# Curvature-based segregation for multi-oriented textures

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## (i) Introduction : Perceptual singularities in smooth ODTs

- A central notion in texture segregation is *feature gradient*.

- Existing results in orientation-based texture segregation (OBTS) link perceptual boundaries to orientation gradients: outstanding orientation gradients signal perceptual singularities and boundaries.

- Most OBTS research is based on piecewise-constant orientation-defined textures (ODTs).

**However**…[Ben-Shahar, *PNAS* 2006]

- Lack of outstanding orientation gradient does not guarantee perceptual coherence.

- In fact, smoothly varying ODTs almost always exhibit salient perceptual singularities.

- These perceptual singularities have no apparent relationship to the orientation gradient.

- Sometimes, perceptual singularities in smoothly-varying ODT are more salient than perpetual singularities from orientation discontinuity.
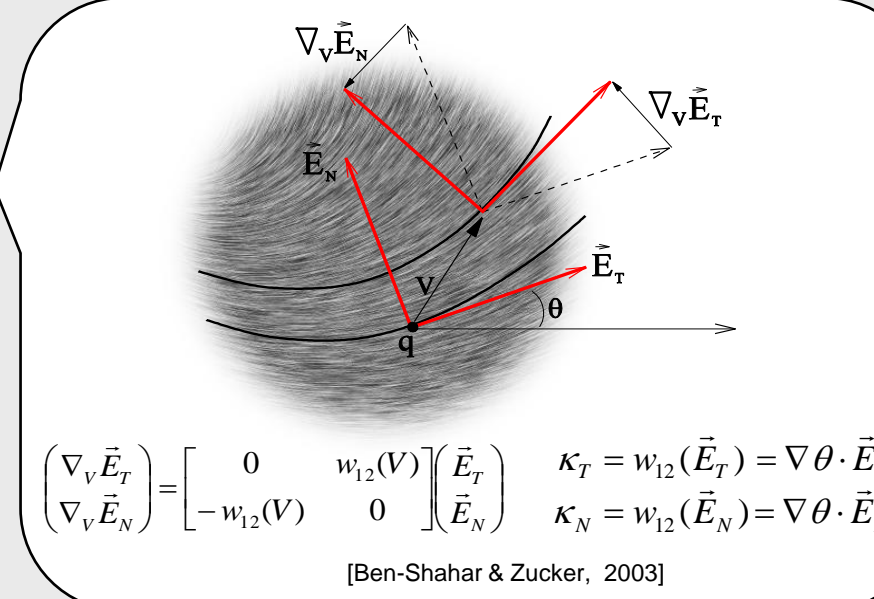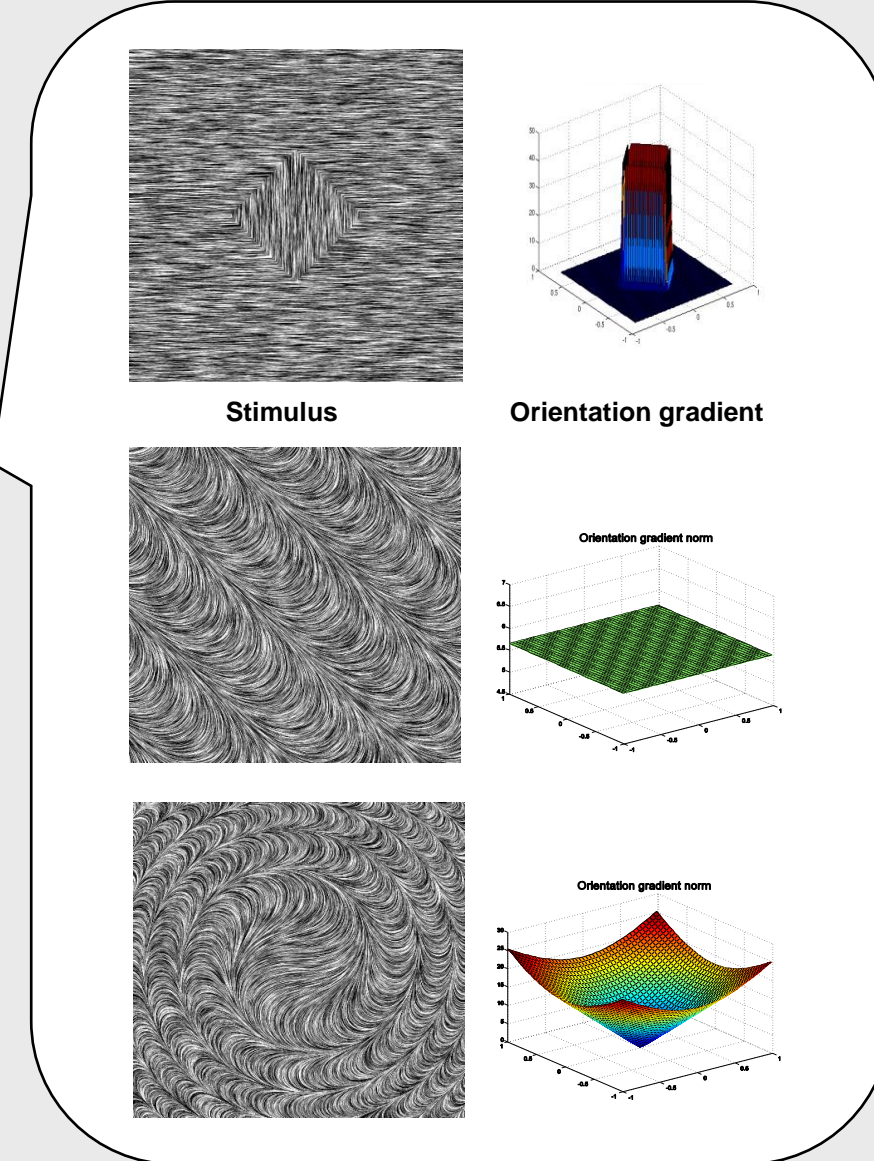


