Sensing and the Imaging Process (IV)

Introduction to Computational and Biological Vision

CS 202-1-5261

Computer Science Department, BGU

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Image formation and representation

Aspects of image formation

1. Geometry
2. Optics

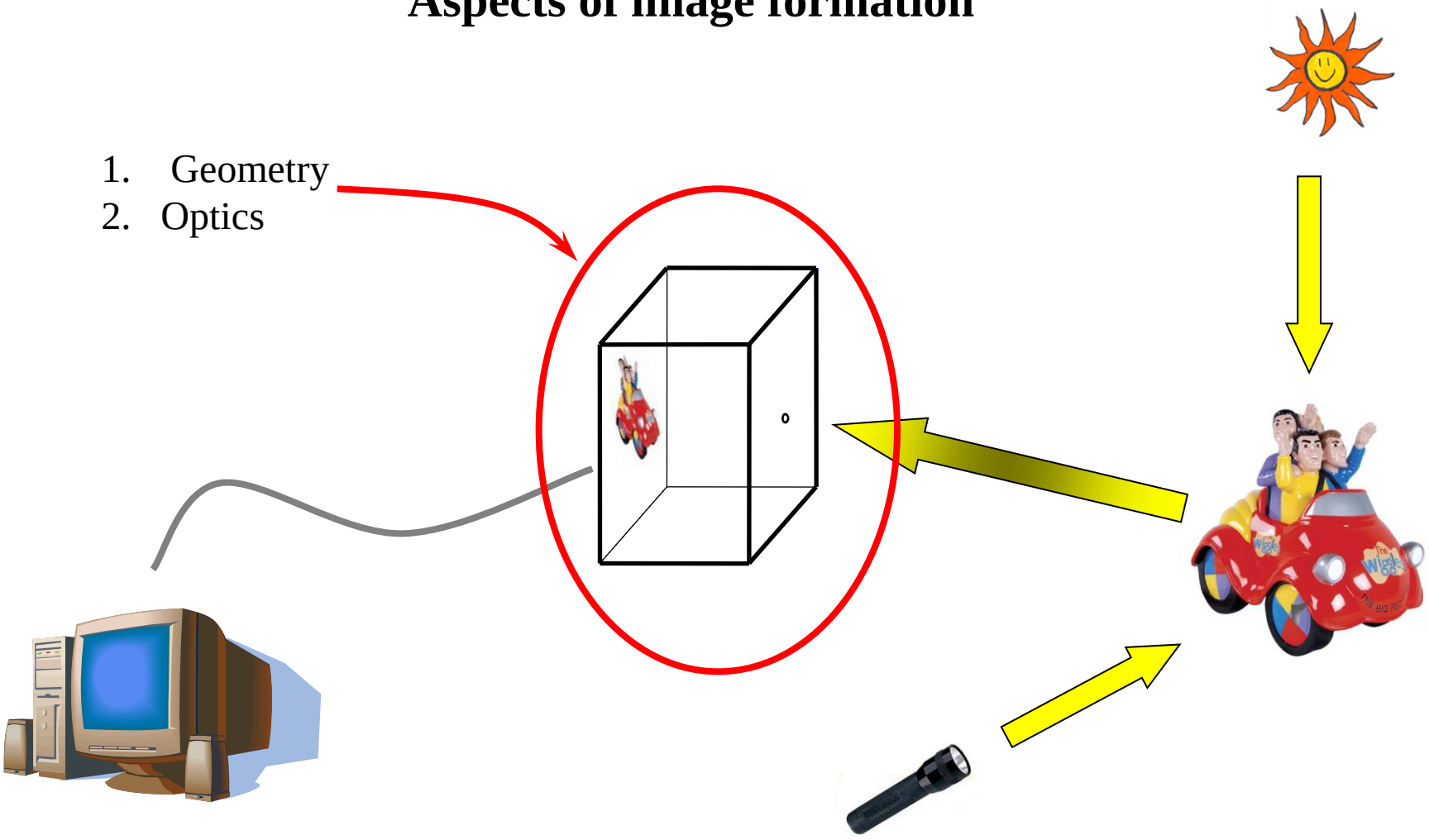


Image formation and representation

Pinhole camera model

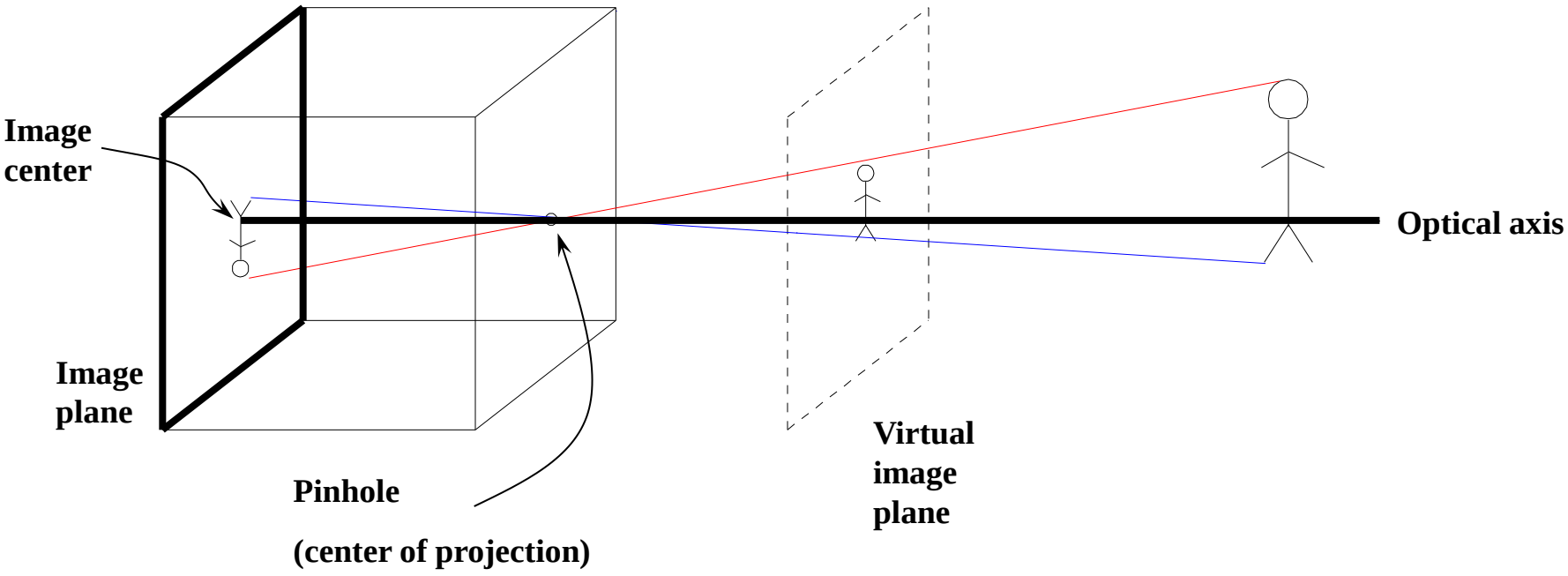
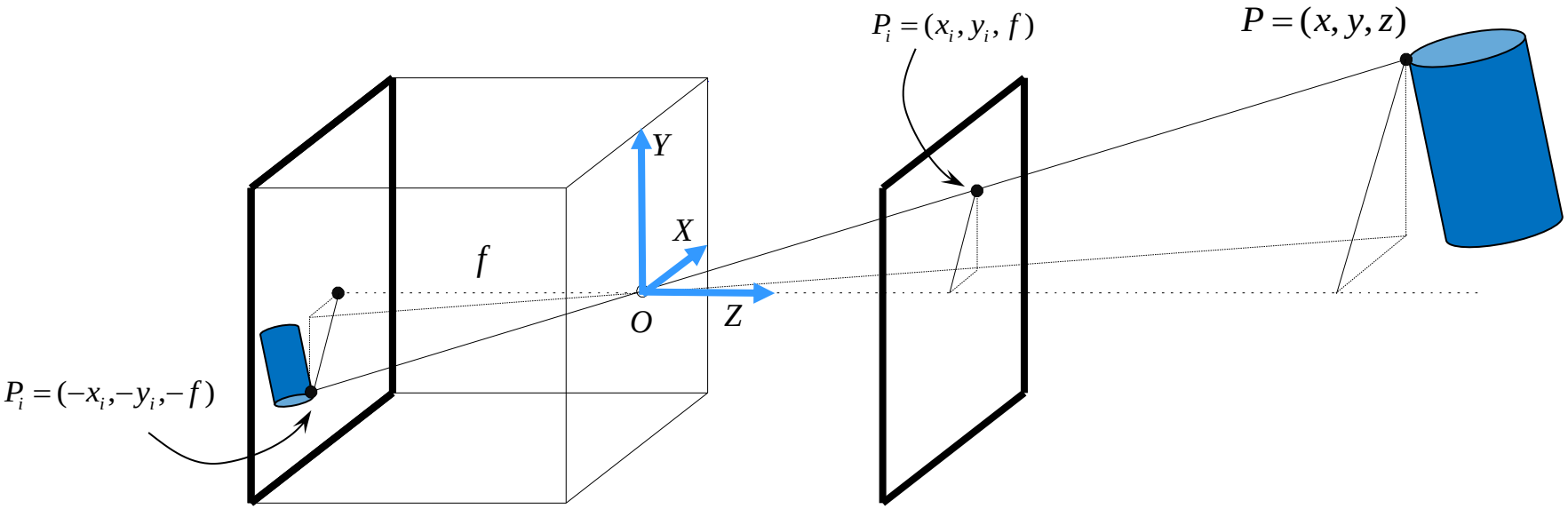


Image formation and representation

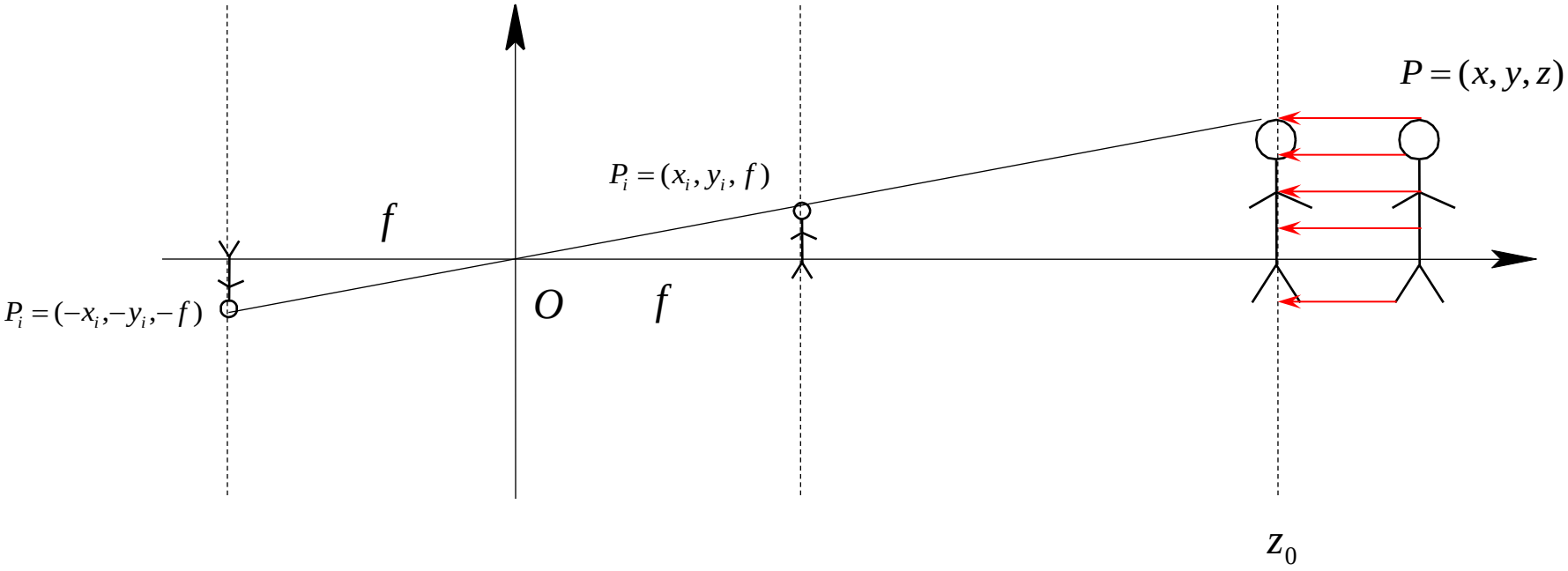
Pinhole camera model: Perspective projection



