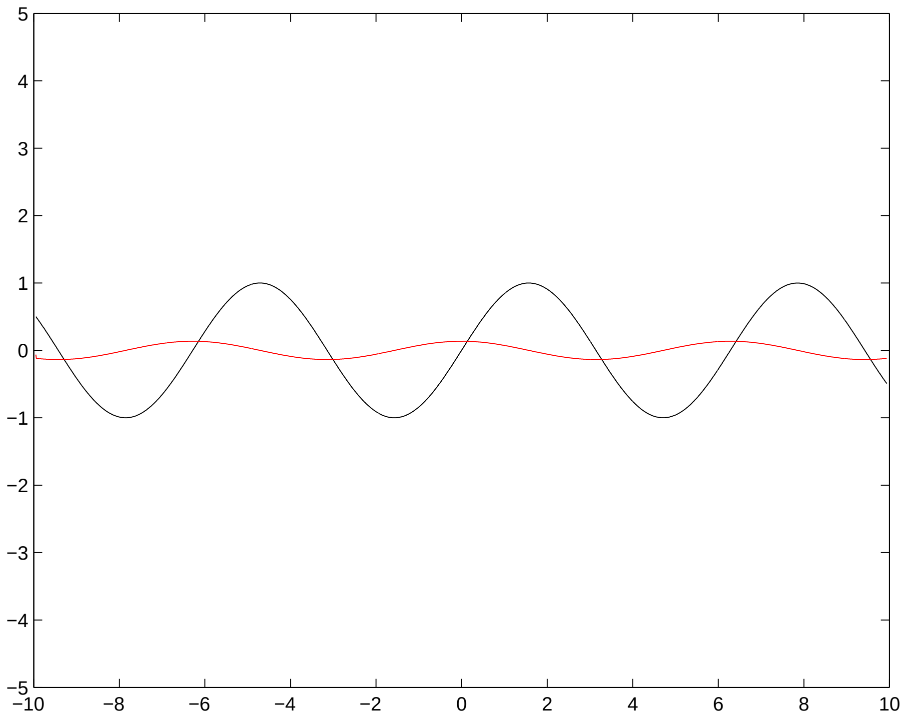


Linear systems and filtering

Linear systems and periodic signals

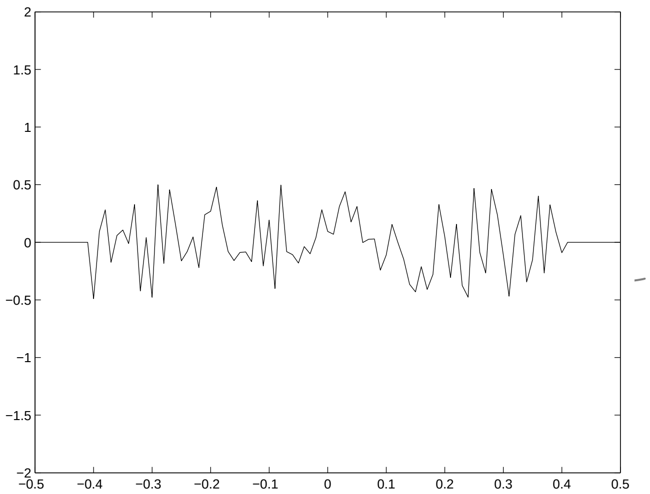


$h(t)$

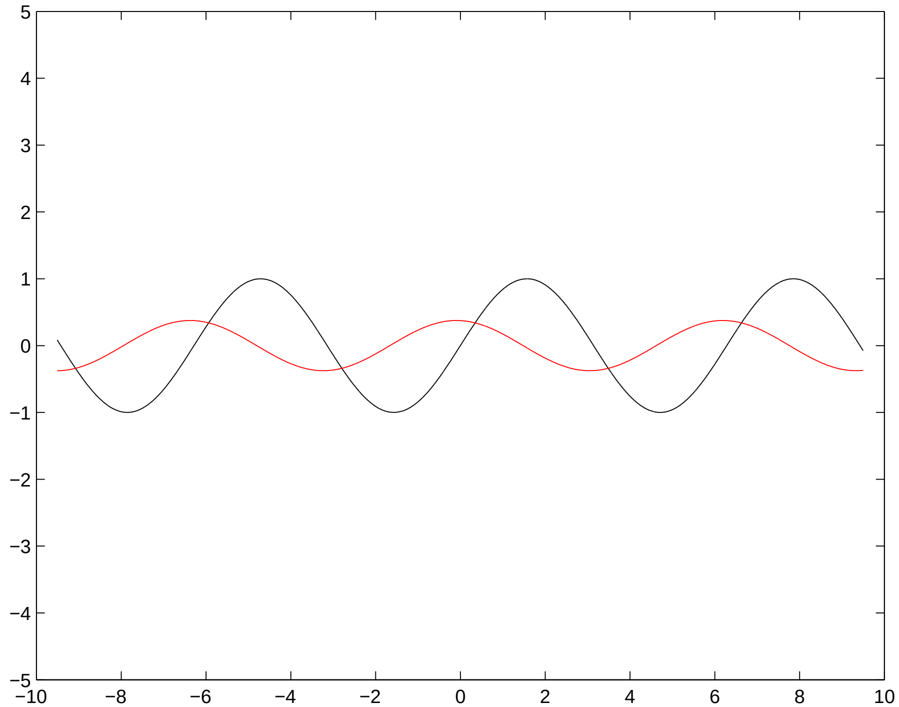


Linear systems and filtering

