

PENETRATION OF METAL-FABRICS COMPOSITES BY SMALL PROJECTILES

L. I. Slepyan and M. V. Ayzenberg-Stepanenko

The Institute for Industrial Mathematics, 4 Yehuda Hanachtom Street, 84311,
P.O.B. 15013, Beer-Sheva 84105, Israel

ABSTRACT. Analytical and numerical solutions are presented, describing dynamics and fracture of composite metal-fabric armour penetrated by small projectiles (bullets and fragments). Comparison of one-dimensional and spatial axisymmetric models for nonlinear dynamics of a ply is conducted and their capabilities for practical calculations are discussed. A computer model for spatial dynamics of composite armour is described as a basis for a calculation tool specially designed to simulate penetration processes in actual metal/fabrics shields. Residual parameters of the projectile after perforation of the primary metal armour are obtained on the basis of experimental data processing and energy considerations. Comparison of calculation results with test data for actual steel-kevlar shields produced by the Israeli Military Industries Co. shows sufficient applicability of the tool.

INTRODUCTION

There are various types of light metal/fabric composite armours consisting of a hard metal or ceramic plate as primary armour and a layer of flexible, extensible fabric plies as a secondary armour. The latter is represented often by kevlar and dyneema. Fiberglass and other composites are used as well. Such an armour is intended to protect light combat or cash-carrying vehicles, security doors, cabins and control rooms in boats and small ships, etc. – against light arms [1].

The role of primary armour is not only to decrease the impact energy, but notably to maximize the area of impact onto the secondary armour. This is achieved by mushrooming the projectile head during penetration of primary armour. Forming a plug serves this purpose as well. The secondary armour is intended to stop the deformed projectile due to the ability of the fabric to consume the remaining kinetic energy of the bullet.

There are many works devoted to description of impact and penetration regimes in ductile, brittle and combined targets. In parallel with purely empirical approaches, which usually have found applications, there are some theoretical approaches, analytical and numerical (see, for example, [2 – 6]). A variety of computer codes are known (see, for example, [5 Chapter 9, 6, 7]).

Computational models for determining the resistance of textile structures to ballistic impact are presented in [8 – 10]. A set of semi-empirical and purely analytical models for determining resistance of a single thread and a thread package has been proposed in [11 – 16]. An analytical solution for the dynamics of a single thread under a concentrated impact [17] has recently been used in [14, 15] to design a simple one-dimensional perforation model.

At least two variables must be determined correctly by a mathematical model: maximal stresses (or another critical variable) and the resistance to the projectile motion. In this sense a weakness of the one-dimensional model is an essential misrepresentation of these values as functions of time. In fact, as can be seen below, they turn out to be invariable under constant projectile velocity and decrease fast with the decrease of the velocity. In contrast, in axisymmetric formulation, which seems to be much closer to reality, these values increase in time under constant velocity. This can compensate for decrease in the resistance force (due to the decrease in velocity) in a considerable part of the projectile path.

In the present paper, comparative results of calculations are presented for one-dimensional and axisymmetric formulations. Furthermore, a three-dimensional axisymmetric model of multi-ply fabric is considered. The investigation has focused on designing a fast computation tool oriented to PC. The tool provides a quantitative basis for computer solution of optimization problems. Calibration of the tool has been done on the basis of a comparison of calculation results and experiments conducted by the Rocket Systems Division (RSD) at the Israeli Military Industries Co.

1. ONE-DIMENSIONAL MODEL FOR FABRIC ARMOUR

A linear or hardening-type nonlinear elastic thread is considered. Initially the thread is at rest, unstressed and placed along a horizontal line. Suddenly a point of it is forced to move in the vertical direction with constant velocity v . Under these conditions, two types of waves propagate along the thread to the right and to the left. One is a longitudinal wave propagating with velocity c , and the other is a transversal wave. The latter has the shape of a growing wedge and D denotes its velocity. It is assumed that $D < c$. This nonlinear problem has a simple analytical solution [17]. It can be described as follows.

The considered domain consists of three regions. In the first, $|s| > ct$ (s is the Lagrange coordinate along the thread and t is time), the thread is horizontal and at rest. In the second, $Dt < |s| < ct$, the thread is also horizontal, but there exists constant tensile stress, $\sigma > 0$. Also, in this region, the particle velocity, v_g , is directed toward the central point. In the third region, $|s| < Dt$, the particle velocity is vertical. Tensile stress in the third regions is the same as in the second.

