Sensing and the Imaging Process (IV)

Introduction to Computational and Biological Vision

CS 202-1-5261

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Image formation and representation

Aspects of image formation

1. Geometry
2. Optics
Image formation and representation

Pinhole camera model

Pinhole (center of projection)

Image plane

Virtual image plane

Optical axis
Image formation and representation

Pinhole camera model: Perspective projection

\[ \overrightarrow{OP_i} = \lambda \cdot \overrightarrow{OP} \quad \Rightarrow \quad \lambda = \frac{x_i}{x} = \frac{y_i}{y} = \frac{f}{z} \quad \Rightarrow \quad \begin{cases} x_i = f \cdot \frac{x}{z} \\ y_i = f \cdot \frac{y}{z} \end{cases} \]
Image formation and representation

Linear approximation: Weak perspective projection

\[
\begin{align*}
\mathbf{p}_i &= (x_i, y_i, f) \\
\mathbf{x}_i &= s \cdot x_i \\
\mathbf{y}_i &= s \cdot y_i
\end{align*}
\]
Image formation and representation

Linear approximation: Orthographic projection

\[ P_i = (x_i, y_i, -f) \]

\[ P = (x, y, z) \]

\[ \begin{align*}
  x_i &= x \\
  y_i &= y
\end{align*} \]
**Image formation and representation**

**Homogeneous coordinates**

\[
(x, y) \iff \begin{bmatrix} x \cdot w, y \cdot w, w \end{bmatrix} \quad w \neq 0
\]

\[
(x, y, z) \iff \begin{bmatrix} x \cdot w, y \cdot w, z \cdot w, w \end{bmatrix} \quad w \neq 0
\]

most often \( w \) is set as \( w = 1 \)

\[
x_i = f \cdot \frac{x}{z}
\]

\[
y_i = f \cdot \frac{y}{z}
\]

\[
x_i = f \cdot \frac{x}{z}
\]

\[
y_i = f \cdot \frac{y}{z}
\]
Image formation and representation

The main problem of vision – recovery of structure is ill defined
Image formation and representation

Aspects of image formation

3. Radiometry
4. Color
Image formation and representation

Aspects of image formation

5. Sampling
6. Quantization
Image formation and representation

Aspects of image formation

1. Geometry
2. Optics
3. Radiometry
4. Color
5. Sampling
6. Quantization
Image formation and representation

Camera intrinsic parameters

\[
\begin{pmatrix}
  x_i \\
  y_i \\
  1
\end{pmatrix} = \begin{pmatrix}
  x_i \\
  y_i \\
  1
\end{pmatrix} = \frac{1}{z} \begin{bmatrix}
  f & 0 & 0 & 0 \\
  0 & f & 0 & 0 \\
  0 & 0 & 1 & 0
\end{bmatrix} \begin{pmatrix}
  x \\
  y \\
  z \\
  1
\end{pmatrix}
\]

\[
x_i = S_x f \cdot \frac{x}{z} = \alpha \frac{x}{z} + x_0
\]

\[
y_i = S_y f \cdot \frac{y}{z} = \beta \frac{y}{z} + y_0
\]

The calibration matrix \( K \)
Image formation and representation

**Camera extrinsic parameters**

Rigid change of coordinates (6 parameters)

\[
P_i = (x_i, y_i, f)
\]

\[
P = R_{3\times3} \cdot P_W + t_{3\times1}
\]

\[
\begin{bmatrix}
P \\
1
\end{bmatrix} = 
\begin{bmatrix}
R_{3\times3} & t_{3\times1} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
P_W \\
1
\end{bmatrix}
\]

\[
F
\]
Image formation and representation

The perspective projection matrix

\[
\begin{pmatrix}
    x_i \\
y_i \\
1
\end{pmatrix}
= \begin{pmatrix}
x_i \\
y_i \\
1
\end{pmatrix}
= \frac{1}{z}
M
\begin{pmatrix}
x \\
y \\
z \\
1
\end{pmatrix}
\]
Image formation and representation

Camera calibration

- Measure enough pairs \( \left( P_i^j; P^j \right) \), \( j = 1..N \geq 6 \)
- Estimate \( M \)
- Estimate the intrinsic and extrinsic parameters

\[
P_i^j = \frac{1}{z} M \cdot P^j
\]
Image formation and representation

Image representation
Image formation and representation

Image representation
(ignoring discretization and quantization)

\[ I(x, y) : R^2 \rightarrow R \]
The issue of representation

Representation:

A *formal* system for making *explicit* certain *entities* or types of *information*, together with the specification of how the system achieves this goal.

Representational tradeoff:

Any particular representation makes certain information (or properties of the represented entities) explicit at the expense of other information (or properties) that is pushed into the background and may be quite hard to recover.

Therefore…

*How information is represented greatly affects how easy it is to do certain things with it.*
Image formation and representation

The issue of representation
Image formation and representation

Image representation

\[ I(x, y): \mathbb{R}^2 \rightarrow \mathbb{R} \]
Image formation and representation

Image representation

Mr. Been Laden