$$\overrightarrow{OP_i} = \lambda \cdot \overrightarrow{OP} \quad \Rightarrow \quad \lambda = \frac{x_i}{x} = \frac{y_i}{y} = \frac{f}{z} \quad \Rightarrow \quad \begin{cases} x_i = f \cdot \dfrac{x}{z} \\ y_i = f \cdot \dfrac{y}{z} \end{cases}$$

Image formation and representation

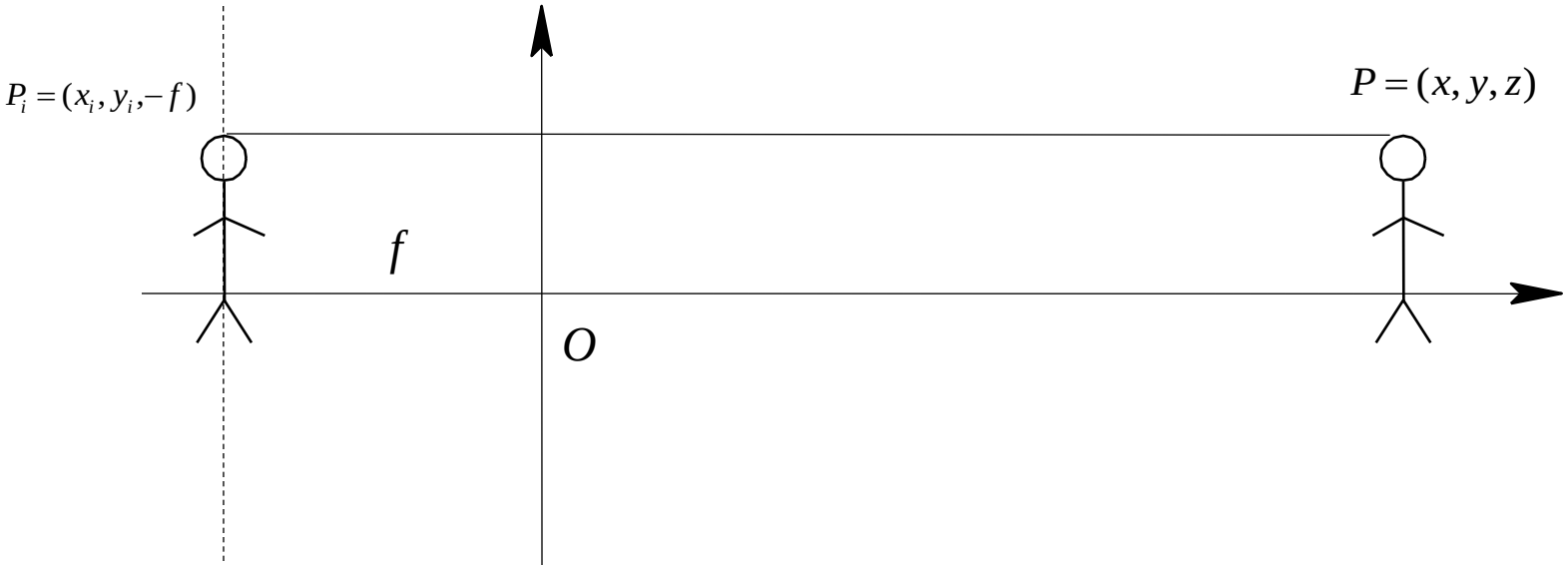
Linear approximation: Weak perspective projection



$$\begin{cases} x_i = s \cdot x \\ y_i = s \cdot y \end{cases}$$

Image formation and representation

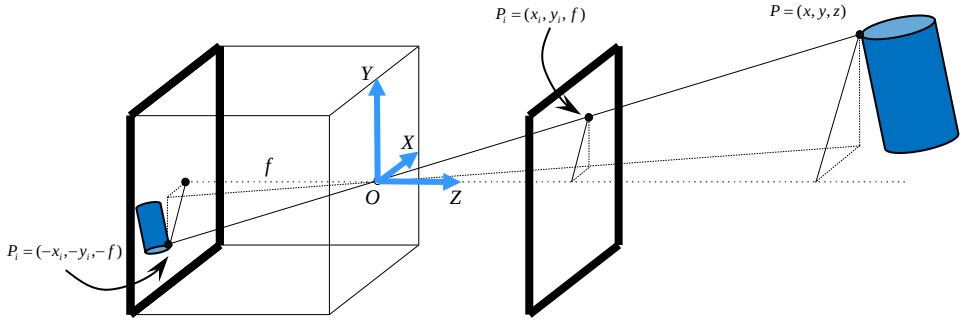
Linear approximation: Orthographic projection



$$\begin{cases} x_i = x \\ y_i = y \end{cases}$$

Image formation and representation

Homogeneous coordinates



$$\begin{aligned} (x, y) &\Leftrightarrow [x \cdot w, y \cdot w, w] & w \neq 0 \\ (x, y, z) &\Leftrightarrow [x \cdot w, y \cdot w, z \cdot w, w] & w \neq 0 \end{aligned}$$

most often w is set as $w = 1$

$$\left.\begin{aligned} x_i &= f \cdot \frac{x}{z} \\ y_i &= f \cdot \frac{y}{z} \end{aligned}\right\} \leftrightarrow \begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \frac{1}{z} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\left.\begin{aligned} x_i &= f \cdot \frac{x}{z} \\ y_i &= f \cdot \frac{y}{z} \end{aligned}\right\} \leftrightarrow \begin{pmatrix} x_i \\ y_i \\ f \end{pmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Image formation and representation