The governing equation for the extensible, flexible thread dynamics is:

$$\left[\sigma(s, t) \frac{\mathbf{R}'}{|\mathbf{R}'|} \right]' = \rho \ddot{\mathbf{R}}(s, t) \quad (1)$$

Here, ρ is initial density, σ is nonnegative tensile stress and \mathbf{R} is the position vector. The primes and dots above denote derivatives with respect to the coordinate along the thread and time respectively. The Lagrange variables are used, then the modulus, $|\mathbf{R}'| = \lambda$, is stretch; $\varepsilon = \lambda - 1$ is strain. In the following, we introduce the modulus E [in general, the secant modulus is meant: $E = E(\varepsilon)$] by the relation $\sigma = E\varepsilon$. In these terms, the wave velocities are

$$c = \sqrt{\frac{E}{\rho}}, \quad D = c \left[\sqrt{\varepsilon(1+\varepsilon)} - \varepsilon \right] \quad (2)$$

$$\sigma = \rho c^2 \varepsilon, \quad Q = 2A\sigma \sin\alpha = 2A\rho c v \left(\left[\sqrt{\varepsilon(1+\varepsilon)} - \varepsilon \right]^2 + \frac{v^2}{c^2} \right)^{-1/2}, \quad (3)$$

Tensile stress, σ , and the external vertical force, Q , are where A is the total cross-section area of the threads which resist to the impact. In its turn, strain satisfies the equation

In the important case of small strain, that is, for a small v/c ratio, the asymptotic relations are valid as follows:

$$2\varepsilon\sqrt{\varepsilon(1+\varepsilon)} - \varepsilon^2 = \frac{v^2}{c^2} \quad (4)$$

Although expressions (3) and (5) are valid only for $v = \text{const}$, they show that the resistance force decreases fast as the velocity decreases. This is a consequence of the one-dimensional

$$\varepsilon \sim 2^{-2/3} \left(\frac{v}{c}\right)^{4/3}, \quad D \sim 2^{-1/3} c \left(\frac{v}{c}\right)^{2/3}, \quad \sigma \sim 2^{-2/3} \rho c^2 \left(\frac{v}{c}\right)^{4/3}, \quad Q \sim 2^{2/3} A \rho c^2 \left(\frac{v}{c}\right)^{5/3} \quad (5)$$

formulation of the problem.

Let σ^* be the limiting tensile stress and σ^{**} the limiting averaged compressive stress. The strength conditions are [we use the asymptotic relations (5)]

$$Q = 2^{2/3} A E^{1/6} \rho^{5/6} v^{5/3} < B \sigma^{**}, \quad \sigma = 2^{-2/3} E^{1/3} \rho^{2/6} v^{4/3} < \sigma^*, \quad (6)$$

where B is the impact area.

Both σ^* and σ^{**} can be critical for armour design. In particular, the resistance, Q , is required to be high enough to satisfy the condition concerning the allowed transversal displacement of the rear. This displacement must be less than the free space behind the armour. Let us then assume that $Q = B \sigma^{**}$ at the beginning of the impact, $t = 0$. In this case, using the expression for Q in (6) for a variable velocity and assuming that all the fibers work simultaneously, one arrives at the equation

$$m \frac{dv}{dt} = B \sigma^{**} \left(\frac{v}{v_0}\right)^{5/3}, \quad (7)$$

where m and v_0 are the projectile mass and its initial velocity (with account taken of the reactive impulse of the plug). The displacement following from this equation is

$$w(t) = \frac{3mv_0^2}{B\sigma^{**}} \left[1 - \left(1 + \frac{B\sigma^{**}t}{v_0} \right)^{-1/2} \right], \quad w_{\max} = w(\infty) = \frac{3mv_0^2}{B\sigma^{**}}. \quad (8)$$

Note that this maximal displacement is six times larger than the theoretical minimum corresponding to the invariable resistance force $B\sigma^{**}$. In fact, the displacement is overestimated, because the one-dimensional problem is considered. As shown below, the resistance force, decreasing due to the bullet deceleration, increases in time due to the axisymmetric geometry of the structure. This leads to an essential decrease in the displacement. Nevertheless, the resistance force tends to zero with the velocity.

2. AXISYMMETRIC FORMULATION AND COMPARATIVE RESULTS

In the axisymmetric formulation, a flexible membrane is considered which does not resist compressive stresses. This means that tangential stresses are at zero, and only radial stresses are still nonzero. In this case, equation (1) takes the form

$$\left[r\sigma(s,t) \frac{\mathbf{R}'}{|\mathbf{R}'|} \right]' = r\ddot{\mathbf{R}}(s,t). \quad (9)$$

It can be rewritten in projections on vertical and radial axis

$$r\rho\ddot{u} = (r\sigma_r)', \quad r\rho\ddot{w} = (r\sigma_z)', \quad (10)$$

where primes denote derivatives with respect to r and

$$\sigma_r = \sigma \cdot (1 + u')/\lambda, \quad \sigma_z = \sigma \cdot w'/\lambda, \quad \sigma = E\varepsilon, \quad \varepsilon = \lambda - 1, \quad \lambda = \sqrt{(1 + u')^2 + (w')^2}. \quad (11)$$