Stimulus  Orientation gradient
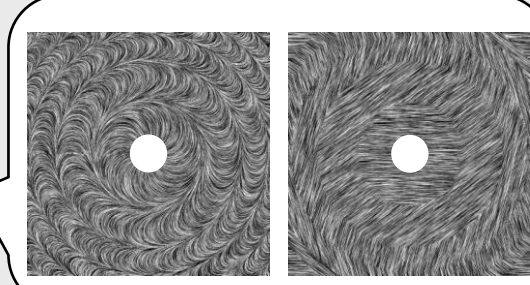
### Visual saliency via curvatures

- A *moving (Frenet) frame* representation leads to two ODT curvatures, one *tangential* ($k_T$) and one *normal* ($k_N$). The pair $\{k_T, k_N\}$ fully generalizes and extends the orientation gradient..

- While neither curvature by itself predicts perceptual singularities and saliency in smoothly-varying ODTs, a measure combining them both does so very accurately [Ben-Shahar, 2006]:

$$PSM(x,y) = \frac{\kappa_N(x,y)^2}{\kappa_T(x,y)^2 + \kappa_N(x,y)^2}$$

$$PS = \bigwedge_{\kappa_T^2 + \kappa_N^2 > \tau} \{Ridges[PSM]\}$$

Rectification operator with perceptual threshold $\tau$



$$\begin{pmatrix} \nabla_v \vec{E}_T \\ \nabla_v \vec{E}_N \end{pmatrix} = \begin{pmatrix} 0 & w_{12}(V) \\ -w_{12}(V) & 0 \end{pmatrix} \begin{pmatrix} \vec{E}_T \\ \vec{E}_N \end{pmatrix} \quad \begin{array}{l} \kappa_T = w_{12}(\vec{E}_T) = \nabla\theta \cdot \vec{E}_T \\ \kappa_N = w_{12}(\vec{E}_N) = \nabla\theta \cdot \vec{E}_N \end{array}$$

[Ben-Shahar & Zucker, 2003]

### A biologically-plausible model
### for curvature-based texture segregation

- The PSM can be calculated from an input image in a biologically plausible manner.

- A three-layer circuit in which both even-symmetric and odd-symmetric receptive fields are used to compute all possible directional derivatives of the dominant orientation, from which the tangential and normal curvatures at each spatial position are selected using nonlinear shunting inhibition [Ben-Yosef&Ben-Shahar, *JOSA* 2008].