The main problem of vision – recovery of structure is ill defined

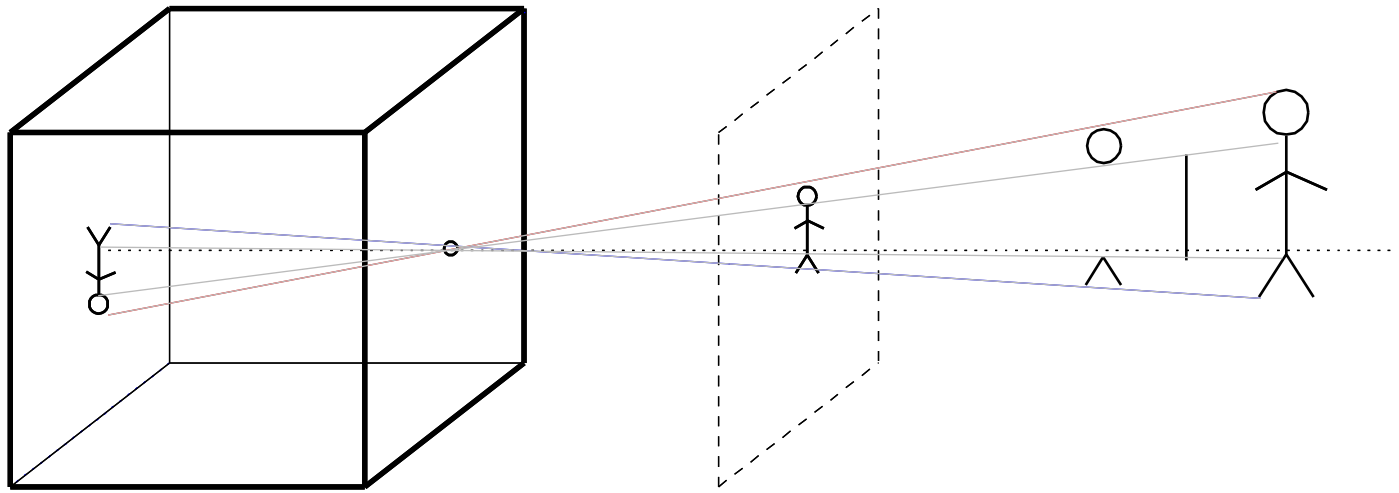


Image formation and representation

Aspects of image formation

3. Radiometry
4. Color

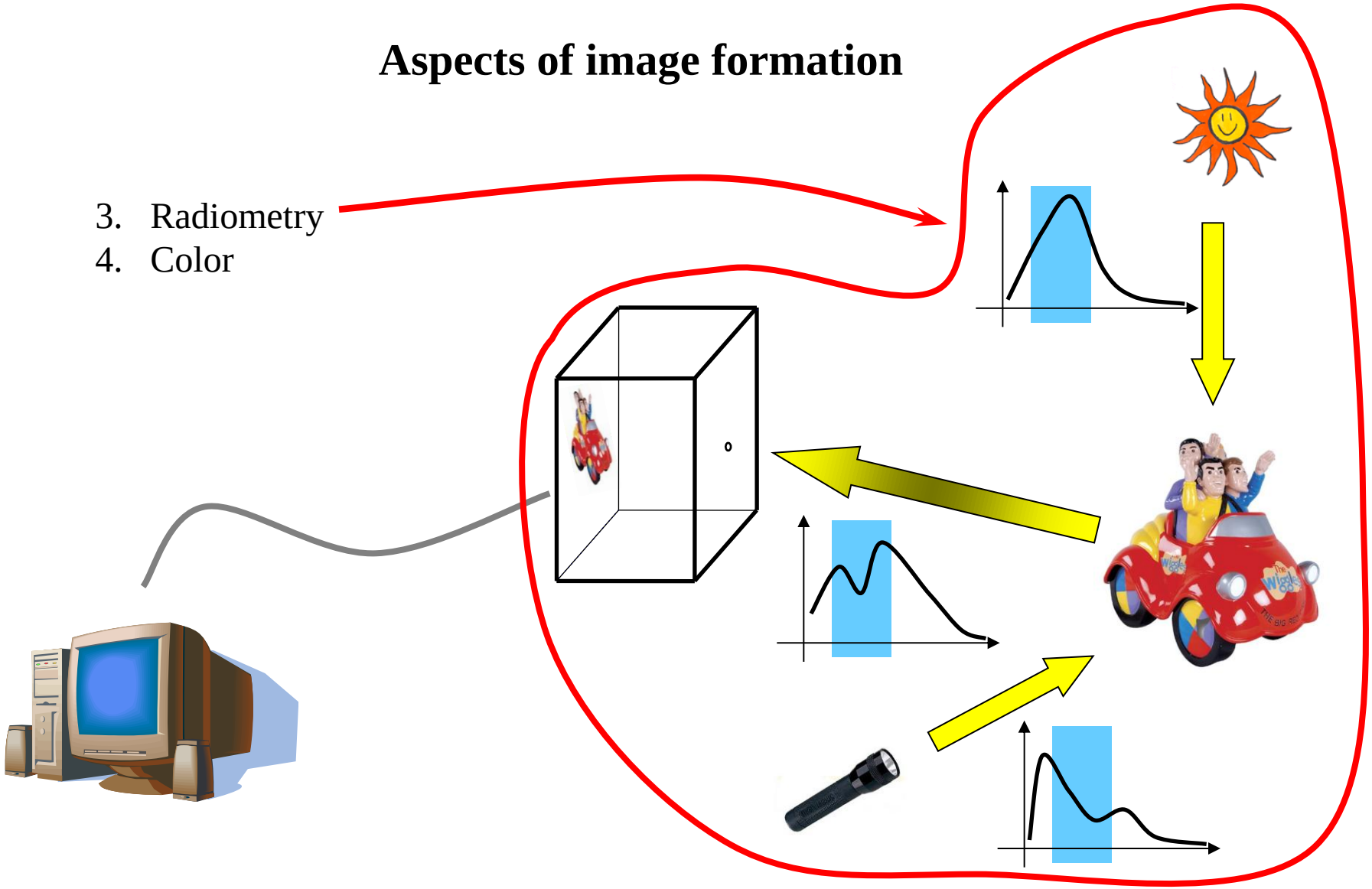


Image formation and representation

Aspects of image formation