The equation for the bullet motion is

$$m\ddot{W} = -Q, \quad (12)$$

where $Q = A\sigma_z$, $A=2\pi r_0 H$, r_0 is the bullet radius, H is the fabrics package thickness. The bullet-fabrics contact condition is

$$w(r_0, t) = W(t). \quad (13)$$

Equations (11) - (13) together with zero initial conditions [except $\dot{W}(0) = v_0$] were numerically calculated using the explicit finite difference scheme. Grid parameters were chosen from the condition of mesh dispersion minimization (see [18, 19]). This approach enables discontinuities and high gradients of the solution to be calculated with good accuracy. Concurrent with this, the one-dimensional problem was simulated.

Some comparative results are presented in Figure 1. The calculations were performed for a constant velocity at $r = r_0$. In Figures 2, 3 and 4 the results correspond to bullet impact onto the fabrics. In the calculated examples the following parameters were used:

bullet: $v_0 = 490$ m/s, $m = 10$ g, $r_0 = 5$ mm;
 fabrics: $H = 10$ mm, $\rho = 2$ g/cm³, $E = 6 \cdot 10^5$ kg/cm² (the linear stress-strain diagram).

These values correspond to an AK-47 bullet and a composite steel-kevlar armour. The initial velocity of the bullet is 730 m/s. After perforation of 2.5 mm thick, 600 BHN primary steel armour, its residual velocity is 600 m/s [see below relation (17)]. With account taken of the plug mass, the velocity becomes 490 m/s.

In contrast to the one-dimensional model where the resistance force Q is constant, in the axisymmetric case it increases with time (see Fig. 1). Calculated reference value $Q_0 = 2.69 \cdot 10^5$ N coincides with that obtained from relations (5). The shapes for one-dimensional and axisymmetric cases do not differ too much.

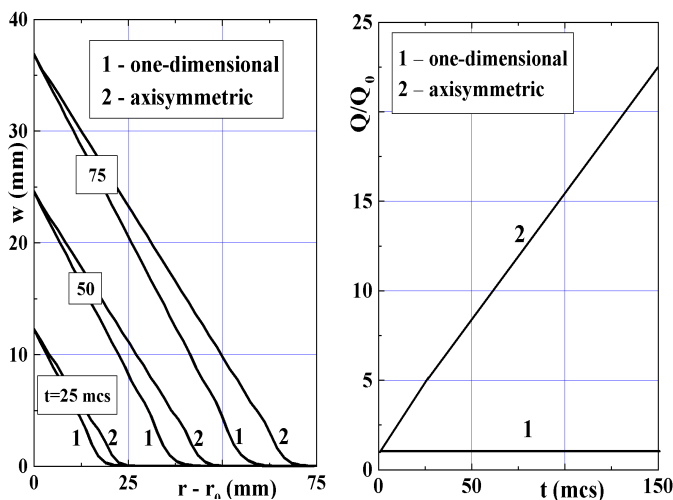


Fig 1 Vertical displacement, w , and resistance force, Q , under constant velocity at $r = r_0$

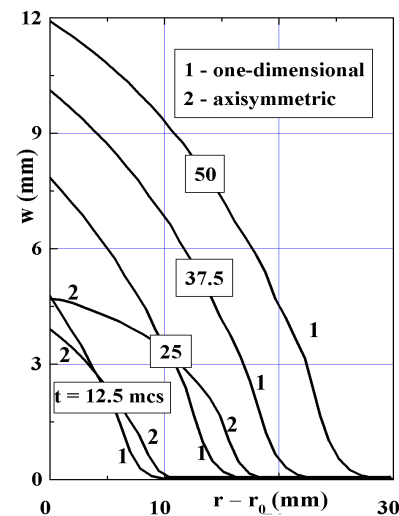


Fig. 2 Vertical displacement, w , under the bullet impact

In the case of the bullet impact, the calculated results – vertical displacements (Fig. 2), the bullet velocity and the resistance force (Fig. 3), and strains (Fig.4) – show significant difference between one-dimensional (a) and axisymmetric (b) models. In the latter case, the bullet is stopped at $t = 25.8$ mcs, and $w_{\max} = 0.49$ cm. In the former, the bullet velocity $v \rightarrow 0$ when $t \rightarrow \infty$, whereas w is integrated and $w_{\max} = 2.67$, which is more than 5 times larger than in the axisymmetric case. This result corresponds to the same initial values Q_0 .