Model outputs
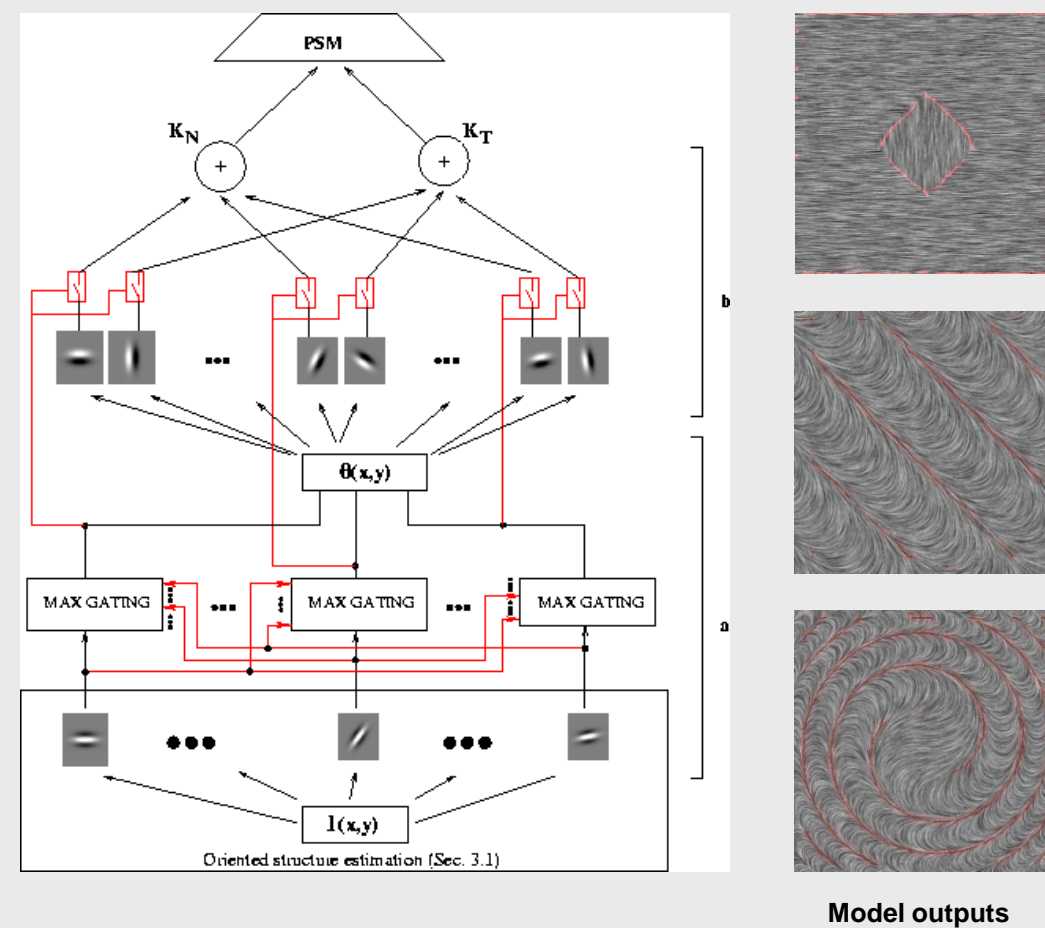
## (ii) Perceptual singularities in multi-oriented textures

A superposition of two different ODTs reveals perceptual singularities in **multi-oriented textures**. This happens in both classical (piecewise constant) and smoothly-varying ODTs. Hence, we hypothesize that any multi-oriented texture (i.e., oriented pattern with more than one dominant orientation at a point), can be decompose to $n$ different piecewise-smooth oriented manifold, each of which gives rise to its own perceptual singularities.

