OUTLINE

Introduction

History and basic concepts

Fuzzy sets and fuzzy logic

Fuzzy clustering

Fuzzy inference

Fuzzy systems

Application examples

"So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality."

Albert Einstein. Geometry and Experience

Fuzzy Logic

Mathematical formalism for representing imprecise knowledge in a human−like way.

− Knowledge representation
− Mathematically formal
− Deals with imprecision

some history...

Multi−valued logic

Jan Lukasiewicz. Three−valued logic (1920)

Other multi−valued logics:

Kurt Gödel, John von Neumann, Donald Kleene

Proto−fuzzy Sets

Max Black. Vagueness, a logic analysis (1937)

Fuzzy Logic

Lotfi Zadeh. Fuzzy Sets (1965)

Jan Lukasiewicz. Three−valued logic (1920)

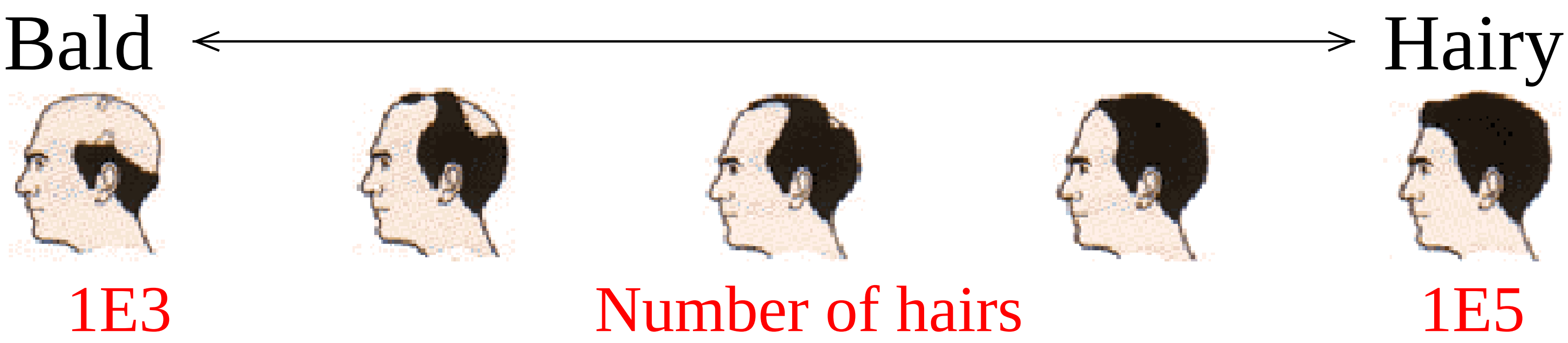
0 = False
1/2 = Possible
1 = True

Sentence: It is possible that tomorrow will rain

Negation: It is possible that tomorrow will not rain

Paradoxes: This sentence is false. True or false?

Max Black. Proto−fuzzy Sets (1937)



Falakros paradox: (belongs to the "sorites paradox" class)

Would you describe a man with one hair on his head as bald? Yes.
Would you describe a man with two hairs on his head as bald? Yes.
You must refrain from describing a man with ten thousand hairs on his head as bald, so where do you draw the line?

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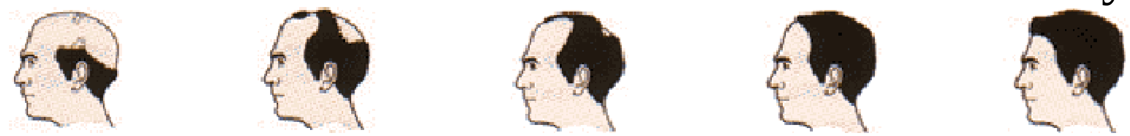
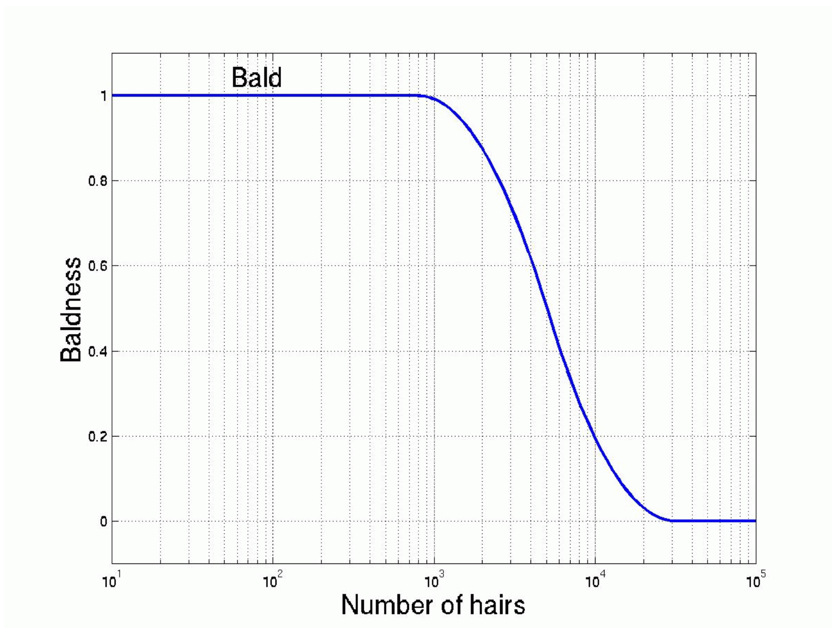
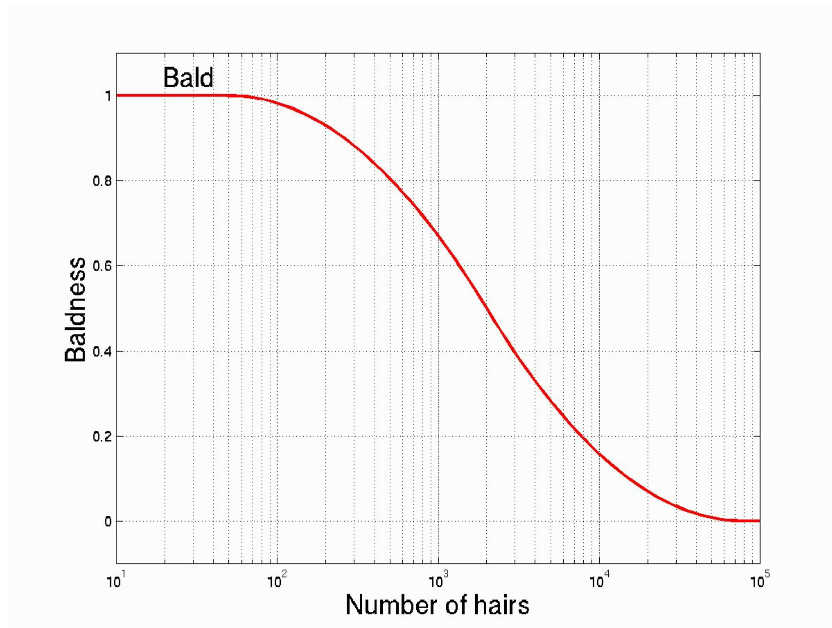
Max Black's sets vs actual fuzzy sets

Bald ⟷ Hairy



1E3 Number of hairs 1E5





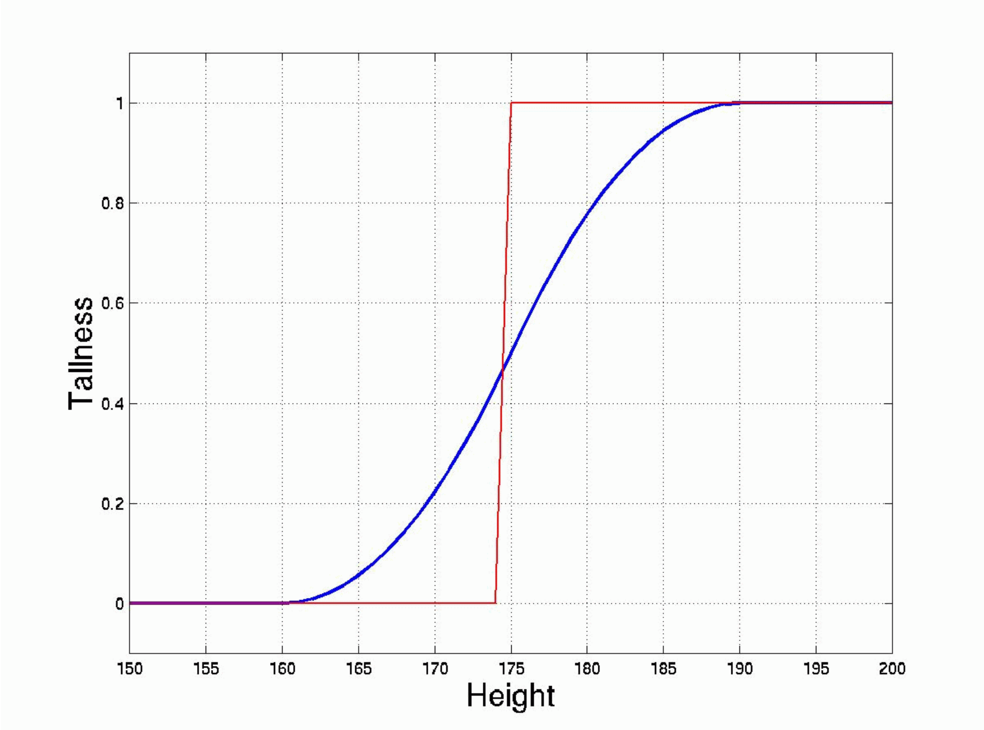
Based on degree of usage

Based on (subjective) degree of truth

Lotfi Zadeh: Father of modern fuzzy logic (1965)