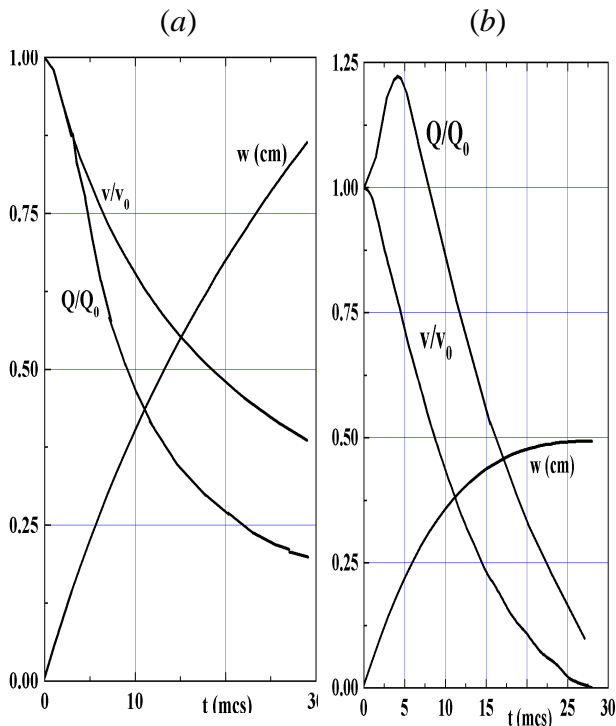


Fig. 3 Bullet displacement, w , velocity, v , and resistance force, Q

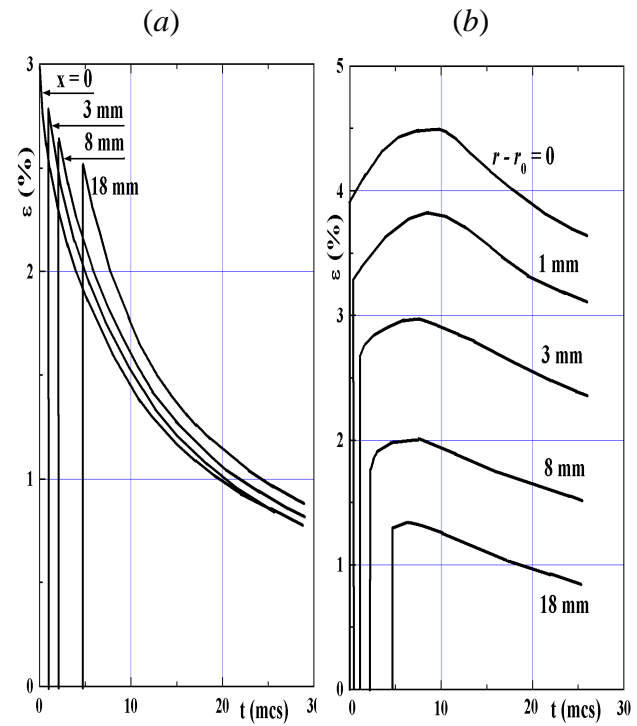


Fig. 4 Strains in cross-sections vs time

3. PENETRATION INTO METAL-FABRIC ARMOURS. CALCULATION TOOL

In this section, the calculation tool simulating penetration/perforation of metal-fabric composite armour by bullets is briefly described. Some examples of calculations are presented and discussed.

The following main parameters are taken into account in the model and calculation tool: bullet weight and geometry, thickness and hardness of the primary armour, weight and number of plies, strength of plies and inter-ply joints (adhesive).

Penetration by the bullet into composite armour is accompanied by a number of phenomena that are simulated by the tool: wave radiation from the bullet-target interaction area, fracture of ply cross-sections, dynamic delamination of plies and inter-ply friction.

We use a likely assumption that the secondary armour does not influence the perforation of the primary armour, and the process under consideration can be divided into two successive independent stages: perforation of the primary armour and penetration into the multi-ply fabrics.

Stage 1. Perforation of thin metal plates

At this stage, the residual bullet velocity after perforation of the primary armour, its exit mass and diameter are estimated from experiments. A simple formula for ballistic limit, v_{bl} , and

residual velocity, v_r , has been proved (see [20,21]) by test data processing on the basis of energy consideration. The tests conducted by the RSD consisted of seven series of a number of shots in each by M-16 and AK-47 bullets onto armour plates of various thicknesses and hardnesses. The test and processing results for M-16 bullets are presented in the Table.