*Two dominant orientations at each point*        *Three dominant orientations at each point*
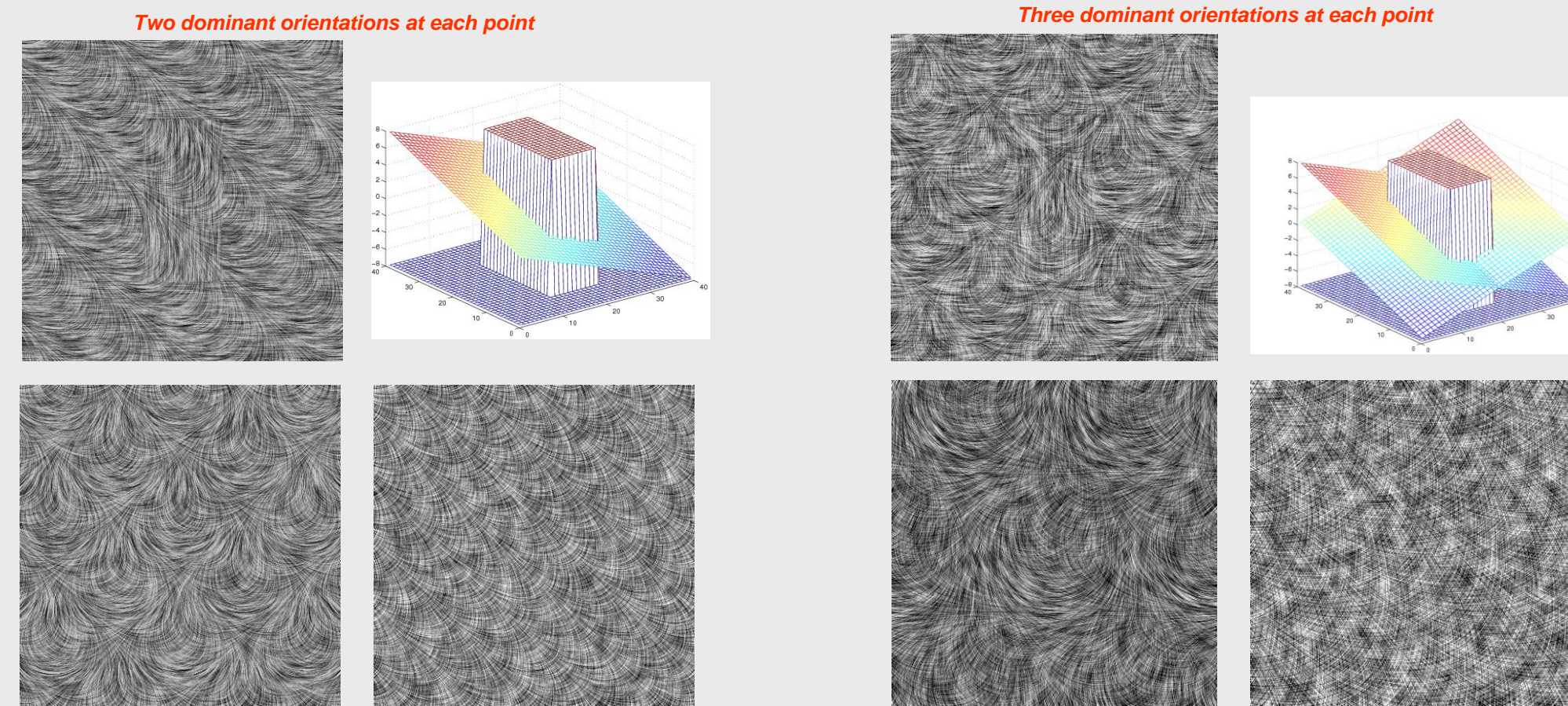


### Predicting perceptual singularities in a multi-oriented texture

1) Estimate the multiple dominant orientations via non-maxima suppression of the response of orientation selective filter bank.

2) Separate the dominant orientations, compute the PSM on each, and combine the results, to predict the perceptual structure in the multi-oriented pattern.



$PSM(\theta_1)$
$PSM(\theta_2)$
$Max_i\{PSM(\theta_i)\}$
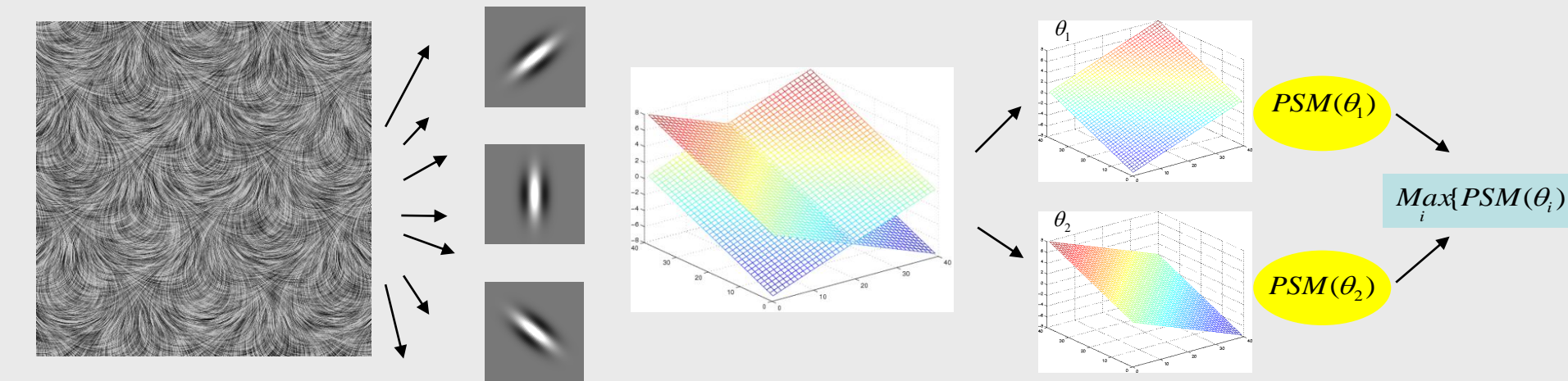
## (iii) A computational model for multi-oriented textures segregation

