Geometric Optimization
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homework assignment no. 3

1. Let P be a set of n points in the plane (called terminals), and let S be a set of k points in the plane (called Steiner points). Let $MST(P)$ denote the minimum spanning tree of P (i.e., of the complete Euclidean graph induced by P).

   (a) Give an example in which the weight of a tree spanning all points in P and some of the points in S can be less than $weight(MST(P))$.

   (b) Let T be a tree of minimum weight spanning all points in P and possibly some of the points in S. Prove that

   $$weight(MST(P)) \leq 2weight(T) \ .$$

2. Let $G = (V, E)$ be a complete graph with exactly two possible edge weights: 1 and 2. Give a 4/3-approximation algorithm for computing a minimum-weight tour in G. Start by computing a *minimum cycle cover* of G, which is a partition of V into cycles of size at least 3 (each vertex of V participates in exactly one cycle), such that the total weight of the cycle edges is as small as possible. This can be done in polynomial time.

   Describe your algorithm, prove its correctness, and establish its approximation ratio.

3. Let P be a set of $n = 2m$ points in the plane, and let $G = (P, E)$ be the complete Euclidean graph induced by P. Our goal is to develop a 3/4-approximation algorithm for the following problem: Compute a tour in G, such that the total weight of its edges is as big as possible. The first step in the algorithm is to compute a *maximum cycle cover* of G, which is a partition of P into cycles of size at least 3 (each point

of P participates in exactly one cycle), such that the total weight of the cycle edges is as big as possible. The second step is to compute a maximum matching of G, which is a partition of P into pairs, such that the total weight of the corresponding m edges is as big as possible. Both steps can be performed in polynomial time.

Describe your algorithm, prove its correctness, and establish its approximation ratio.

4. Let P be a set of n points in the plane. Find in $O(n \log n)$ time a largest empty disk that is defined by a pair or triplet of points of P.

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