Table

N series	H (m)	BHN	v_0 (m/s)	v_r (m/s)	$v_{r.c.}$ (m/s)	$v_r/v_{r.c.}$
1	0.0082	505	1012	391	383	0.98
2	0.0060	505	998	639	596	0.94
3	0.0048	525	970	624	655	1.05
4	0.0045	700	967	471	503	1.06
5	0.0041	595	971	628	652	1.04
6	0.0032	595	969	742	732	0.99
7	0.0029	595	973	802	762	0.95

Here H and BHN are the plate thickness and the Brinell hardness number, which were invariable for the current test series, while the bullet initial velocity, v_0 , and the residual velocity, v_r , are averaged values over all the shots in the current series. In the data processing the ratio of energies mv_0^2 and $BHN \cdot H \cdot d^2$ (d is the bullet caliber) was used, and the following dependence was proved:

$$v_{r.c.}^2 = v_0^2 - v_{bl}^2, \quad v_{bl}^2 = \alpha \frac{BHd^2}{m}. \quad (14)$$

Formula (14) gives the best approximation to test results if the empirical constant α is:

$$\alpha = 2.47 \cdot 10^7 \text{ for an M-16 bullet } (m = 0.0036 \text{ kg, } d = 0.0056 \text{ m}), \text{ and}$$

$$\alpha = 1.95 \cdot 10^7 \text{ for an AK-47 bullet } (m = 0.0097 \text{ kg, } d = 0.0076 \text{ m}).$$

The last column of the Table shows good accuracy of the theoretical approximation.

Stage 2. Dynamics and penetration of the multi-ply fabric armour

The mechanical model of the multi-ply is as follows. The fabrics is assumed to consist of a number of thin flexible plies connected by an adhesive. The following conditions and assumptions are considered to be valid in the problem formulation:

- the plies are thin membranes as described in Section 2;
- the tension stress-strain diagram $\sigma = \sigma(\varepsilon)$ of a ply is not specified in advance;
- a ply fails under the condition that the tensile strain achieves a given limiting value;
- the adhesive functions in tension-compression and shear as massless viscoelastic springs;
- the normal and shear stress-strain diagrams in the adhesive are not specified in advance;
- normal/shear flaking is realized under the condition that normal/shear stresses reach given limiting values;
- dry friction between plies realizes after shear flaking under compressive stresses.

Multi-ply package dynamics is described by the axisymmetric model (10) modified with account taken of the inter-ply interaction:

$$\rho_f \ddot{u}_j = r^{-1} (h/h_s) (r\sigma_r)_j' + h_s^{-1} (S_r^{j+1,j} - S_r^{j,j-1}),$$

$$\rho_f \ddot{w}_j = r^{-1} (h/h_s) (r\sigma_z)_j' + h_s^{-1} (N_z^{j+1,j} - N_z^{j,j-1}), \quad (15)$$

where $u_j(r,t)$ and $w_j(r,t)$ are radial and vertical displacements of the j -th ply, ρ_f is the fabrics density, h is the thickness of the “dry” ply, $2h_s(r,t)$ is the current distance between the $j+1$ -th and $j-1$ -th plies, S_r and N_z are r - and z -projections of sums of normal and tangential stresses in the adhesive between neighboring plies.

In-ply stress σ_j in a current ply cross-section is calculated from the following expression:

$$\sigma_j = \begin{cases} \sigma_j(\varepsilon), & (\varepsilon \leq \varepsilon_{j,lim}) \cap (\lambda_j \geq 1) \cap [t < t_j(r)] \\ 0, & (\lambda_j < 1) \cup [t \geq t_j(r)] \end{cases}, \quad (16)$$

where λ_j is elongation (11), $\varepsilon_{j,lim}$ is the limiting strain, and $t_j(r)$ is the moment of fracture of a current cross-section of the j -th ply. Stresses in the adhesive correspond to similar relations.

An algorithm and corresponding computer tool for calculation of this problem have been designed on the basis of an explicit finite difference scheme. Stability and mesh dispersion minimization conditions are revealed in the calculation process. The tool, as was examined, has fast CP-time (for examples below, it is in the interval from 3 to 8 min for PC-586). Calibration of the tool has been completed by the comparison of calculation results with tests conducted by the RSD, and, in this way, effective moduli and strengths of plies and adhesive were evaluated.

Some results of numerical simulations of penetration into metal-fabric armour are presented in Figures 5, 6 and 7 (see also examples in [22]). The primary armour is hard steel plate ($H = 4\text{mm}$, $\text{BHN}=500$). The secondary multi-ply armour is manufactured of (a) Kevlar-49 and (b) Kevlar-29 fabrics with the same thickness (with account taken of adhesive): 0.5 mm , the same areal density of the ply: 0.5 kg/m^2 , and tensile strength limit: 2.75 GPa . The Young moduli are different: (a) 131 GPa and (b) 62 GPa .

Configurations are shown in Fig. 5 of two shields of Kevlar-49 and Kevlar-29 and of the same above-mentioned primary armour under penetration by M-16 bullets (the ply position corresponds to boundaries between light and dark strips). The latter stops the bullet at $t = 77\text{ mcs}$ and 31 plies are broken through, while the former is completely perforated at $t = 34\text{ mcs}$ with the bullet residual velocity, $V_{res} = 337\text{ m/s}$.