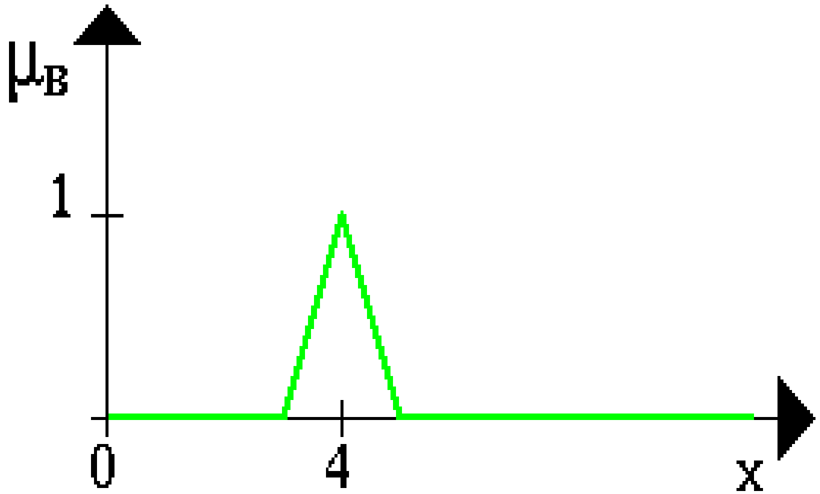
Key concept: Partial membership (μ)

"Tall" membership		
Height	Crisp	Fuzzy
1.60	0	0.00
1.65	0	0.06
1.70	0	0.22
1.75	1	0.50
1.80	1	0.78
1.85	1	0.94
1.90	1	1.00

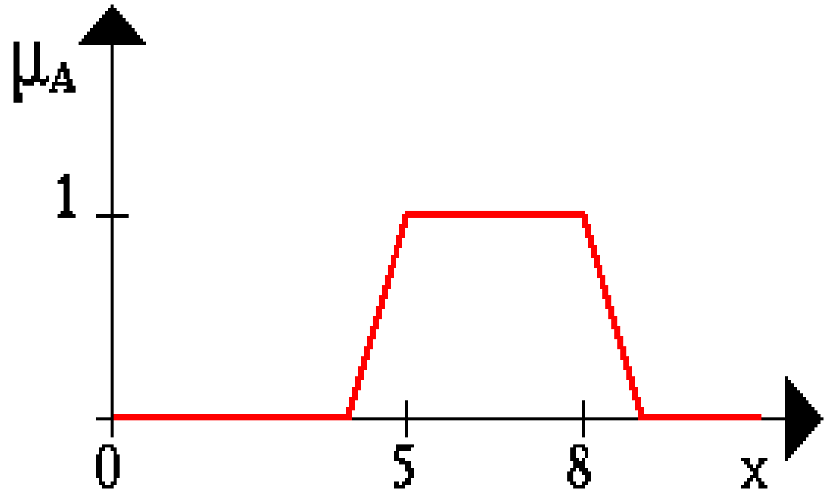


Fuzzy numbers and fuzzy quantities

... about 4 kg of beans ...



... between 5 and 8 weeks ...



As they assign a membership value μ to a given real value x, they are called membership functions

Fuzzy−set operations

Emptiness:

Sets with no members (i.e. $\forall x, \mu(x)=0$)

Complement:

How much do items not belong

Containment:

What groups belong to other groups

Intersection:

How much are items in BOTH sets

Union:

How much are items in EITHER sets

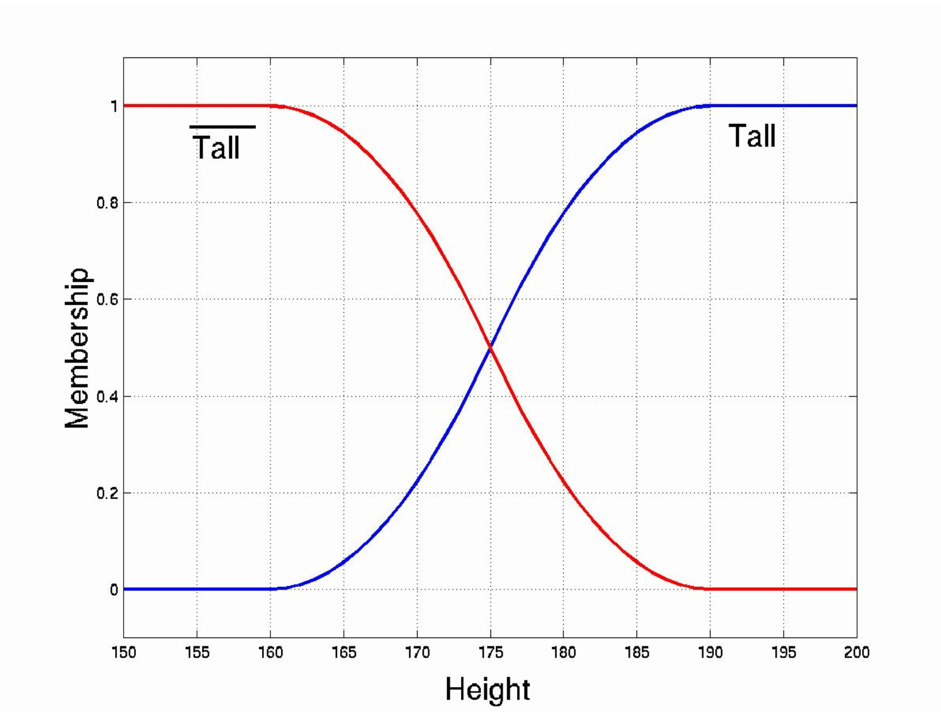
Containment: What groups belong to other groups

i.e. each element on the subset belong to the larger set

$$B \subseteq A \text{ iff } \forall x, \mu_B(x) <= \mu_A(x)$$

Complement: How much do items not belong?

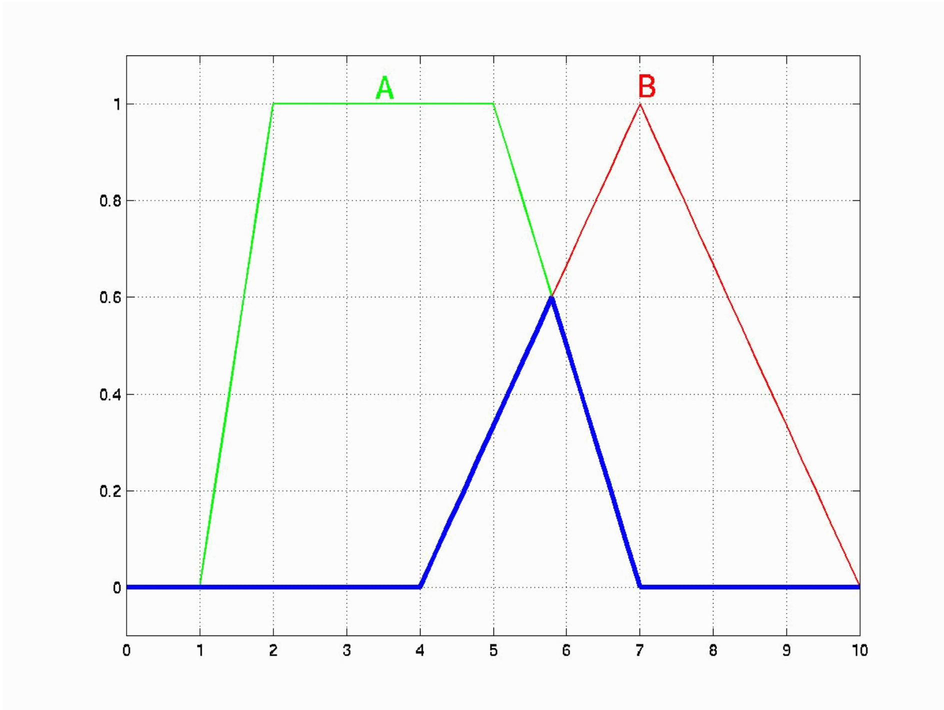
i.e. how far are them from full membership?



$$\mu_{\bar{F}}(x) = 1 - \mu_F(x)$$

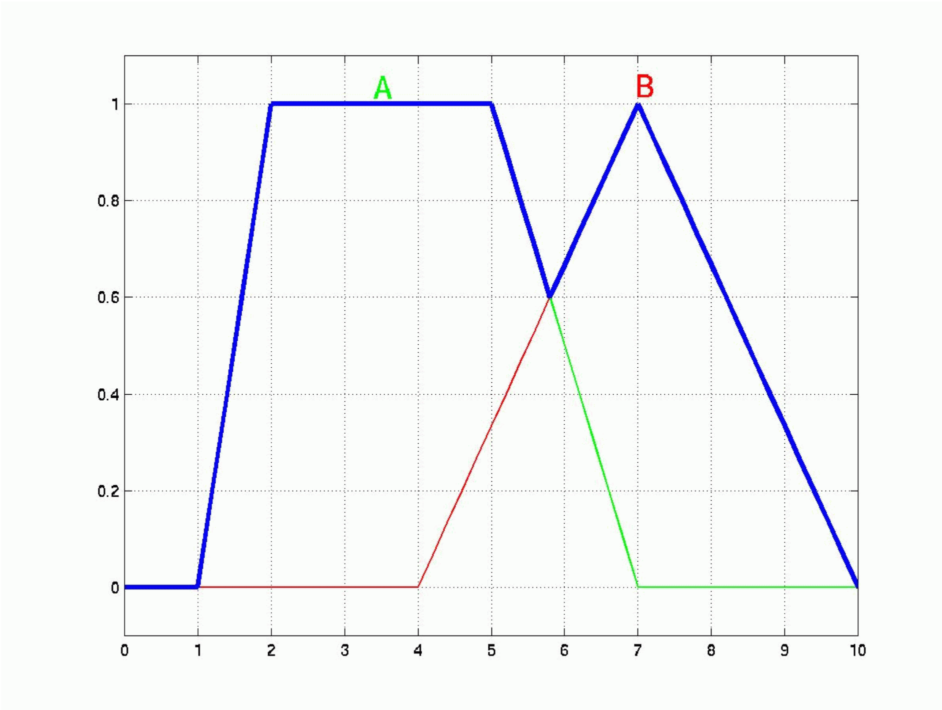
Intersection: How much are items in both sets?

i.e. the degrees of membership both sets share



$$\mu_{A \cap B}(x) = \min\{\mu_B(x), \mu_A(x)\}$$

Union: How much are items in either set?