Linear systems and periodic signals

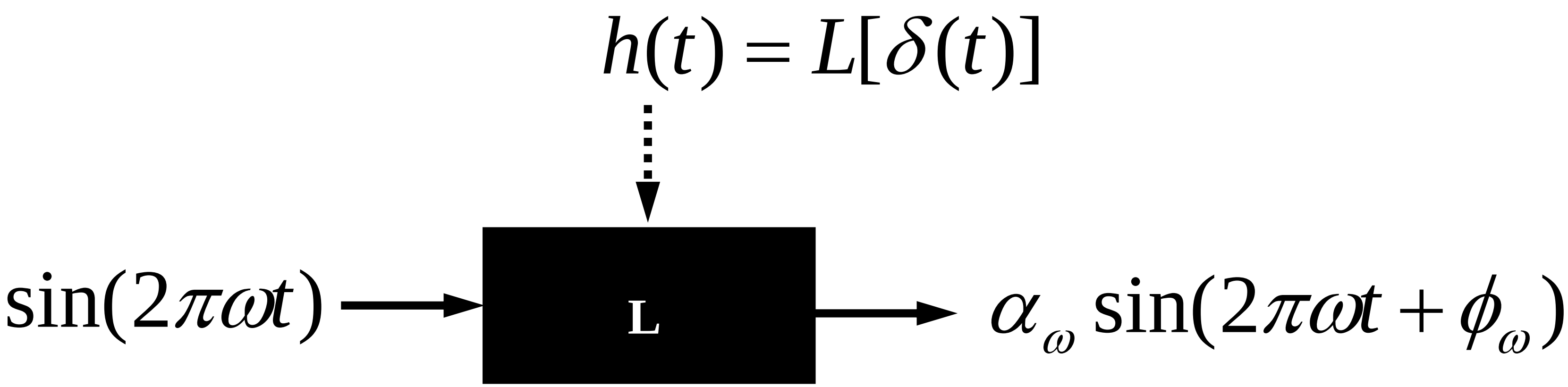


~~**I bet I got you now…**~~



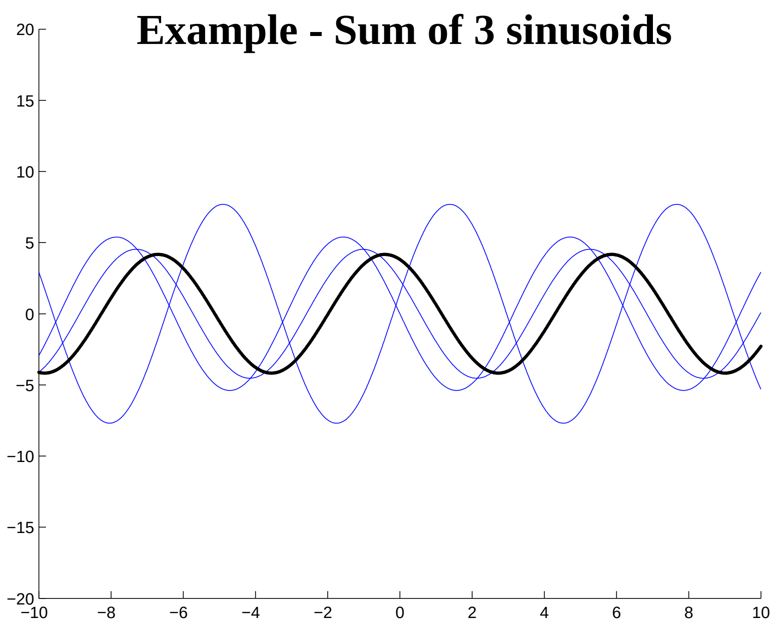
Linear systems and filtering

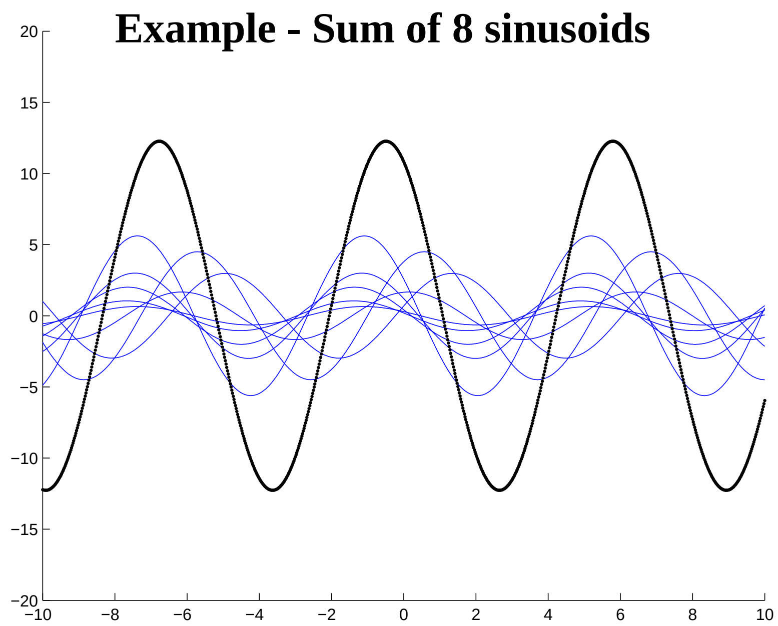
Linear systems and periodic signals



Linear systems and filtering

Linear systems and periodic signals





$$\sin(\omega t+\phi_1)+\sin(\omega t+\phi_2)=2\cos\left(\frac{\phi_1-\phi_2}{2}\right)\sin\left(\omega t+\frac{\phi_1+\phi_2}{2}\right)=const\cdot\sin(\omega t+\bar{\phi})$$