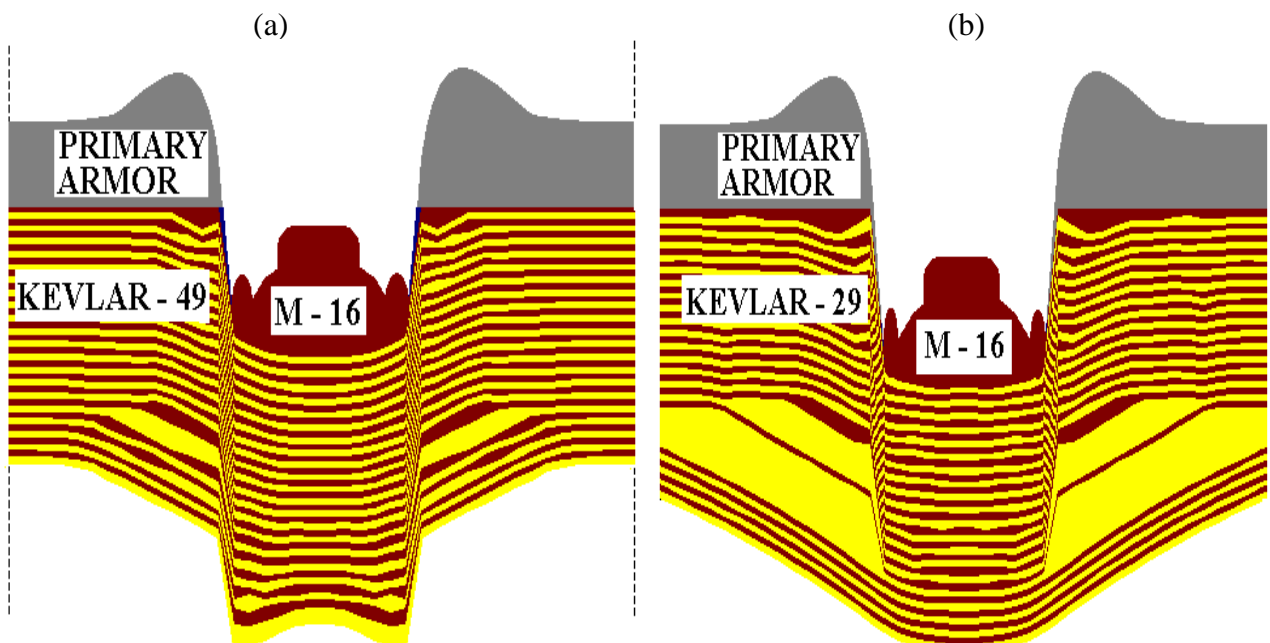


Fig. 5 Steel-kevlar composite targets ($n_p = 40$) vs. an M-16 bullet

The same configurations are shown in Fig. 6 for the case of these armours penetrated by AK-47 bullets. The shield of Kevlar-29 stops the bullet at $t = 82.5$ mcs and 33 plies were broken through, the shield of Kevlar - 49 was perforated at $t = 36.5$ mcs with $V_{res} = 267$ m/s.