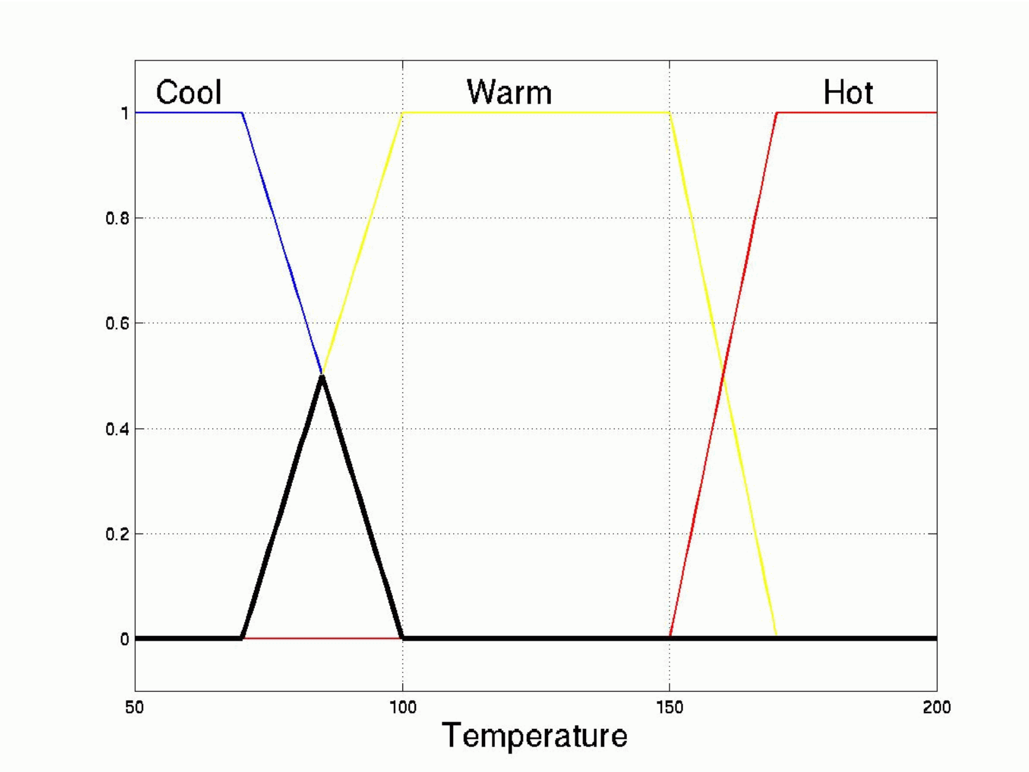
$$\mu_{A \cup B}(x) = \max\{\mu_B(x), \mu_A(x)\}$$

Equivalence between sets and logic

Sets	Logic
Membership John's height belongs to "Tall"	Truth John is tall
Complement	Negation
Intersection	And
Union	Or

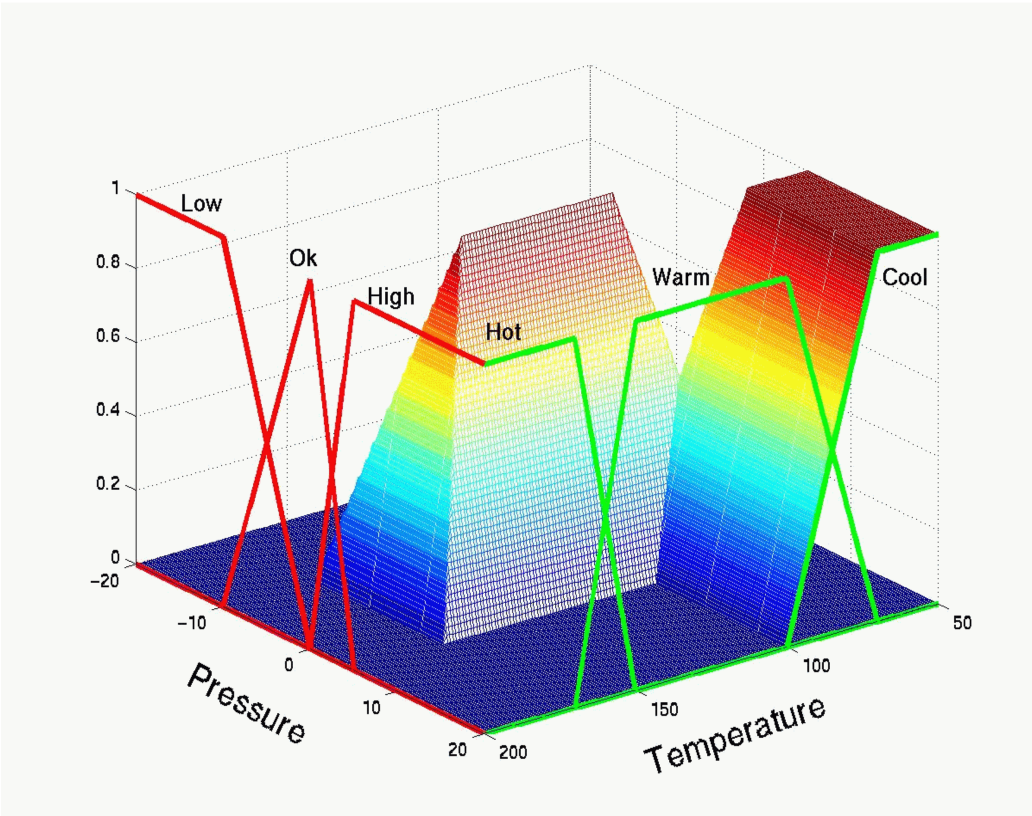
1−D vs 2−D fuzzy logic operations (1)

1−D: Temperature is cool AND warm

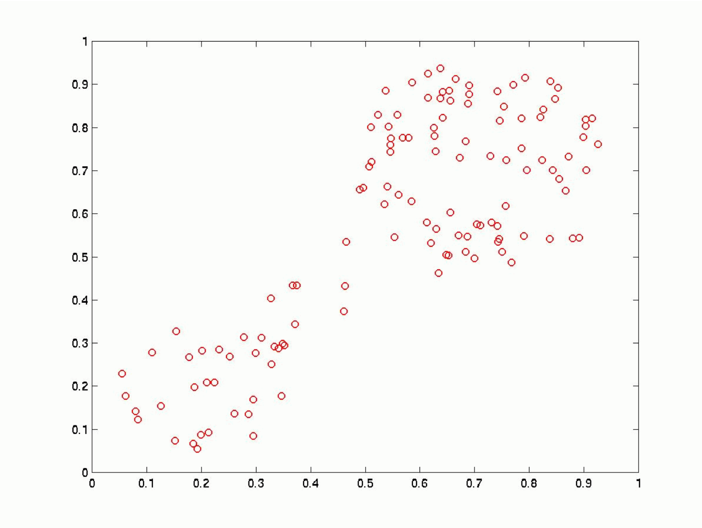


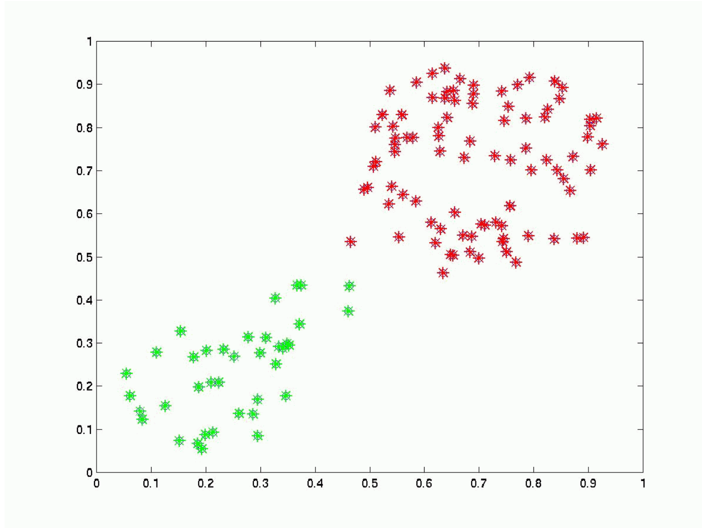
1−D vs 2−D fuzzy logic operations (2)

2−D: Temperature is Cool AND Pressure is High
Temperature is Warm and Pressure is Ok



