Toward a Theory of Shape from Specular Flow

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Shape from Specular Reflections
Shape from Specular Reflections
[Whiteout, by Anish Kappor]
Shape from Specular Reflections
Shape from Specular Reflections

[Image of a 3D model and two interpretations]

Interpretation 1
- Shape 1
- Scene 1

Interpretation 2
- Shape 2
- Scene 2

[Fleming et al., 2004]
Human Perception

[Fleming et al., 2004]
Shape from Specular Reflections

- Consider relative motion
- Motion induces specular flow
Outline

• Related Work
• 2D Shape Reconstruction
• 3D Shape Reconstruction
• Future Work
Related Work

Point Light Sources
• [Longuet-Higgins 1960]
• [Blake and Bulthoff 1991]
• [Oren and Nayar 1997]

Calibrated Environments
• [Savarese, Chen, Perona 2005]
• [Bonfort, Sturm, Gargallo 2006]
Natural Lighting and Motion

Related Work

• [Waldon and Dyer, 1993]: Qualitative shape

• [Roth and Black, 2006]: Parametric shapes (for example spheres, ellipsoids)

Our Work

• Quantitative reconstruction

• General smooth surfaces
Shape from Specular Flow in 2D

Orthographic image plane $I(x)$

Far-field illumination environment $E(\theta)$

Surface profile $f(x)$
Shape from Specular Flow in 2D

Orthographic image plane
$I(x)$

Far-field illumination environment
$E(\theta)$

$I(x) = E(\theta(x))$

Related via surface:
$$\tan\left(\frac{\theta(x)}{2}\right) = -f_x(x)$$

Surface profile
$f(x)$
Shape from Specular Flow in 2D

Specular flow
\[ u = \frac{dx}{dt} \]

Environment motion
\[ \omega = \frac{d\theta}{dt} \]

Related via surface:
\[ \tan \left( \frac{\theta(x)}{2} \right) = -f_x(x) \]
Shape from Specular Flow in 2D

\[ \tan\left(\frac{\theta(x)}{2}\right) = -f_x(x) \]

\[ \omega = \frac{d\theta}{dt} = \frac{d\theta}{dx} \frac{dx}{dt} = \frac{d\theta}{dx} u \]

\[ 2u(x)f_{xx} + \omega f_x^2 + \omega = 0 \]

\[ f_x(x) = \tan \left( -\frac{\omega}{2} \int_{x_0}^{x} \frac{d\lambda}{u(\lambda)} + C \right) \]
Shape from Specular Flow in 2D

Specular flow
\[ u = \frac{dx}{dt} \]

Environment motion
\[ \omega = \frac{d\theta}{dt} \]

\[ f_x(x) = \tan \left( -\frac{\omega}{2} \int_{x_o}^{x} \frac{ds}{u(s)} + C \right) \]
Shape from Specular Flow in 2D

\[ 2u(x)f_{xx} + \omega f_x^2 + \omega = 0 \]

\[ f_u(x) = \tan\left( \frac{\omega}{2\kappa(x)} \sqrt{\int_0^x \omega d\lambda - \left( \int_0^x f_x(x) \right)^2} + C \right) \]

Profile curvature
Summary: 2D

• Analytic solution for shape in unknown lighting
• Enabled by:
  – Far-field camera and environment
  – Observed environment motion
• Handles general (smooth) surfaces, including points with zero curvature

Up next: three dimensions
Shape from Specular Flow in 3D

Illumination environment $E(\theta, \phi)$

Image plane

$I(x, y)$

$S = (x, y, f(x, y))$

$E(\theta, \phi) = (2\theta, \phi) = (\alpha, \beta)$

$\hat{\mathbf{n}} = (\theta, \phi)$

$\hat{\mathbf{r}} = (2\theta, \phi)$

$x$ $y$ $z$
Shape from Specular Flow in 3D

\[ \mathbf{u}(x, y) \]

\[ \omega(\alpha, \beta) \]
Shape from Specular Flow in 3D

\[
\tan(\alpha) = \frac{2\|\nabla f\|}{1-\|\nabla f\|^2}, \quad \tan(\beta) = \frac{f_y}{f_x}
\]

\[
\omega = \frac{d(\alpha,\beta)}{dt} = \frac{\partial(\alpha,\beta)}{\partial(x,y)} \frac{d(x,y)}{dt} = \frac{\partial(\alpha,\beta)}{\partial(x,y)} u
\]

\[
\omega = Ju
\]

\[
\begin{pmatrix}
\frac{fx f_{xx} + fy f_{xy}}{\|\nabla f\| \cdot (1+\|\nabla f\|^2)} & \frac{fx f_{xy} + fy f_{yy}}{\|\nabla f\| \cdot (1+\|\nabla f\|^2)} \\
\frac{fx f_{xy} - fy f_{xx}}{2\|\nabla f\|^2} & \frac{fx f_{yy} - fy f_{xy}}{2\|\nabla f\|^2}
\end{pmatrix}
\]
Singularities in Specular Flow

\[ \omega = J u \quad \rightarrow \quad u = J^{-1} \omega \]

\[ \text{Det}(J) = \frac{2K(1 + \| \nabla f \|^2)}{\| \nabla f \|} \]
Special Case:
Rotation about the View Direction

\[ \omega = Ju \]

\[ \omega'(\alpha \beta \omega_z) = \omega(\omega_\alpha(\alpha, \beta), \omega_\alpha(\alpha, \beta)) \]

\[ \omega_\alpha(\alpha, \beta) = \omega \sin \alpha_0 \sin(\beta_0 - \beta) \]

\[ \omega_\beta(\alpha, \beta) = \omega(\cos \alpha_0 - \sin \alpha_0 \cos(\beta - \beta_0) \cot \alpha) \]
Special Case: Rotation about the View Direction

Integral curves of specular flow
Special Case:
Rotation about the View Direction

Integral curves of specular flow
Special Case:
Rotation about the View Direction

\[
\omega = Ju
\]

\[
Du(\|\nabla f\|^2) = 0
\]

\[
Du(\angle \nabla f) = \frac{2\omega_z}{\|u\|}
\]

\[
\nabla f
\]
Proof of Concept

ACQUISITION

IMAGE

MEASURED FLOW

RECOVERED SURFACE

initial data

initial data
Comment: Measuring Specular Flow

Multi-resolution
Horn & Schunck

Black & Anandan

Flow Orientation

PARABOLIC CURVE
Conclusion…Future Work

Summary

• General equation relating shape and specular flow
• Analyzed solutions in some cases (2D and view axis rotation)
• General surfaces, including points with zero Gaussian curvature

Future Work

• General environment motion
• Object motion
• Estimating specular flow