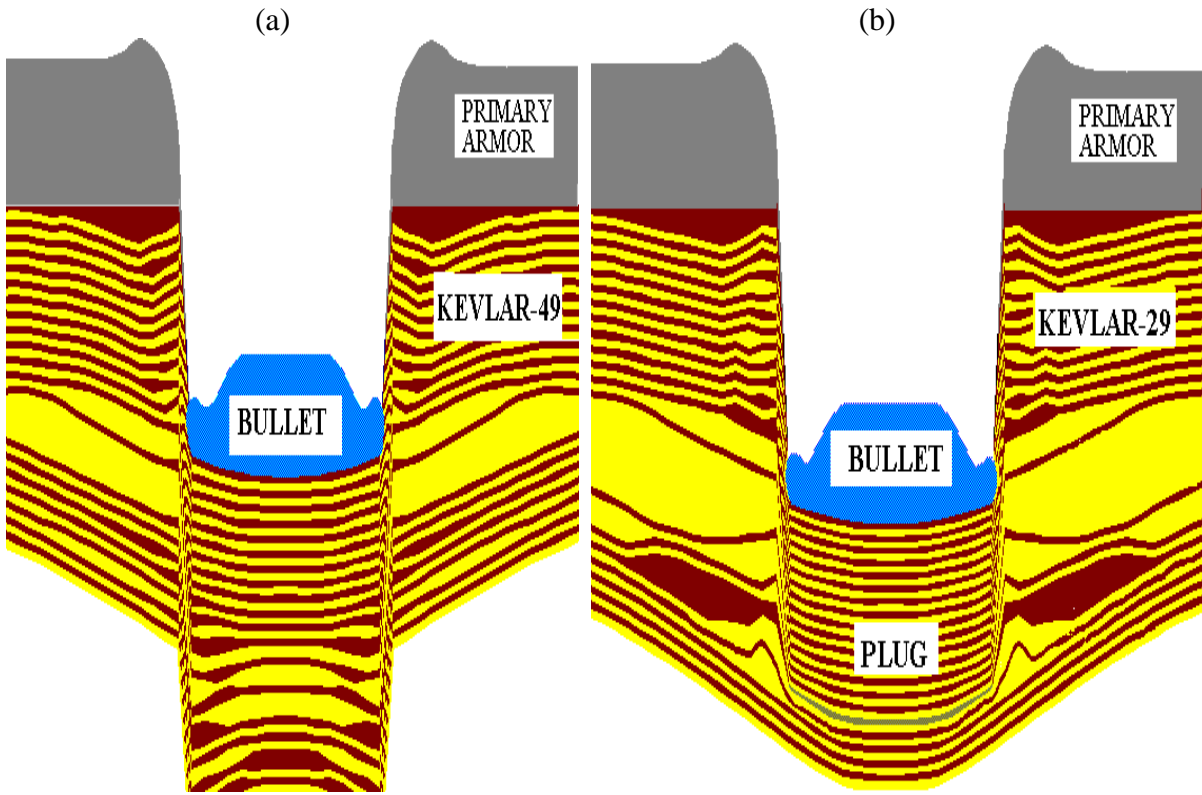


Fig. 6 Steel-kevlar composite targets ($n_p = 40$) vs. an AK-47 bullet

Comparative estimation of the stopping power of these shields vs the ply number can be seen in Fig. 7. Here $V_{res}(0) = v_0$ is the bullet residual velocity after perforation of the primary layer, n_p is the ply number in the package, n_p^* is the number of perforated plies.

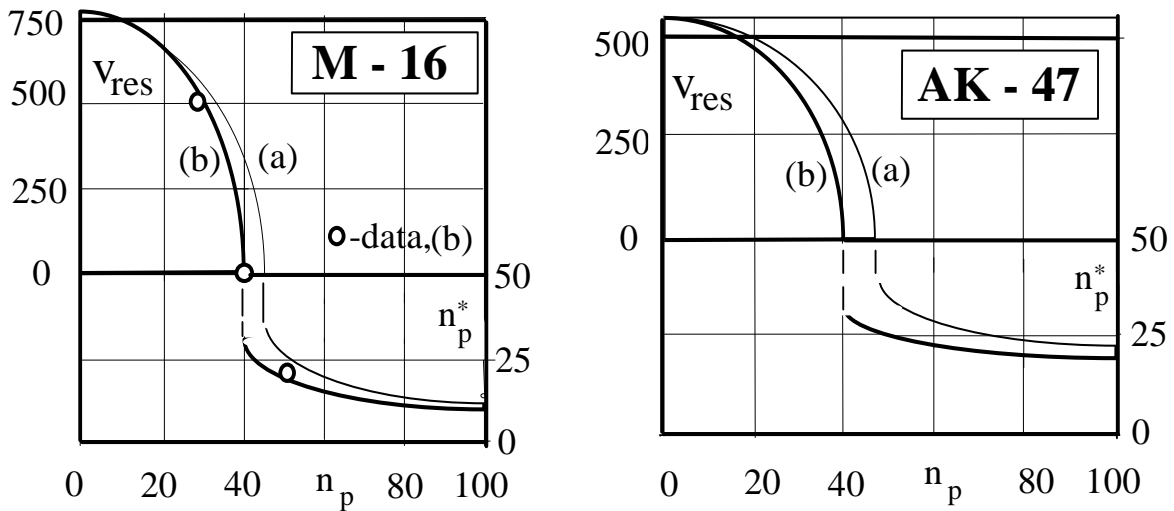


Fig. 7 Shield stopping power vs. the number of plies

It can be seen that the stopping power of the pliable fabrics (b) proves better than that of the rigid one (a). Comparison of calculation results with test data for actual steel-kevlar shields produced by the Israeli Military Industries Co. shows good prediction ability of the tool.

ACKNOWLEDGEMENTS: The authors are indebted to Dr. A. Pridor for his constant attention to the work, to Dr. N. Farber and Dr. Y. Yeshurun for the technical collaboration and the opportunity to use the test data. This research was supported by grants No. 9673-1-96 from the Ministry of Science, Israel, and No. 94-00349 from the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel.