Fuzzy Clustering

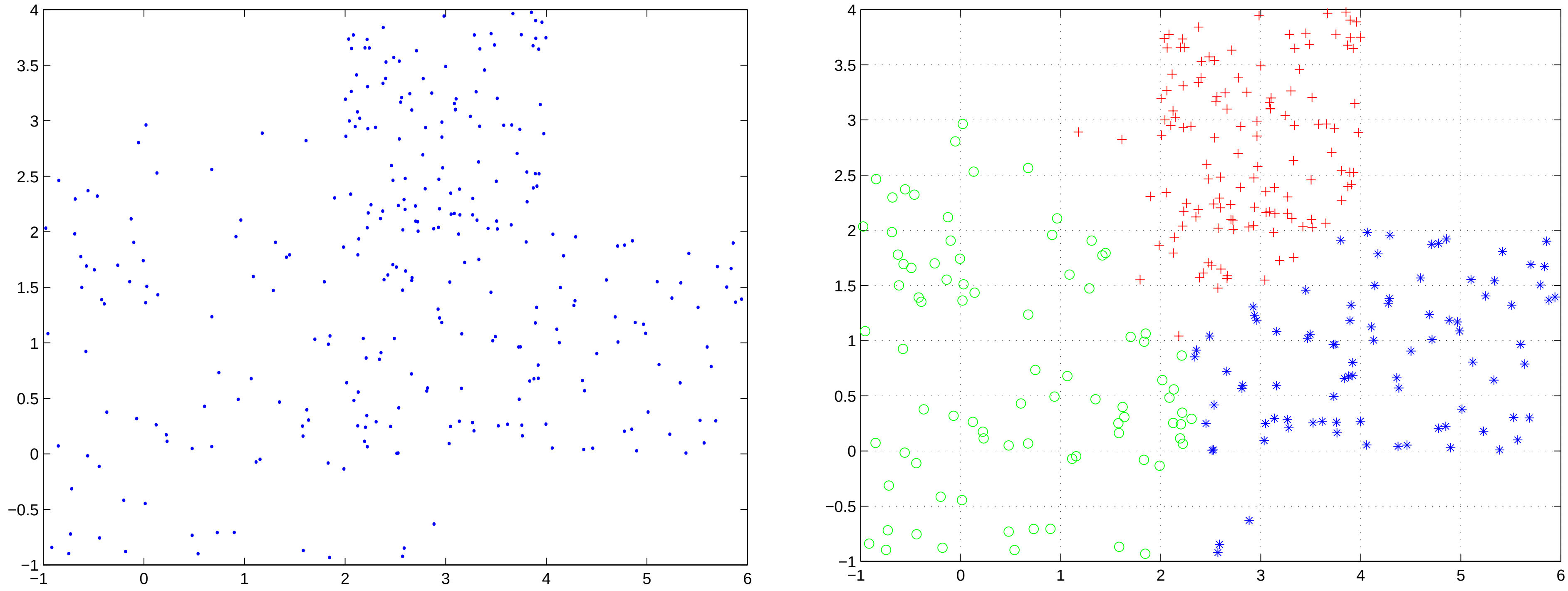




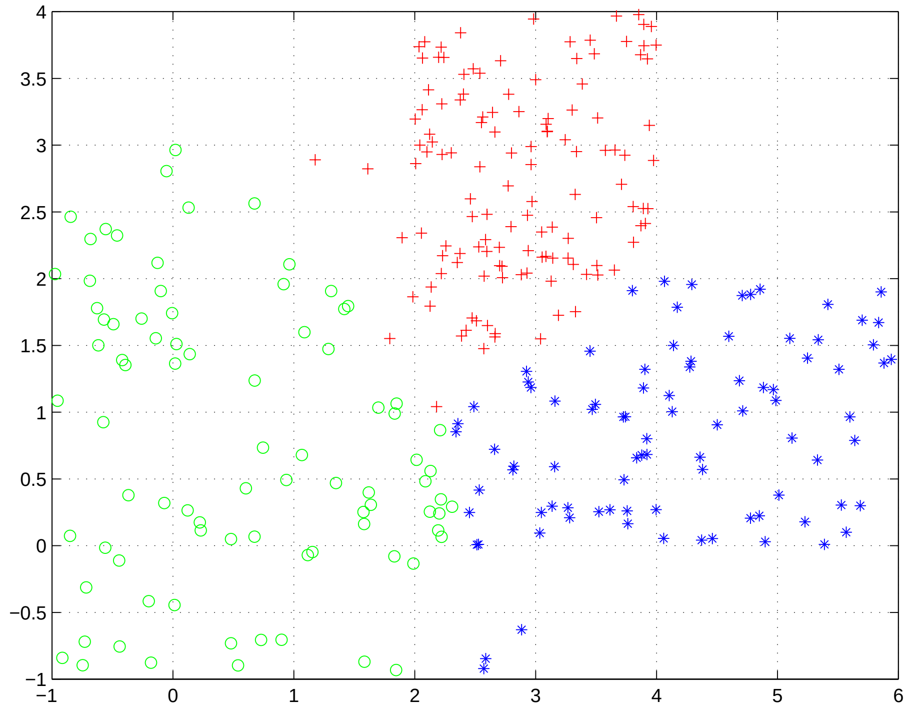
Cluster analysis: To partition a given set of data into clusters having the following properties:

Intra−cluster homogeneity
Data in the same cluster, as similar as possible

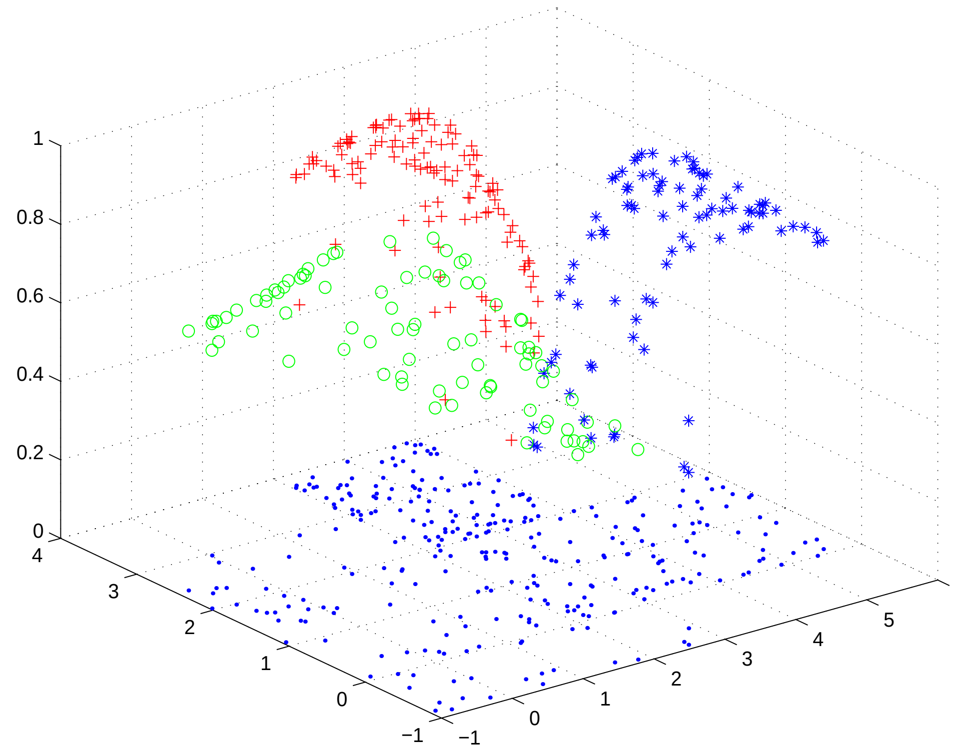
Inter−cluster heterogeneity
Data in different clusters, as different as possible



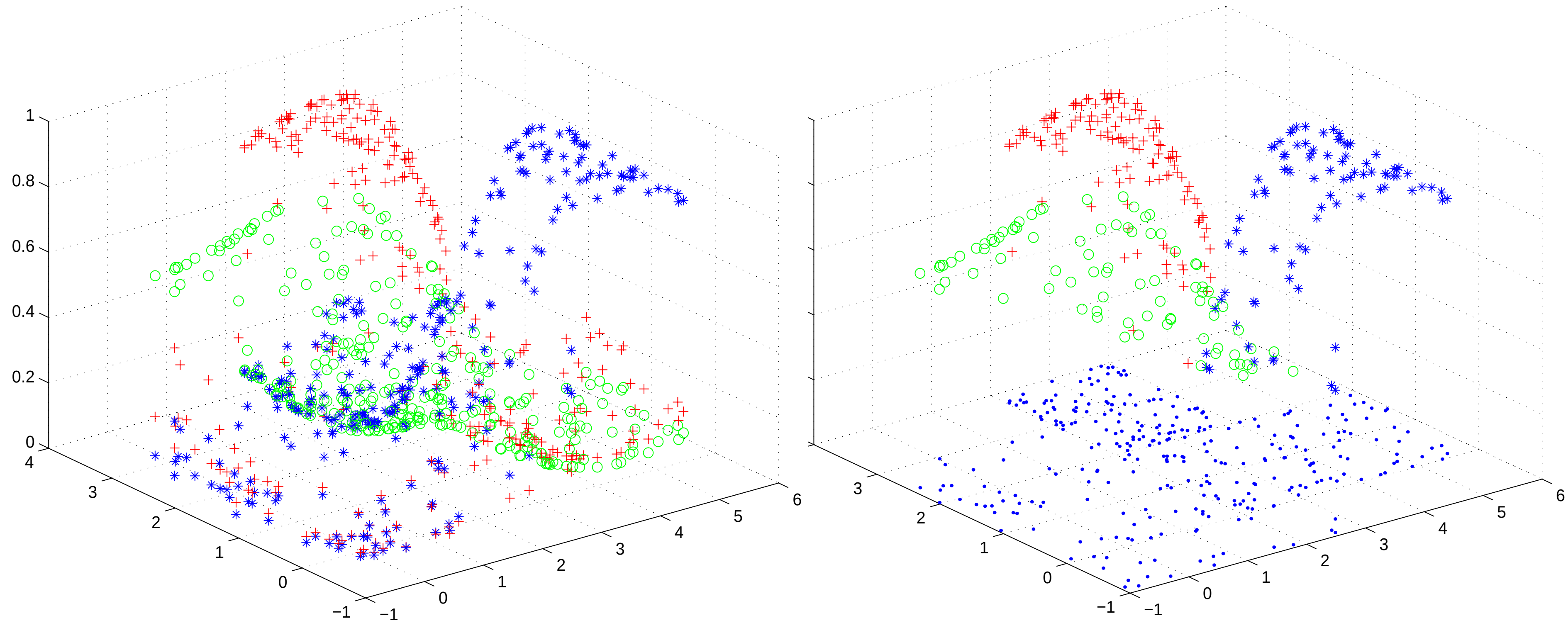
4
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
−1
0
1
2
3
4
5
6



−1
0
1
2
3
4
5
6
−0.5
0.5
1.5
2.5
3.5



0
0.2
0.4
0.6
0.8
1
−1
0
1
2
3
4
5
6



1
0.8
0.6
0.4
0.2
0
4
3
2
1
0
−1
−1
0
1
2
3
4
5
6

How do humans explain their decisions?

* By enumerating some (partially) fullfilled conditions

Why did you choose this car?

− It is big enough for my family, but not too much...
− Its gas consumption is not high, but the car is still fast...
− I prefer clear colors, but not white...
− This mark is well known...

How do humans explain their decisions?