5. Sampling
6. Quantization

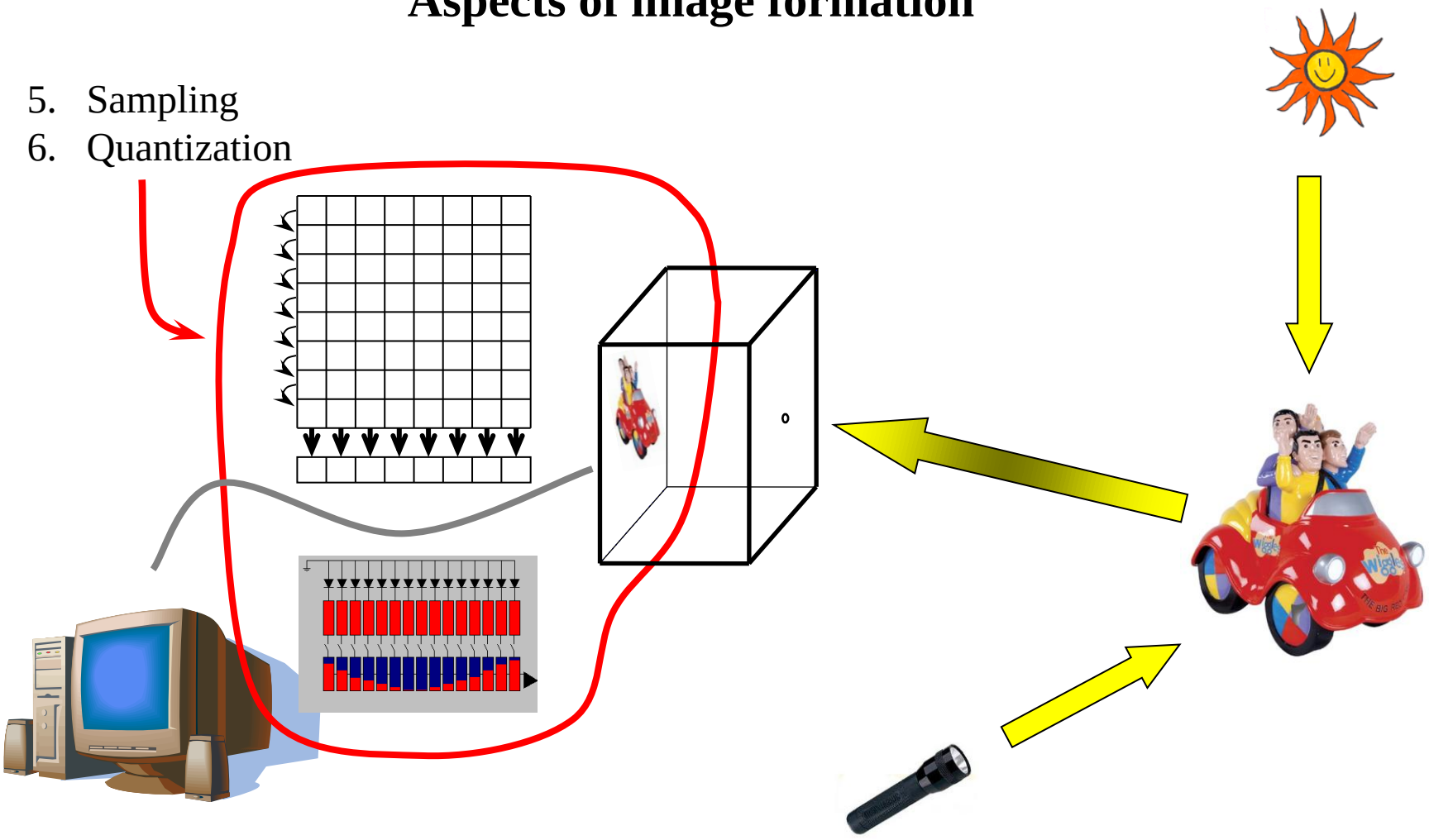


Image formation and representation

Aspects of image formation

1. Geometry
2. Optics
3. Radiometry
4. Color
5. Sampling
6. Quantization

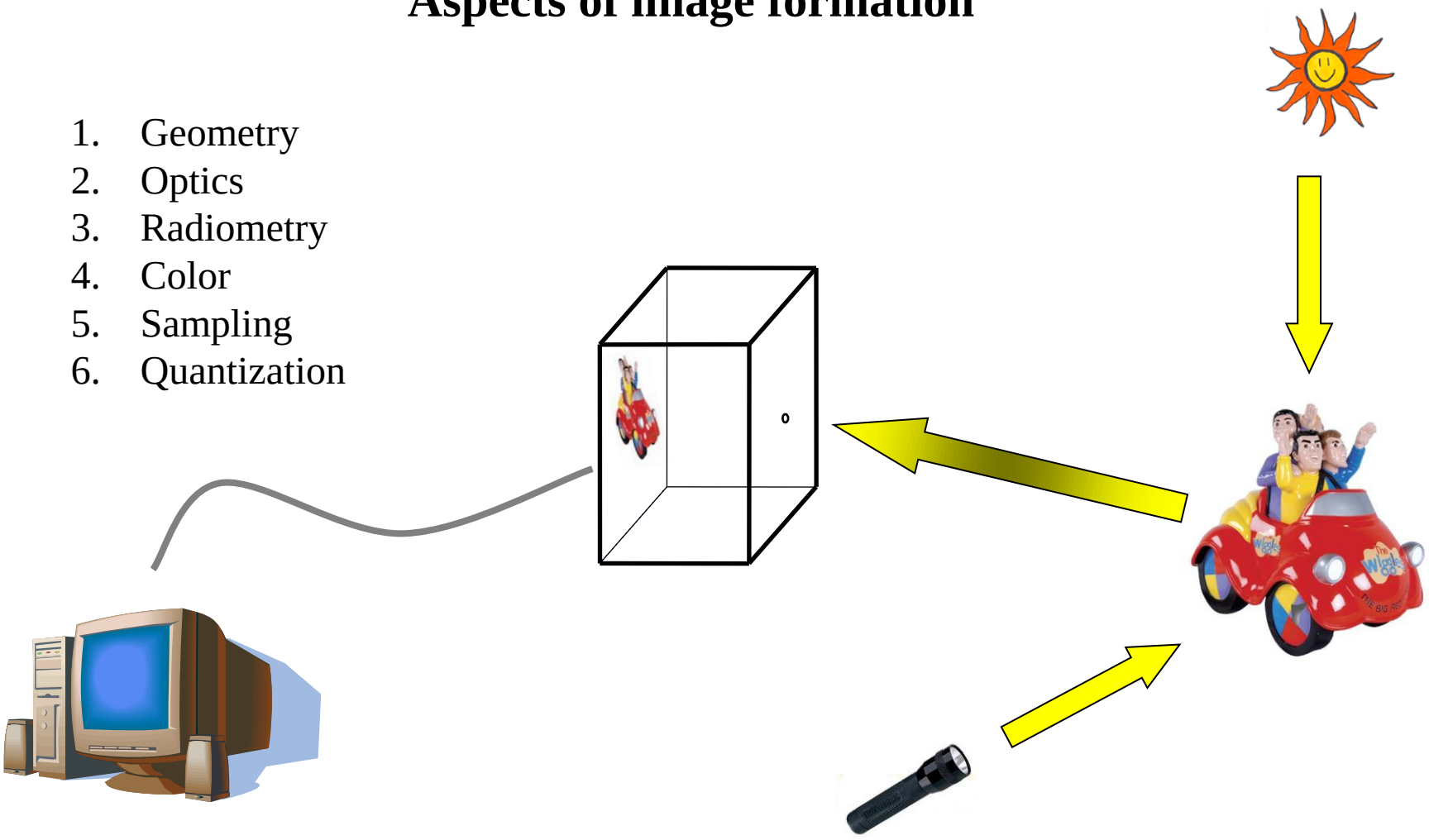
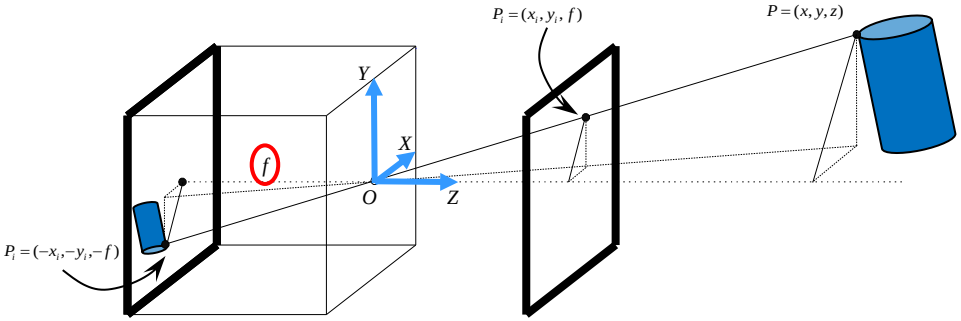


