1. Let $D$ be a set of disks in the plane. A subset $D'$ of $D$ is an independent set of $D$, if for any two disks $d_1, d_2 \in D'$, $d_1 \cap d_2 = \emptyset$. $D'$ is a maximum independent set of $D$, if it is an independent set of $D$ and if $|D'| \geq |D''|$ for any other independent set $D''$ of $D$.

Give a simple constant factor approximation algorithm for computing a maximum independent set of $D$.

2. Let $D$ be a set of $n$ disks of diameter 1 in the plane. Give a PTAS for computing a maximum independent set of $D$. (Use the shifting strategy.)

Hint: Draw a grid by drawing the horizontal line $y = i$ and the vertical line $x = i$, for each integer $i$. You may assume that none of the centers of the disks in $D$ lies on a grid line. Fix an integer $k > 0$ (as a function of $\varepsilon$). For each pair of integers $(i, j)$, such that $0 \leq i, j \leq k-1$, let $D_{i,j}$ be the subset of disks obtained by removing all disks that are intersected by a vertical line $x = l$, where $(l \mod k) = i$, or by a horizontal line $y = l$, where $(l \mod k) = j$. Prove that the independent set of at least one of these $k^2$ subsets is of size at least $(1 - \varepsilon)|OPT|$, and show how to compute it in polynomial time.

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