REFERENCES

1. Ballistic Performance of Protective Armor In: *Composite Products for Ballistic Protection*. Information Bulletin of the Rocket Systems Division. TAAS – Israeli Military Industries Co. (1993).
2. Wilkins, M. L. (1978) Mechanics of Penetration and Perforation. *Int. J. Engng. Sci.* **16**, 793-807.
3. Wilkins, M. L. (1980) Computer Simulation of Penetration Phenomenon. In *Ballistic Materials and Penetration Mechanic* (R. C. Laible ed.). Elsevier, 225-252
4. Anderson, C. E. and Bodner, S. R. (1988) Ballistic Impact: The Status of Analytical and Numerical Modeling. *Int. J. Impact Engng.* **7**, 9 - 35.
5. Zukas, J. A. (ed.) (1990) *High Velocity Impact Dynamics*, Wiley, N.-Y.
6. Cortes, R., Navarro, C., Martinez, M.A., Rodriguez, J. and Sanchez-Galvez, V. (1992) Numerical Modelling of Normal Impact on Ceramic Composite Armours. *Int. J. Impact Engng.* **12**, 639 - 651.
7. Johnson, G. R. and Cook, V.H. (1993) Lagrangian EPIC Code Computations for Oblique, Yawed-Rod Impact onto Thin-Plate and Spaced-Plate Targets at Various Velocities. *Int. J. Impact Engng.* **14**, 373-383.
8. Roylance, D., Wilde, A. and Tocci, G. (1973) Ballistic Impact of Textile Structures. *Textile Res. J.* **43**, 1, 41 – 49.
9. Cunniff, P. M. (1992) An Analysis of the System Effects in Woven Fabrics under Ballistic Impact. *Textile Res. J.* **62**, 9, 495 – 509.
10. Ting, J., Roylance, D., Chi, C. H. and Chitragad, B. (1993) Numerical Modelling of Fabric Panel Response to Ballistic Impact. In: Proc 25th Int. SAMPE Techn. Conf. Oct. 26-28, 1993, 384-392.
11. Vinson, J. R. and Zukas, J. A. (1975) On the Ballistic Impact of Textile Body Armour. *J. Appl. Mech.* **6**, 263 – 268.
12. Shim, V. P. W., Tan, V. B. C. and Tay, T. E. (1995) Modelling Deformation and Damage Characteristics of Woven Fabric under Small Projectile Impact. *Int. J. Impact Engng.* **16**, 4, 585-605.
13. Navarro, C., Rodrigues, J. and Cortes, R. (1994) Analytical Modelling of Composite Panels Subjected to Impact Loading. *Colloque C8 J. Phys. IV* **4**, 9, C8-515– C8-520.
14. Cunniff, P. M. (1996) A Semiempirical Model for the Ballistic Impact Performance of Textile-based Personnel Armour.. *Textile Res. J.* **66**, 1, 45 – 49.

15. Chocron-Benloulou, I. S., Rodriguez, J. and Sanchez-Galvez, V. (1997) A Symple Analytical Model to Simulate Textile Fabric Ballistic Impact Behavior. *Textile Res. J.* **67**, 7, 520 - 528.
16. Chocron-Benloulou, I. S., Rodriguez, J. and Sanchez-Galvez, V. (1997) A Symple Analytical Model for Ballistic Impact in Composites. *Colloque C3 J. Phys IV* **7**, C3-821–C3-826.
17. Smith, J. C., MkCrackin, F. L. and Schiefer, F. H. (1958) Stress-Strain Relationships in Threads Impacted Transversely. *Textile Res. J.* **28**, 4, 34-41.
18. Ayzenberg-Stepanenko, M. V. (1981) Numerical Experiment on Fracture Dynamics of a Layered Composite. *Mechanics of Composites*, 1, 53 - 60.
19. Ayzenberg-Stepanenko, M. V. (1992) Non-Stationary Wave Propagation in Composite Structures. *Proc. 24th Israel Conf. Mech. Engng.*, sec. 4.8.2, 1-3.
20. Slepyan, L., Ayzenberg-Stepanenko, M., Pridor, A., Shtemler Yu. and Vainberg, Ya. (1995) Light Armor Penetration by Small Arms. *The Institute for Industrial Mathematics and the Rocket Systems Division of TAAS*. Report, July 1995, Beer-Sheva, Israel.
21. Pridor, A., Slepyan, L., Ayzenberg-Stepanenko, M. and Shtemler Yu. (1998) Engineering Problems of Penetration: Advancements at the Institute for Industrial Mathematics, Beer-Sheva. *Proc. 27th Israel Conf. Mech. Eng.*, 341-343.
22. Slepyan, L. and Ayzenberg-Stepanenko, M. (1997) Superplastic Protective Structures. In *Progress in Industrial Mathematics*. B.G. Teubner, Stuttgart, 252-259.