Image formation and representation

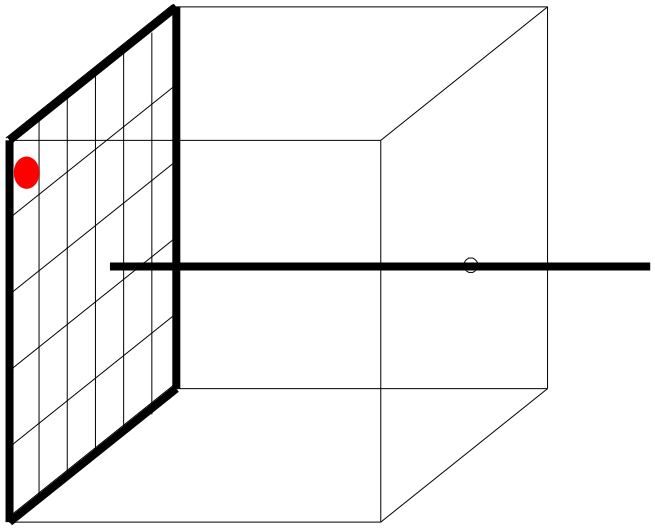
Camera *intrinsic* parameters



$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \frac{1}{z} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x_i = S_x f \cdot \frac{x}{z} = \alpha \frac{x}{z} + x_0$$