* By proposing some (fuzzy) behavior rules

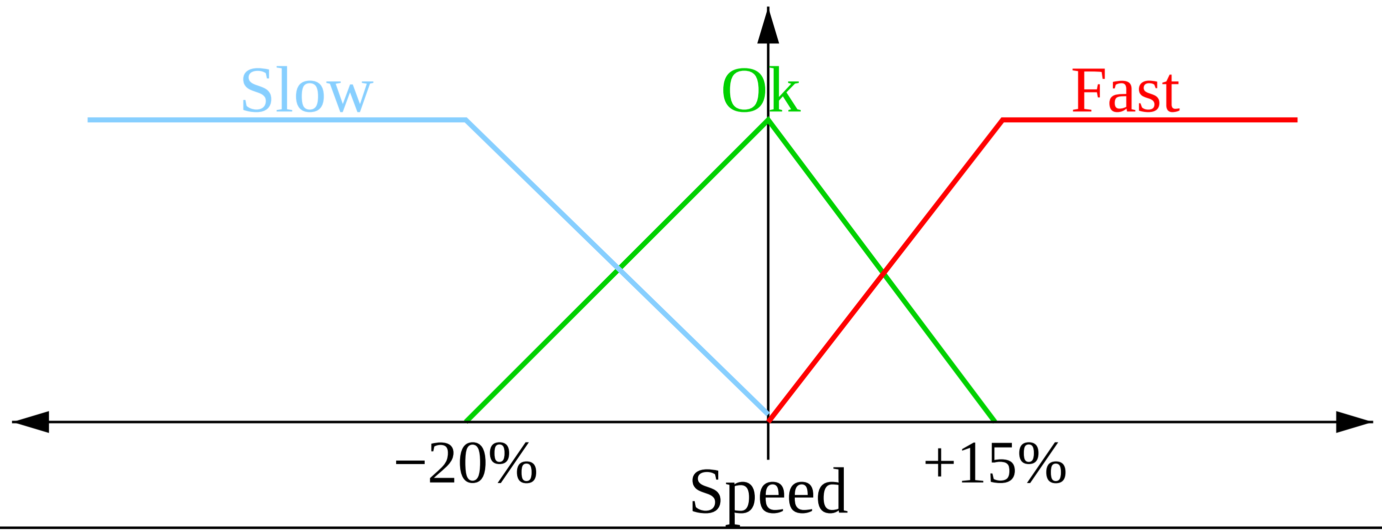
How do you decide to push or to release your car's accelerator?

Release if car is fast(er than desired) and speed is constant or rising,
push if car is slow(er) and speed is constant or slowing down;
else do nothing.

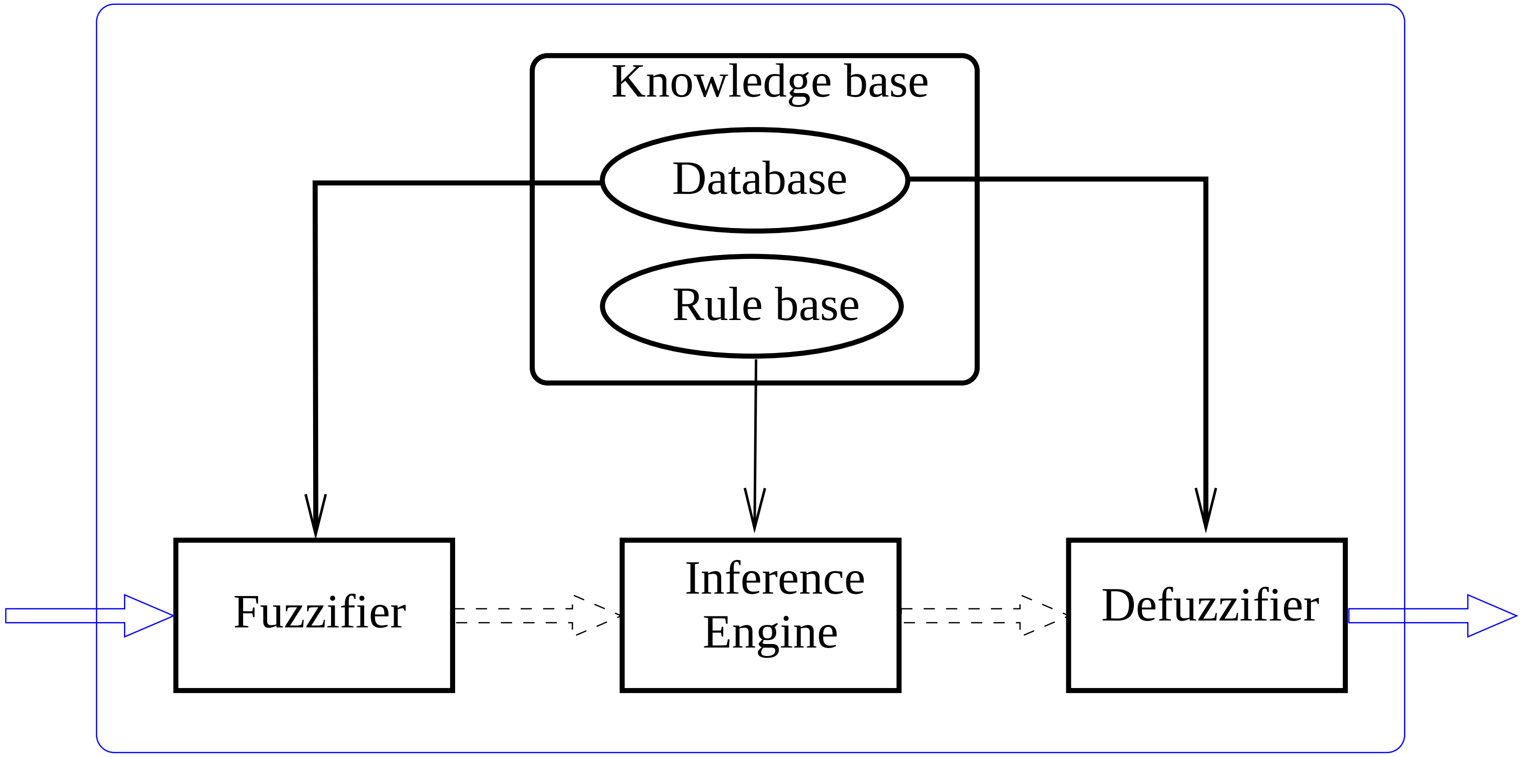
	Slowing	Constant	Rising
Fast	N	R	R
Ok	N	N	N
Slow	P	P	N

Linguistic or "fuzzy" variables

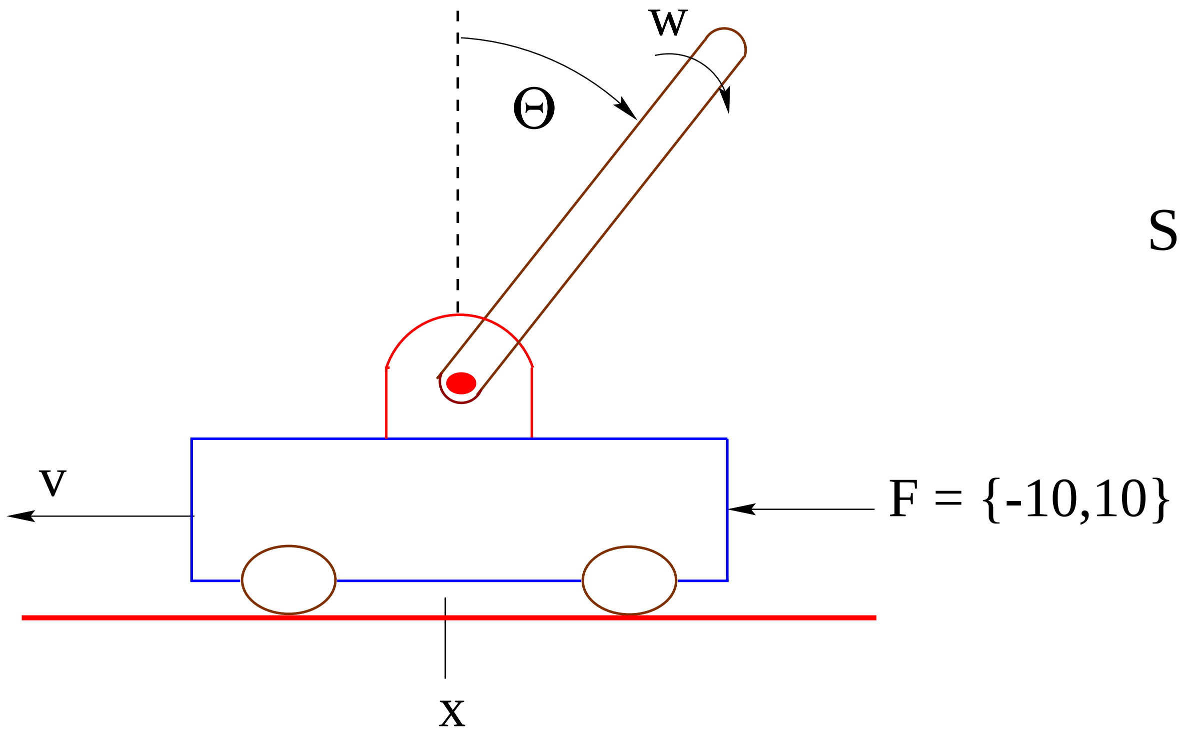
Name:	The "thing" being qualified (e.g. speed)
Labels:	"Linguistic" values or adjectives (e.g. slow, fast)
Membership functions:	assign membership values to real, measured values.