Linear systems and filtering

Periodic signals and The Fourier transform

$$x(t) = \int_0^{\infty} A_\omega \sin(2\pi\omega t + \theta_\omega) d\omega$$

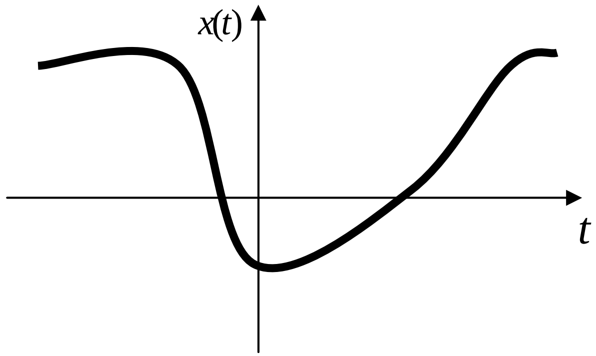
$$F(\omega) = F[x(t)] \Rightarrow \{A(\omega), \theta(\omega)\}$$

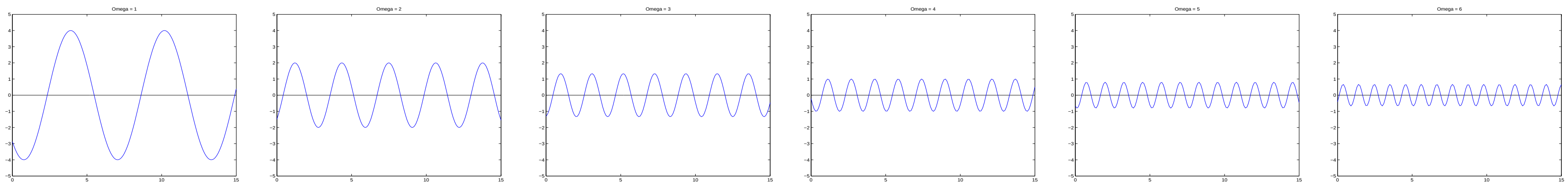
$$x(t) = \int_{-\infty}^{\infty} F(\omega) \cdot e^{i2\pi\cdot\omega\cdot t} d\omega$$

$$F(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-i2\pi\cdot\omega\cdot t} dt$$

Linear systems and filtering

Periodic signals and The Fourier transform





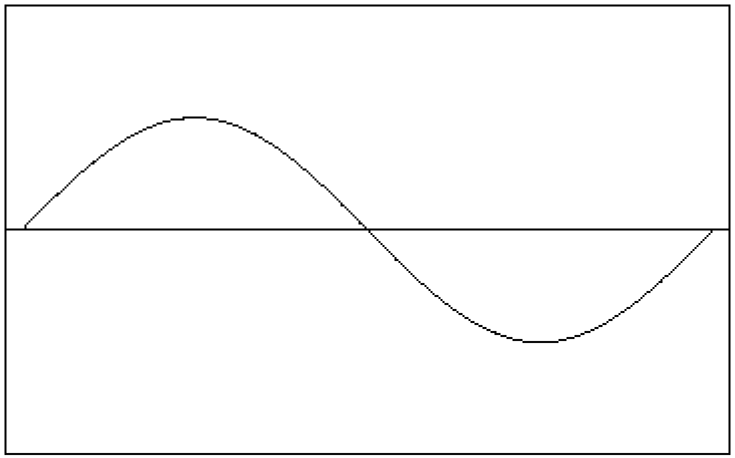
…

$$\begin{aligned}\omega &= 1\\ A_\omega &= 4\\ \theta_\omega &= 0.8\pi\end{aligned} \qquad \begin{aligned}\omega &= 2\\ A_\omega &= 2.8\\ \theta_\omega &= 0.1\pi\end{aligned} \qquad \begin{aligned}\omega &= 3\\ A_\omega &= 2.5\\ \theta_\omega &= 0.3\pi\end{aligned} \qquad \begin{aligned}\omega &= 4\\ A_\omega &= 1.8\\ \theta_\omega &= 0.3\pi\end{aligned} \qquad \begin{aligned}\omega &= 5\\ A_\omega &= 1.5\\ \theta_\omega &= 0.35\pi\end{aligned} \qquad \begin{aligned}\omega &= 6\\ A_\omega &= 1.2\\ \theta_\omega &= -0.2\pi\end{aligned} \quad \cdots$$