$$y_i = S_y f \cdot \frac{y}{z} = \beta \frac{x}{z} + y_0$$

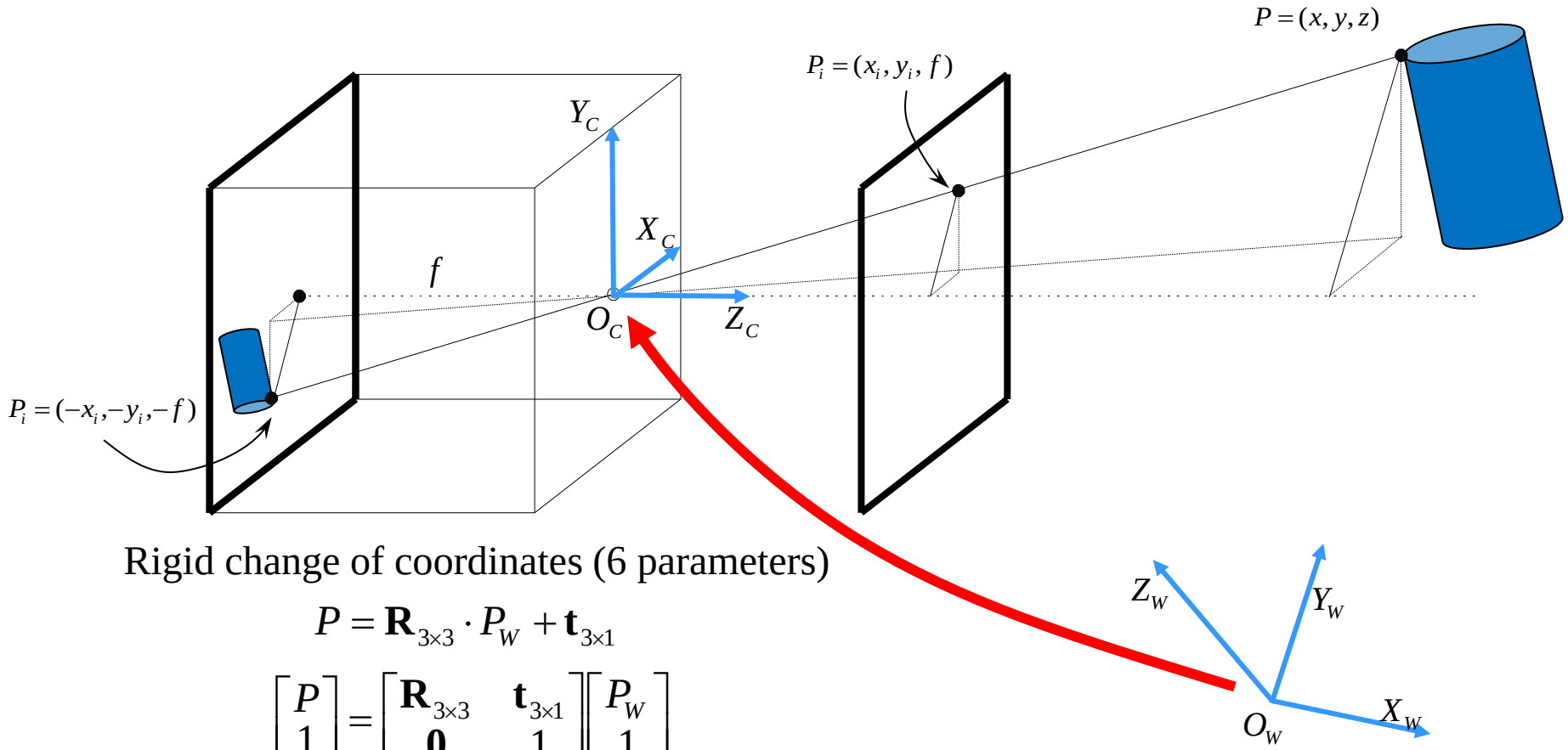


$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \frac{1}{z} \underbrace{\begin{bmatrix} \alpha & 0 & x_0 & 0 \\ 0 & \beta & y_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The *calibration matrix* **K**

Image formation and representation

Camera *extrinsic* parameters



Rigid change of coordinates (6 parameters)

$$P = \mathbf{R}_{3\times3} \cdot P_W + \mathbf{t}_{3\times1}$$

$$\begin{bmatrix} P \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{R}_{3\times3} & \mathbf{t}_{3\times1} \\ \mathbf{0} & 1 \end{bmatrix}}_{\mathbf{F}} \begin{bmatrix} P_W \\ 1 \end{bmatrix}$$

Image formation and representation

The perspective projection matrix

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \frac{1}{z}\mathbf{M}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{M}_{3\times 4} = \begin{bmatrix} \alpha\mathbf{r}_1^T - \alpha\cot\theta\,\mathbf{r}_2^T + x_0\mathbf{r}_3^T & \alpha t_1 - \alpha\cot\theta\, t_2 + x_0 t_3 \\ \frac{\beta}{\sin\theta}\mathbf{r}_2^T + y_0\mathbf{r}_3^T & \frac{\beta}{\sin\theta}t_2 + y_0 t_3 \\ \mathbf{r}_3^T & t_3 \end{bmatrix}$$

Pixel scaling (int)

Camera translation (ext)

Camera rotation (ext)

CCD XY skew (int)

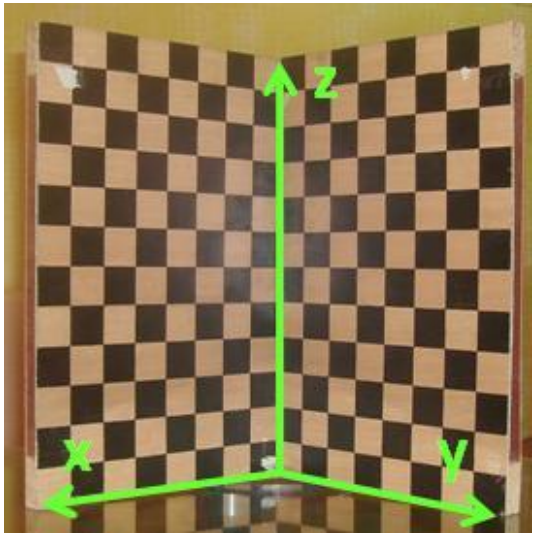
Camera center (int)