Fuzzy Inference System



Inverted Pendulum Problem



State of the System

$S = \{x, v, \theta, w\}$

Inverted Pendulum Control: Rules

IF angle is zero AND angular velocity is zero THEN speed is zero

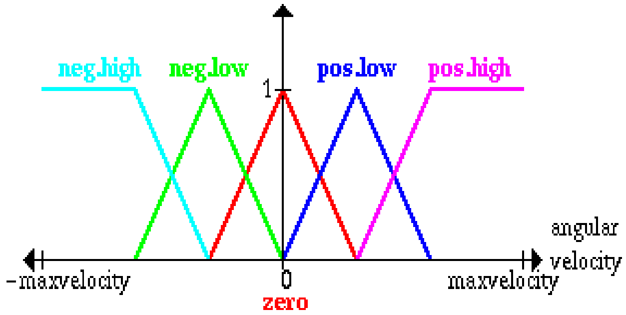
IF angle is zero AND angular velocity is pos_low THEN speed is pos_low

IF angle is neg_low AND angular velocity is zero THEN speed is neg_low

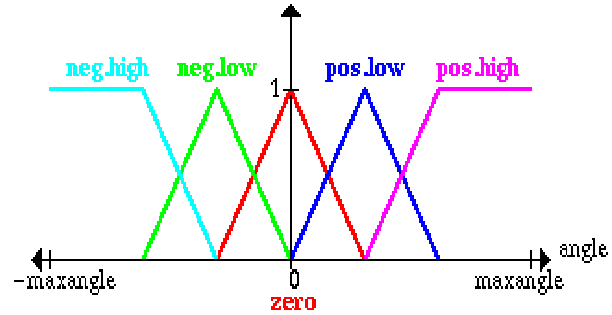
speed		angle				
		NH	NL	Z	PL	PH
ang. velocity	NH			NH		
	NL			NL	Z	
	Z	NH	NL	Z	PL	PH
	PL		Z	PL		
	PH			PH		

Inverted Pendulum Control: Variables

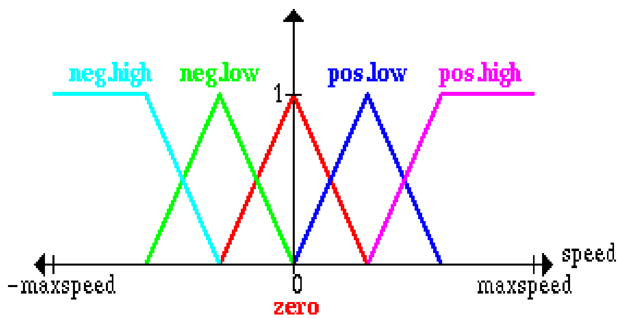
Input1: Angular velocity



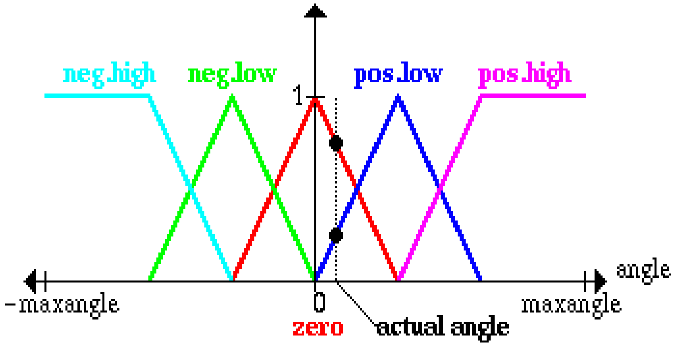
Input2: Angle

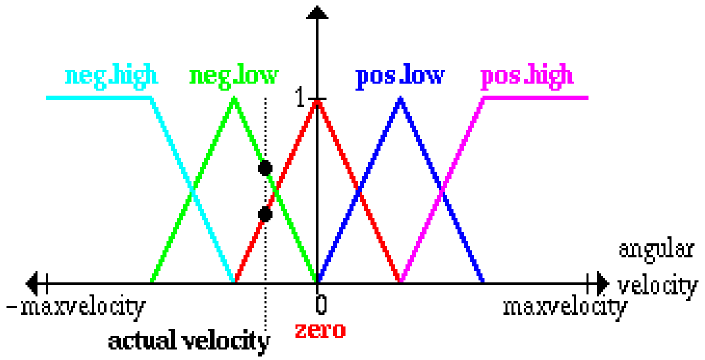


Output: Speed



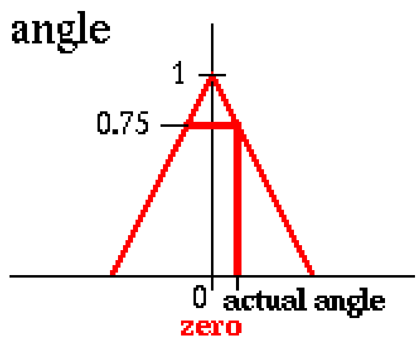
Inverted Pendulum Control: Input measures





Inverted Pendulum Control: Inference 1 (Fuzzyfication)

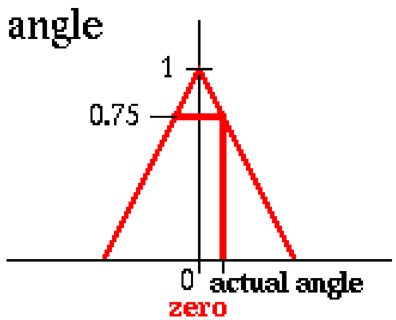
IF angle is zero AND angular velocity is zero THEN speed is zero



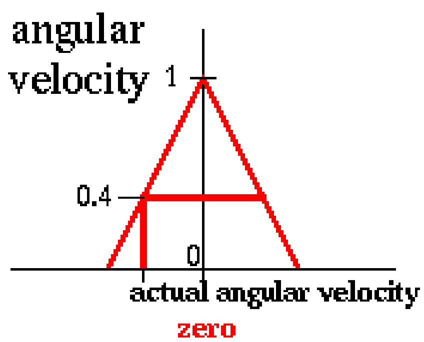
We realize that our actual value belongs to the fuzzy set "zero" to a degree of 0.75.

Inverted Pendulum Control: Inference 2 (Fuzzyfication)

IF angle is zero AND angular velocity is zero THEN speed is zero



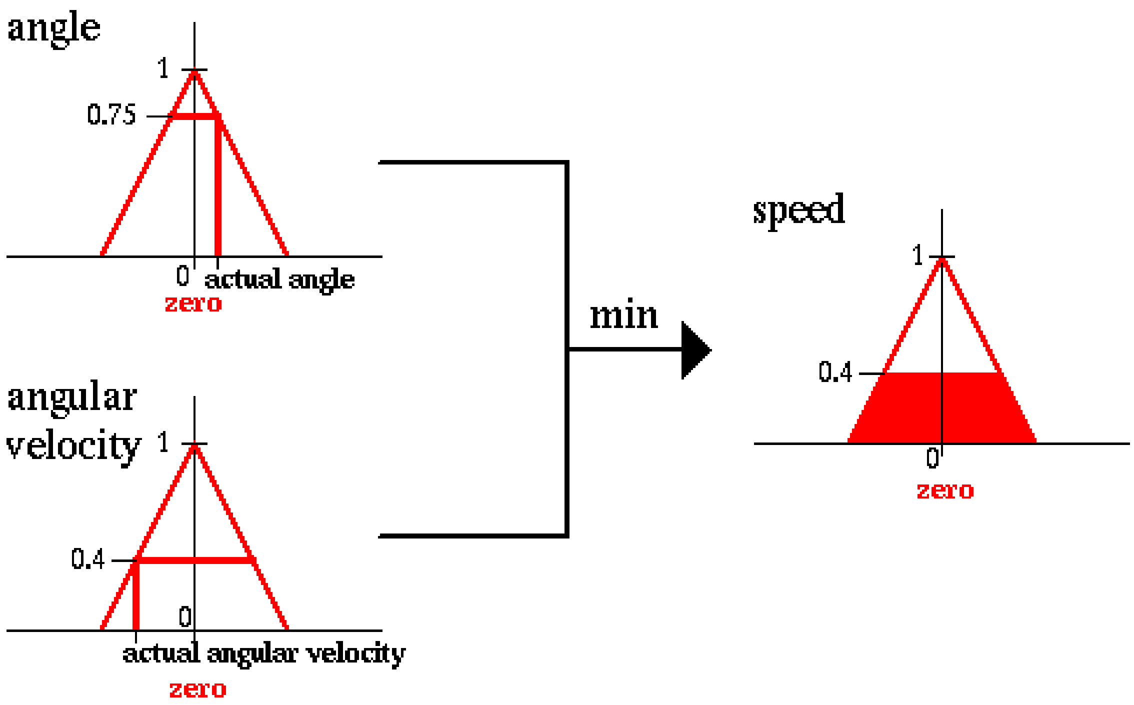
We realize that our actual value belongs to the fuzzy set "zero" to a degree of 0.75.



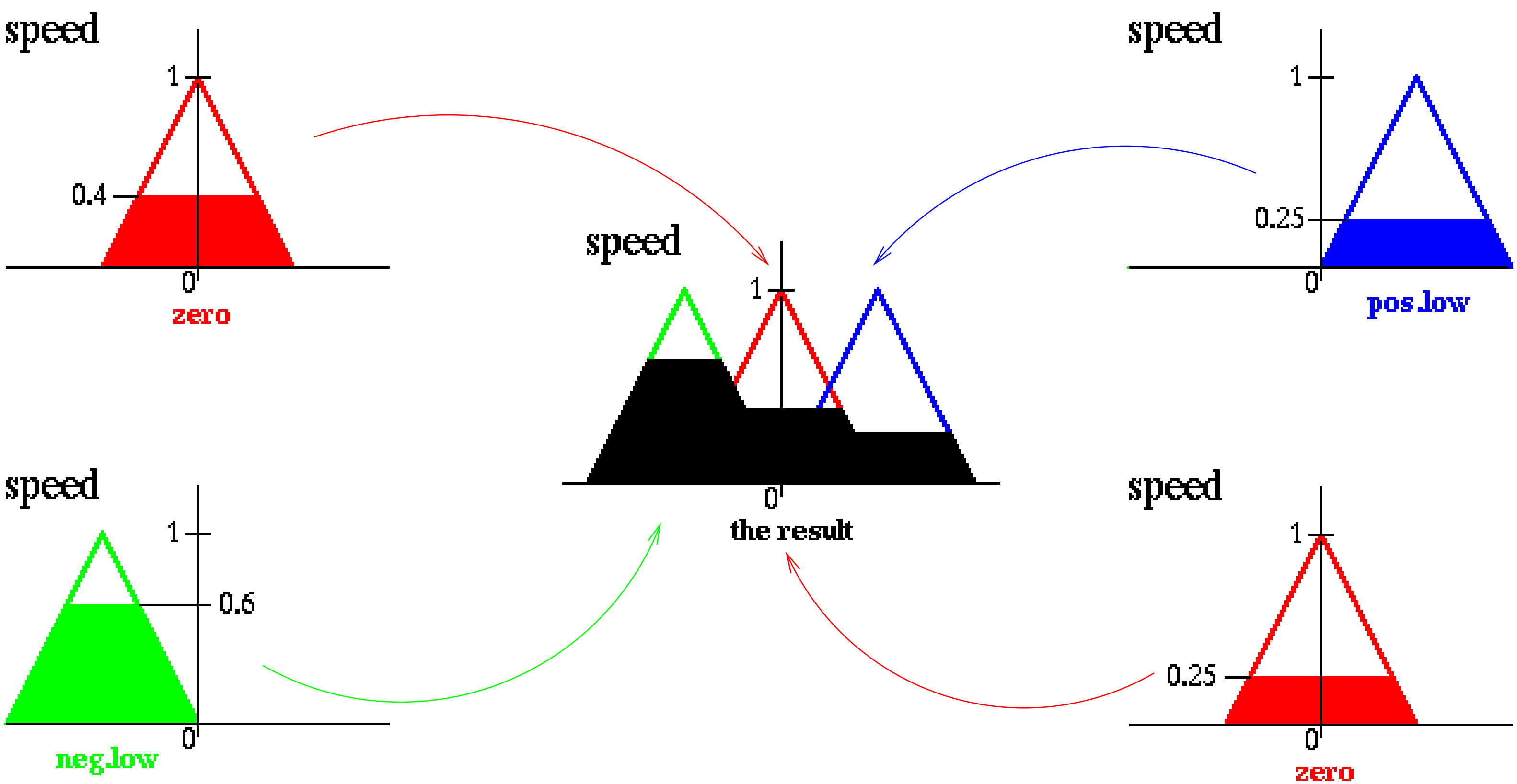
We realize that our actual value belongs to the fuzzy set "zero" to a degree of 0.4.