Linear systems and filtering

Periodic signals and The Fourier transform

$$x(t) = \int_0^\infty A_\omega \sin(2\pi\omega t + \theta_\omega) d\omega$$



Linear systems and filtering

Periodic signals and The Fourier transform

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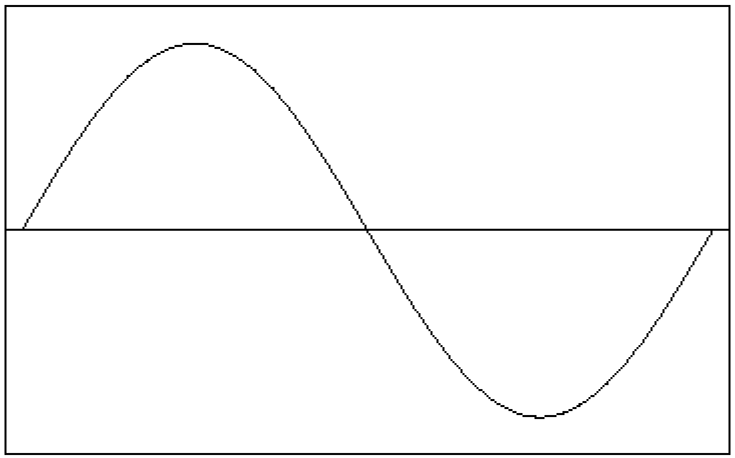
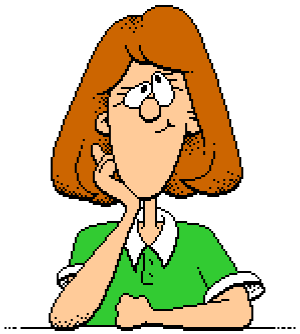


Linear systems and filtering

Linear systems and periodic signals

1: $\sin(2\pi\omega t) \longrightarrow$ **L** $\longrightarrow \alpha_\omega \sin(2\pi\omega t + \phi_\omega)$



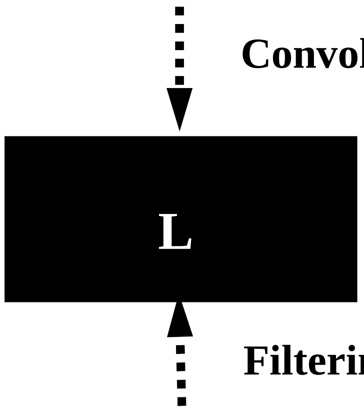
2: $$x(t) = \int_0^\infty A_\omega \sin(2\pi\omega t + \theta_\omega)\, d\omega$$