Main idea: The computation of the orientation derivatives (i.e, curvatures) is done *without* separating and grouping the orientation measurements to smooth oriented manifolds. Instead, this goal is achieved By *transforming* the problem to a dual space, where derivatives can be computed as a solution of a differential equation.

Transformation to a dual space:

$$\begin{aligned} f_1(\theta_1,\theta_2) &= \theta_1 \cdot \theta_2 \\ f_2(\theta_1,\theta_2) &= \theta_1 + \theta_2 \end{aligned} \quad\Rightarrow\quad \theta \cdot (f_2 - \theta) = f_1 \quad\Rightarrow\quad$$

**PDE**
$$\nabla\theta = \frac{\theta \cdot \nabla f_2 - \nabla f_1}{2\theta - f_2}$$

From which curvatures are calculated directly from the PDE:

$$\kappa_T(\theta_i) = \nabla\theta_i \cdot \vec{e}_\theta = \frac{\theta \cdot \nabla f_2 - \nabla f_1}{2\theta - f_2} \cdot (\cos\theta_i, \sin\theta_i)$$

$$\kappa_N(\theta_i) = \nabla\theta_i \cdot \vec{e}_{\theta+90} = \frac{\theta \cdot \nabla f_2 - \nabla f_1}{2\theta - f_2} \cdot (-\sin\theta_i, \cos\theta_i)$$

Since both curvatures share the same denominator, we define two modified curvatures:

$$\bar\kappa_T(\theta_i) \equiv (\theta_i \cdot \nabla f_2 - \nabla f_1) \cdot (\cos\theta_i, \sin\theta_i)$$

$$\bar\kappa_N(\theta_i) \equiv (\theta_i \cdot \nabla f_2 - \nabla f_1) \cdot (-\sin\theta_i, \cos\theta_i)$$

The PSM is calculated and the "union" of all PSMs at position (x,y) is being taken by MAX:

$$PSM(\theta_i) = \frac{\bar\kappa_N^2(\theta_i)}{\bar\kappa_T^2(\theta_i) + \bar\kappa_N^2(\theta_i)} \qquad PSM = \max_i\{PSM(\theta_i)\}$$

This can be extended and generalize for arbitrary number of N overlapping orientation manifolds.

N=3:
$$\begin{aligned} f_1(\theta_1,\theta_2,\theta_3) &= \theta_1 \cdot \theta_2 \cdot \theta_3 \\ f_2(\theta_1,\theta_2,\theta_3) &= \theta_1 \cdot \theta_2 + \theta_1 \cdot \theta_3 + \theta_2 \cdot \theta_3 \\ f_3(\theta_1,\theta_2,\theta_3) &= \theta_1 + \theta_2 + \theta_3 \end{aligned} \quad\Rightarrow\quad \theta^3 - f_3\theta^2 + f_2\theta - f_1 = 0$$

Arbitrary N:
$$f_k(\theta_1,\theta_2,...,\theta_n) = \sum_{c \in C_N^k} \prod_{i \in c} \theta_i$$

## (iv) Model outputs

*Three dominant orientations:*      *Synthetic results (orientations are given rather than measured)*      *Synthetic results for four dominant orientations:*      *Experimental results for classical texture stimuli:*



$Max\{PSM(\theta_i)\}$

$Max\{PSM(\theta_i)\}$