Inverted Pendulum Control: Inference 3 (Implication)

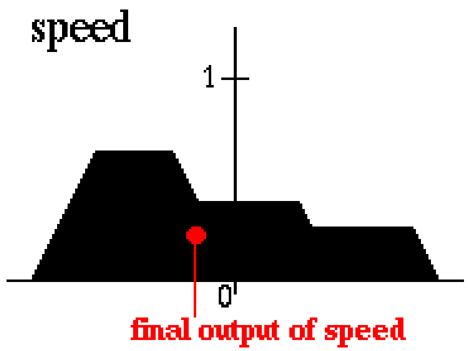
IF angle is zero AND angular velocity is zero THEN speed is zero



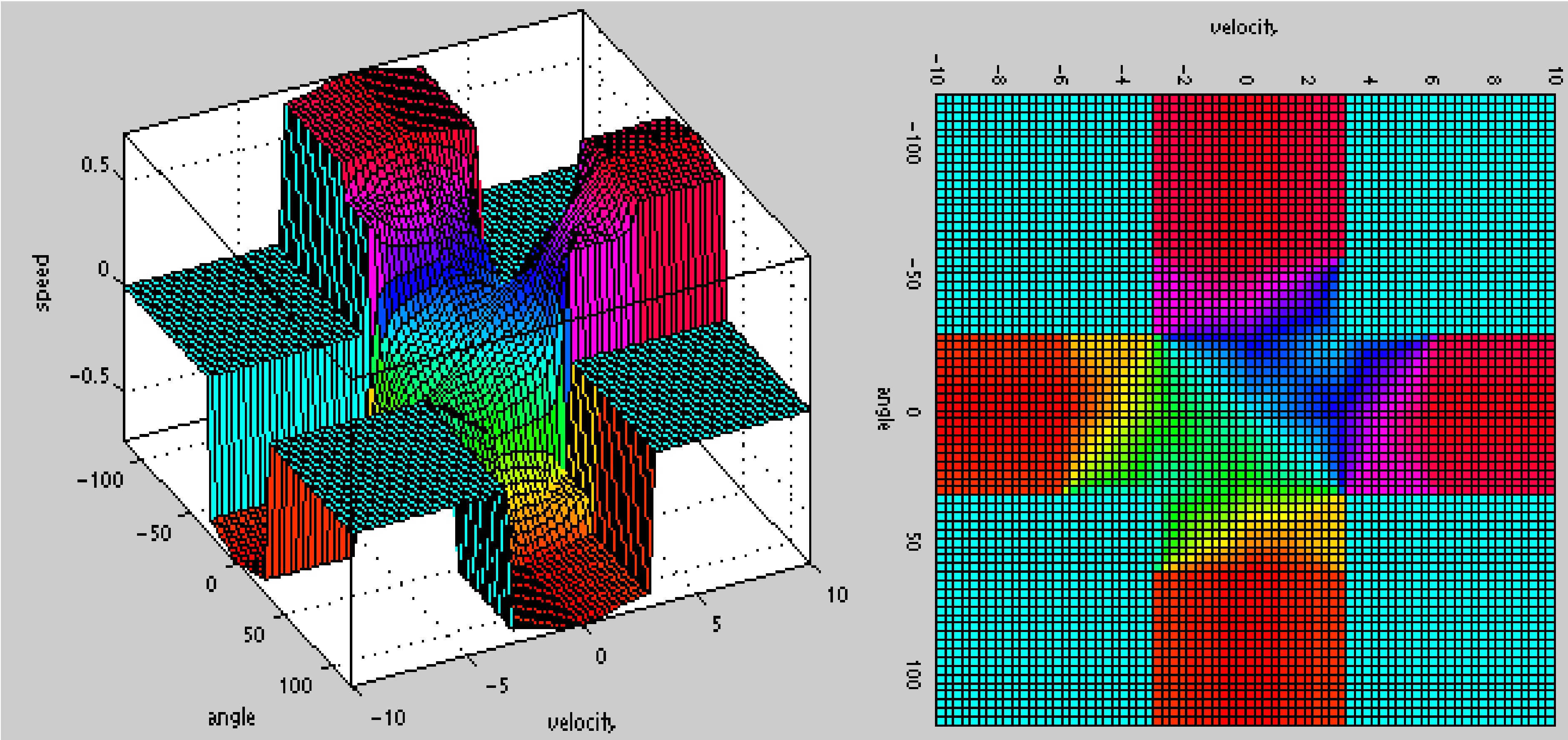
Inverted Pendulum Control: Inference 4 (Aggregation)



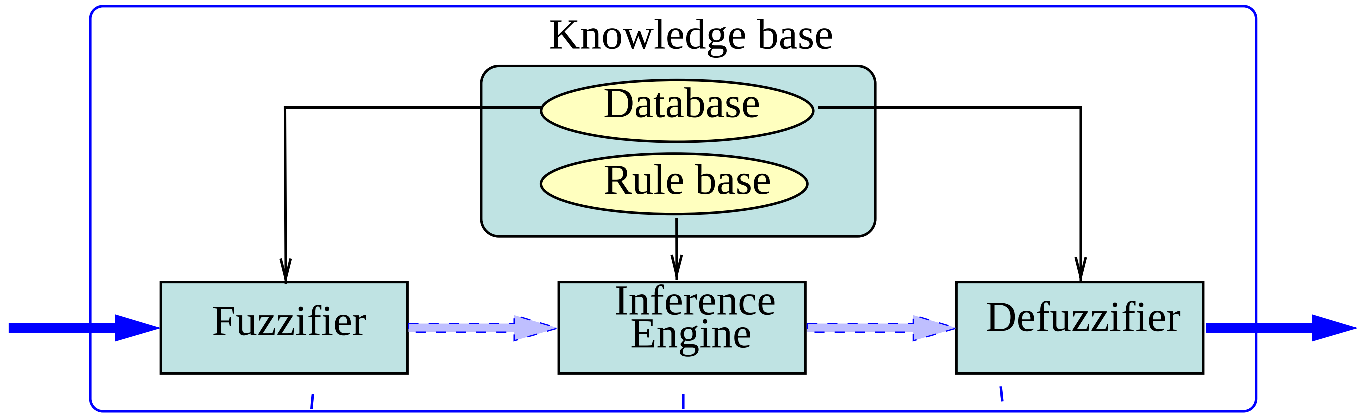
Inverted Pendulum Control: Inference 5 (Defuzzyfication)

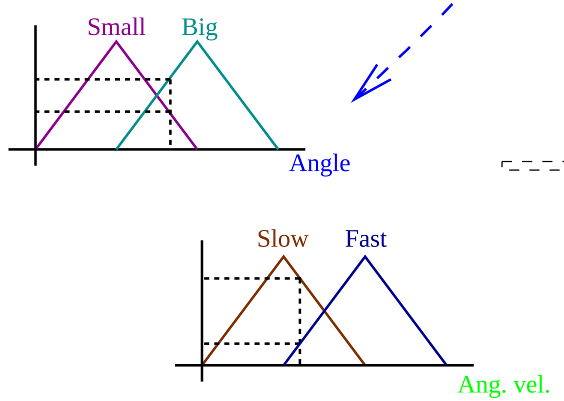


Inverted Pendulum Control: Response

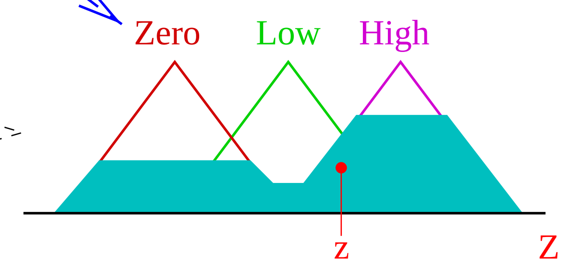


Fuzzy Inference System





Angle	Ang. vel	Speed
Small	Slow	Zero
Small	Fast	Low
Big	Slow	High
Big	Fast	Zero



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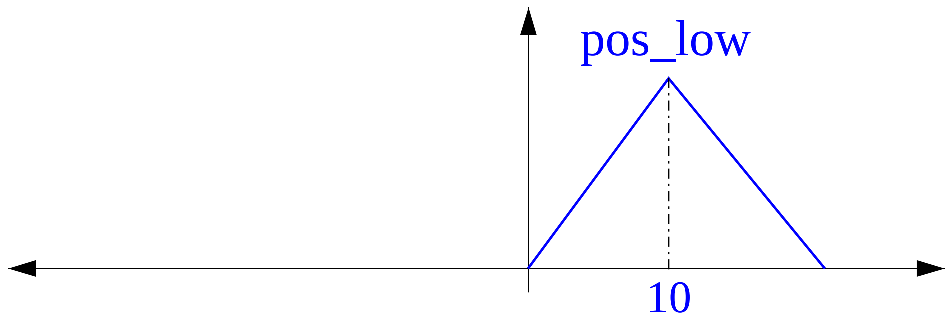
3 types of fuzzy rule consequents