Linear systems and filtering

Linear systems and periodic signals

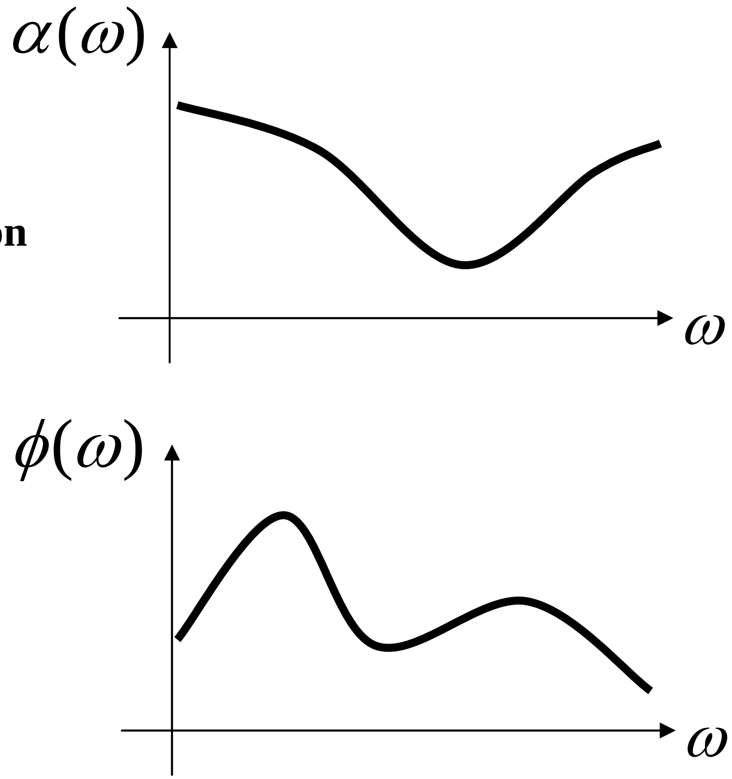
$$h(t) = L[\delta(t)]$$

Convolution representation



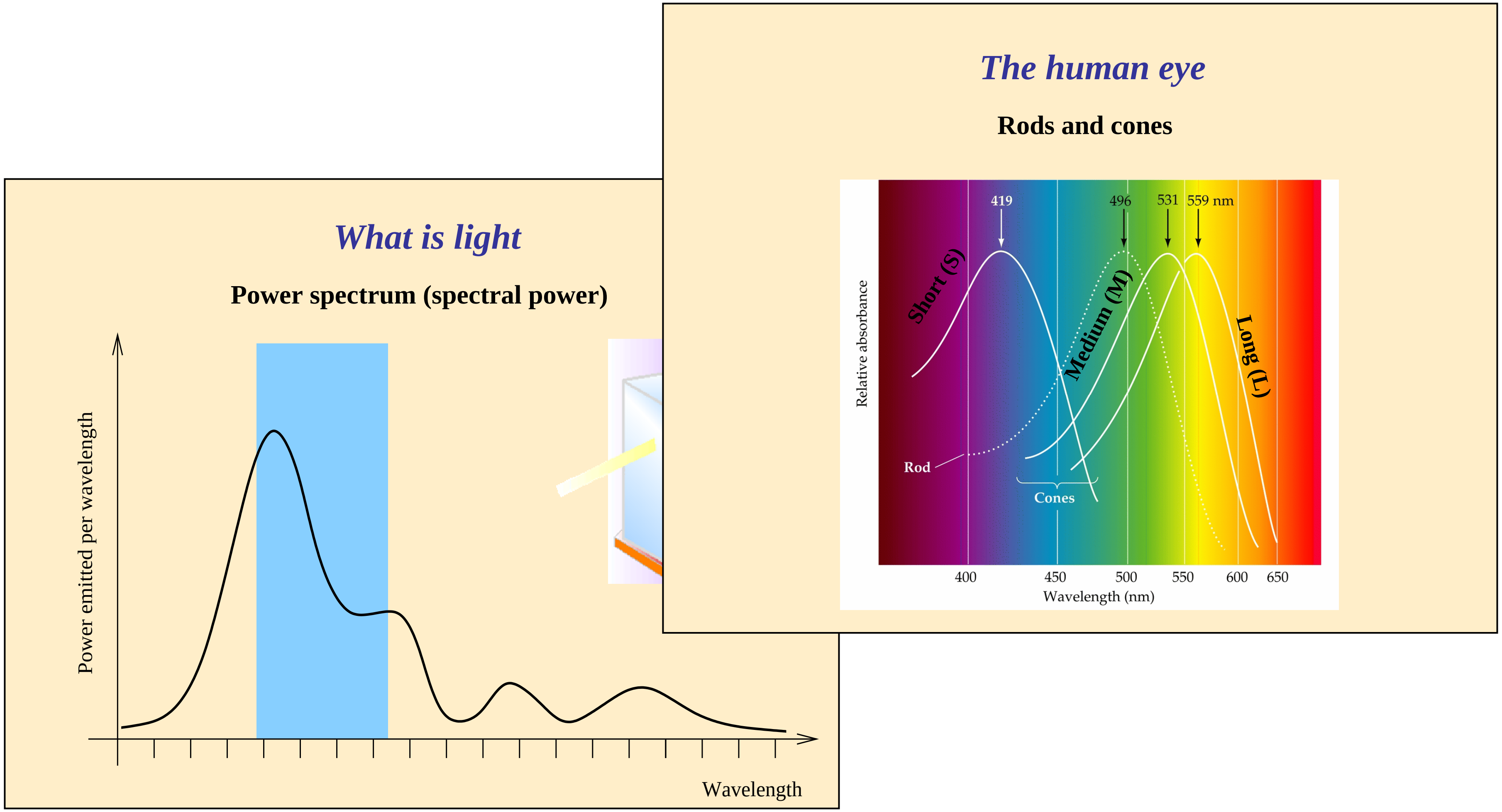
Filtering representation

$$\{\alpha(\omega), \phi(\omega)\}$$



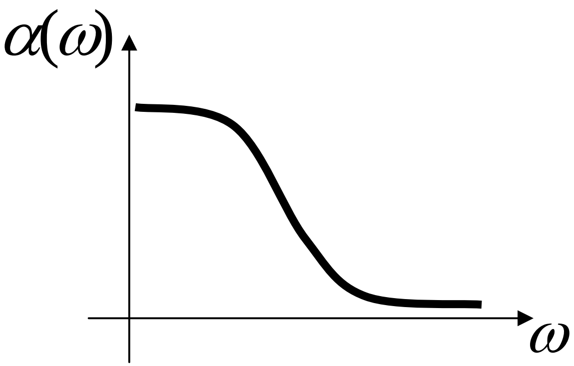
Linear systems and filtering

Linear systems and periodic signals

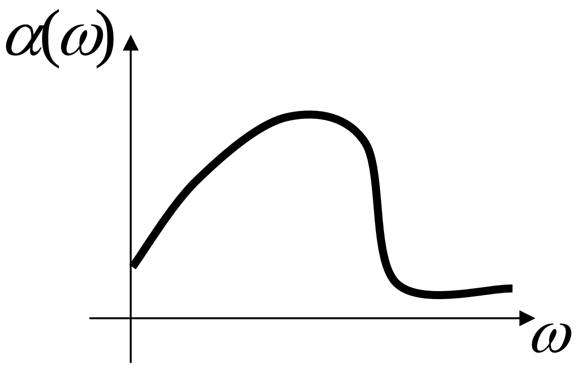


Linear systems and filtering

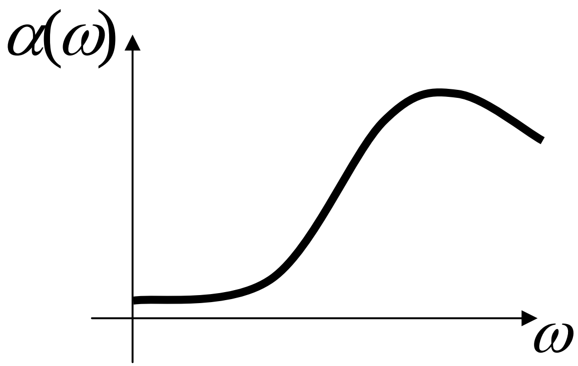
Linear filtering and the Modulation Transfer Function (MTF)



Low pass filter



Band pass filter



High pass filter

Linear systems and filtering

Linear systems and periodic signals

<u>Assume</u>

the human visual system is linear

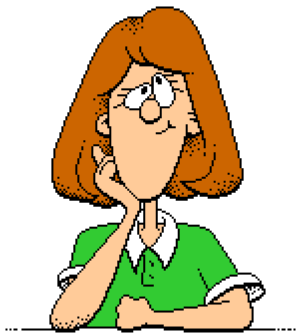
<u>Design</u>

an experiment that retrieves that MTF of the human visual system

Adapt…

Linear systems and filtering

Linear systems and periodic signals



Is the human visual system linear?