Image formation and representation

Camera calibration

- Measure enough pairs $\left(P_i^j ; P^j\right) \quad j = 1..N \geq 6$
- Estimate **M**
- Estimate the intrinsic and extrinsic parameters

$$P_i^j = \frac{1}{z}\mathbf{M} \cdot P^j$$



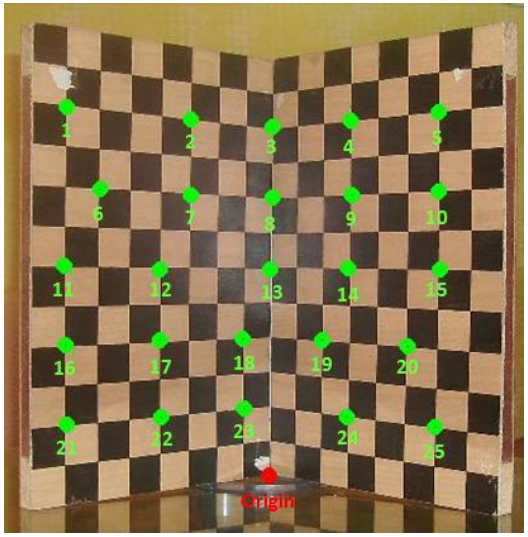


Image formation and representation

Image representation

32	48	57	98	142	158	160	151	155	153
161	160	155	163	159	166	159	176	153	156
143	116	99	114	100	105	132	148	132	108
124	124	110	93	96	78	38	42	66	62
197	196	199	200	196	200	199	195	203	201
150	144	125	119	118	108	144	164	227	81
162	163	171	174	172	165	175	171	193	188
201	152	184	110	89	136	119	100	120	183
93	137	155	173	172	164	162	159	171	157
203	179	174	173	138	117	100	107	107	118
188	145	111	169	160	135	107	74	59	63
198	193	182	192	198	203	200	196	201	192
183	179	156	128	128	157	169	174	159	208
161	165	174	164	163	164	156	157	156	162
130	154	173	112	190	174	153	179	187	161
46	85	89	68	167	154	155	163	155	158
191	198	191	205	192	190	156	105	106	122
180	170	200	160	166	184	159	144	113	75
192	192	190	195	191	190	195	194	196	196
168	151	171	185	181	170	149	167	174	177
157	156	171	153	147	157	162	159	156	163
112	162	168	155	180	188	172	162	186	185
67	67	49	69	92	87	111	156	160	158
202	193	195	196	198	196	193	196	198	162
192	198	191	201	201	197	198	188	173	144
155	156	198	198	194	190	192	194	192	190
182	171	163	161	162	176	186	195	161	154

Image formation and representation

Image representation

(ignoring discretization and quantization)

$$I(x, y) : R^2 \rightarrow R$$

Image formation and representation

The issue of representation

Representation:

A *formal* system for making *explicit* certain *entities* or types of *information*, together with the specification of how the system achieves this goal.

Representational tradeoff:

Any particular representation makes certain information (or properties of the represented entities) explicit at the expense of other information (or properties) that is pushed into the background and may be quite hard to recover.

Therefore…

How information is represented greatly affects how easy it is to do certain things with it.

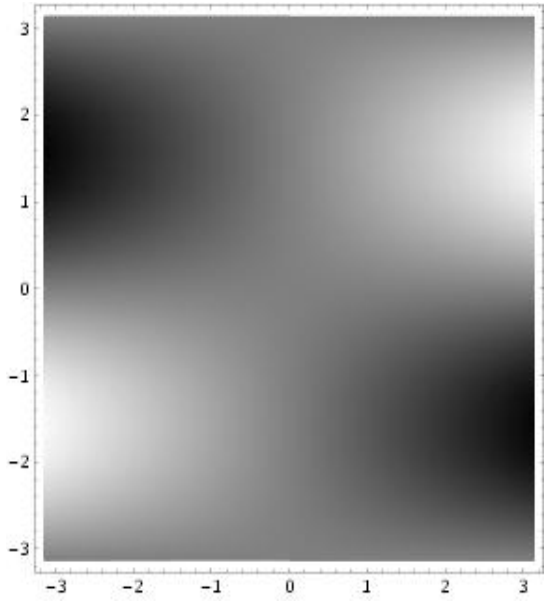
Image formation and representation

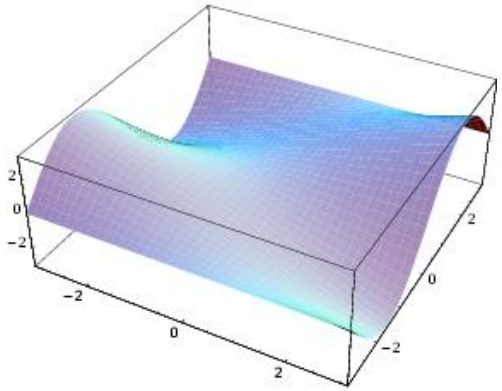
The issue of representation

Image formation and representation

Image representation

$$I(x, y) : R^2 \rightarrow R$$





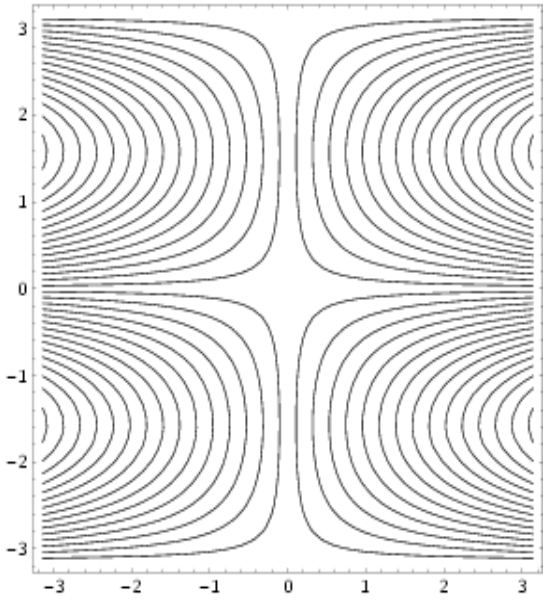


Image formation and representation

Image representation