IF angle is zero AND angular velocity is pos_low THEN speed is pos_low

Mamdani−type:
membership functions



Sugeno−type:
linear function of inputs

pos_low = 0.1*angle + 0.2* velocity + 10

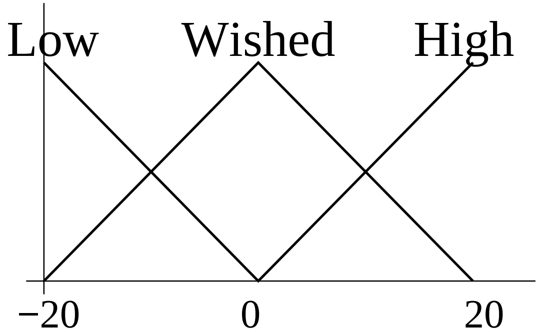
Singleton−type
constant values

pos_low = 10

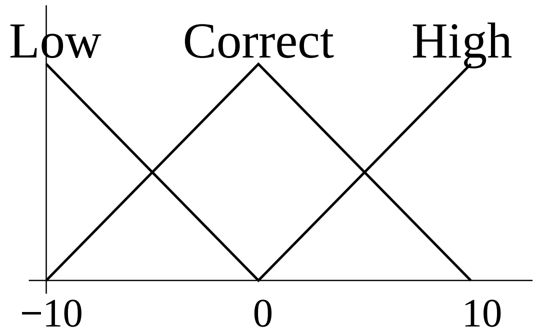
Fuzzy temperature controller

Temperature \ Pressure	Low	Correct	High
High	NS	NB	NB
Wished	Z	Z	NB
Low	PB	PS	NB

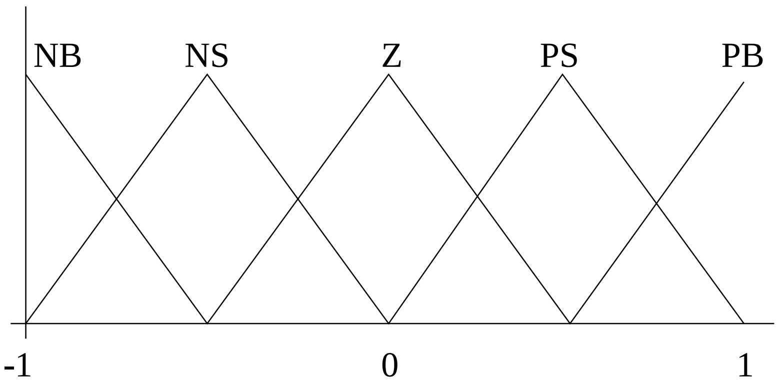
Temperature

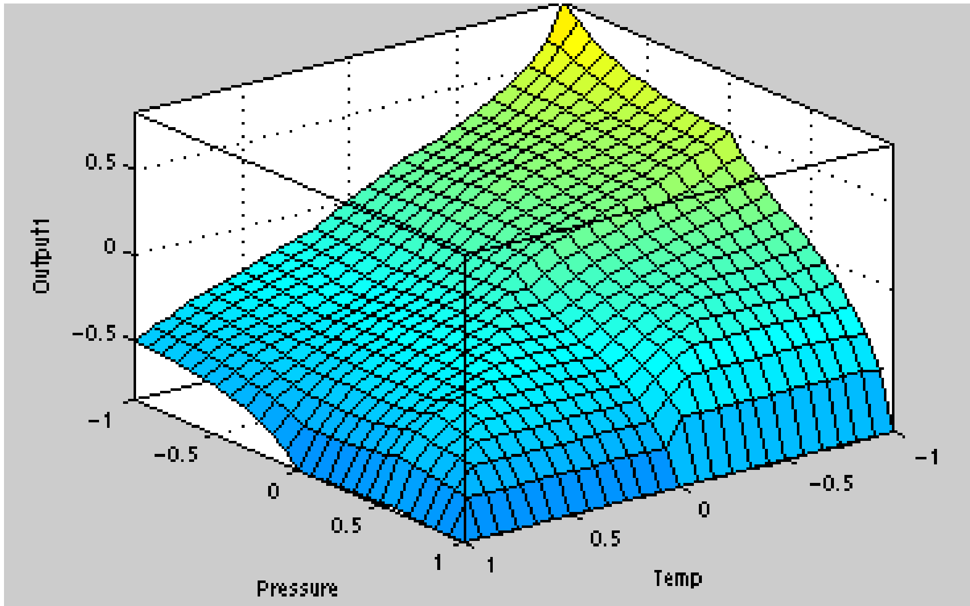


Pressure



Mamdani-type fuzzy temperature controller





Sugeno-type fuzzy temperature controller

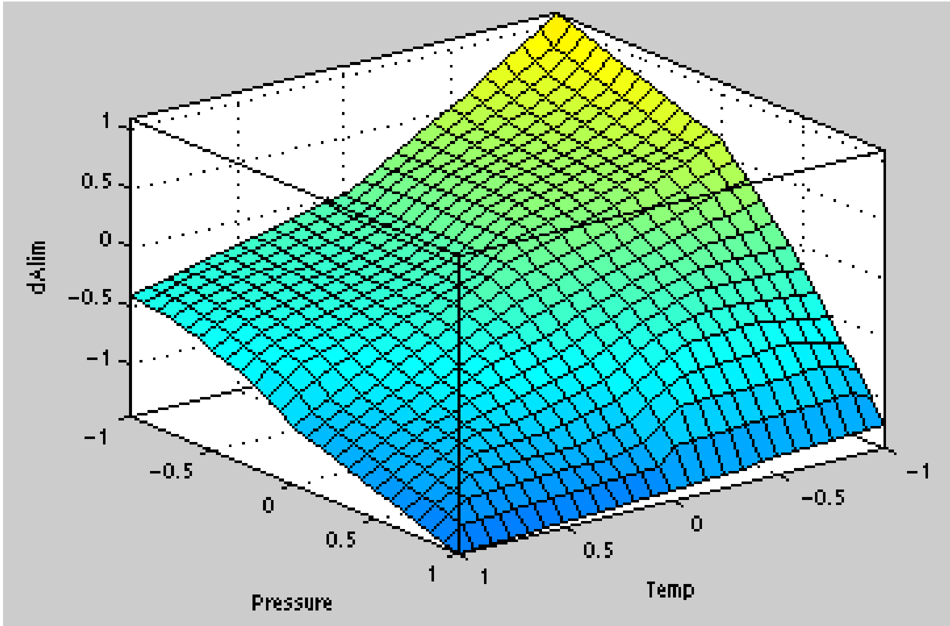
PB = -0.1T +1

PS = -0.01T + 0.5

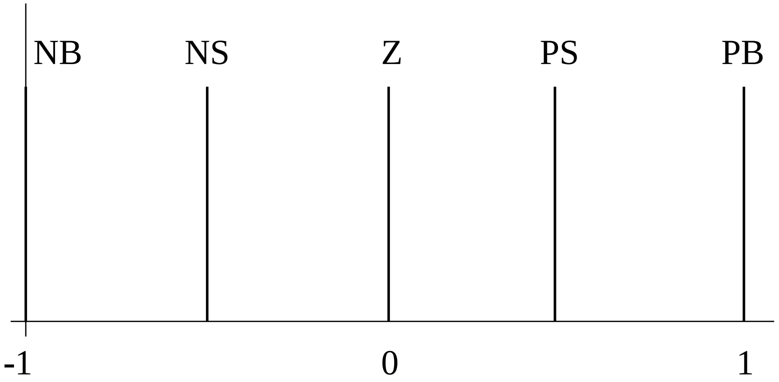
Z = 0

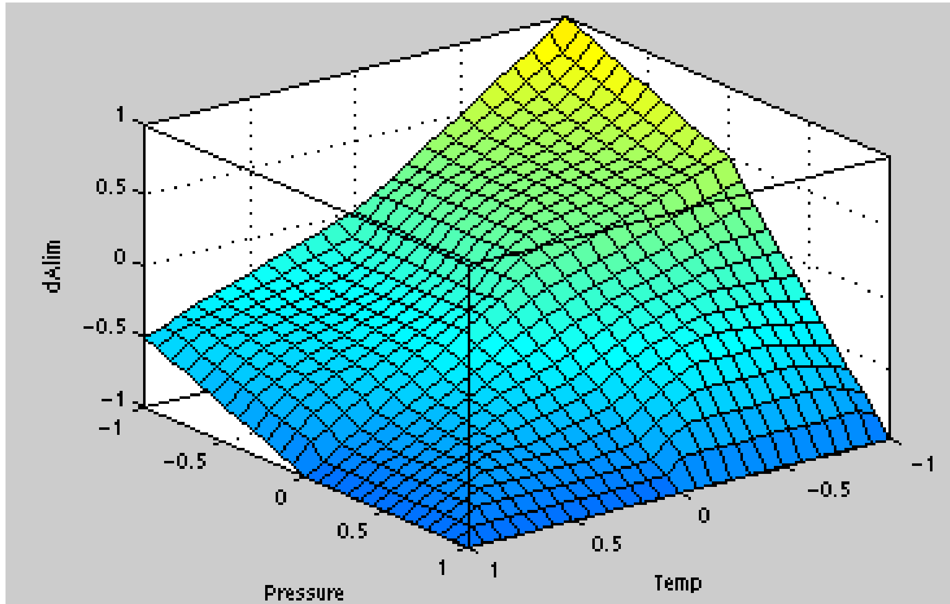
NS = -0.01T - 0.1P -0.5

NB = -0.1T -0.5P -1



Singleton-type fuzzy temperature controller





Comparison among the three systems