Automata and Logic on Infinite Objects

Due: 4/11/18

Unless otherwise stated, all claims should be proven.

Question 1. (20 points)
Let $\Sigma = \{a, b\}$ and $\Sigma' = \Sigma \cup \{\#\}$. Let
\[ L_n = \{u_1\#u_2\#u_3\#\ldots \in (\Sigma^n\#)^\omega \mid \exists \infty i \in \mathbb{N} \text{ s.t. } u_i \neq u_{i+1} \}. \]
That is, $L_n$ is subset a language of $\omega$-words over $\Sigma'$ which are partitioned to words over $\Sigma$ of size $n$ separated by the $\#$ sign. A word $u_1\#u_2\#u_3\#\ldots$ is in $L_n$ iff there are infinitely many indices $i$ such that the $i$-th sub-word $u_i$'s different than the following sub-word $u_{i+1}$. Describe an NBW $N$ accepting $L_n$ or if you prefer, two NBWs $N_1$ and $N_2$ of size $O(n)$ such that the intersection of their accepted languages is $L_n$.

Question 2. (20 points)
Prove or refute (provide a counterexample):
1. Every NBW $A$ has an equivalent NBW $A'$ with a single initial state.
2. Every NBW $A$ has an equivalent NBW $A'$ with a single accepting state.

Question 3. (24 points)
Let $G = (\Sigma, Q, Q_0, \delta, \{F_1, F_2, \ldots, F_k\})$ be a generalized Büchi automaton (NGBW) with $n$ states. Construct an NBW with $O(nk)$ states recognizing the same language.

Question 4. (24 points)
Let $\Sigma = \{a, b\}$ and let $L_k = \{w \in \Sigma^\omega \mid \text{both } a \text{ and } b \text{ appear at least } k \text{ times in } w\}$. Let $L_k,\omega = L_k \cap \Sigma^\omega$ and $L_k,\ast = L_k \cap \Sigma^\ast$. An NFW for the language $L_k,\omega$ needs at least $k^2$ states.
1. Show that $L_k,\omega$ can be recognized by an NBW with at most $2k + 5$ states.
2. Show that $L_k,\omega$ can be recognized by an NCW with at most $3k + 5$ states.

Hint: Recall that $\Sigma = \{a, b\}$. What does this entail about finitely/infinitely many occurrences of $a$ or $b$ letters in words from $L_k,\omega$?

Question 5. (24 points)
Let $B$ be an NBW. Let $B'$ be the NFW obtained from $B$ by treating it as an NFW. Prove or give a counterexample:
1. $[B] = \mathcal{R}_{\text{ref}}([B'])$
2. $[B] = \mathcal{R}_{\text{ref}}([B'])$ iff $[B] \in \mathbb{D}_{\text{DBW}}$