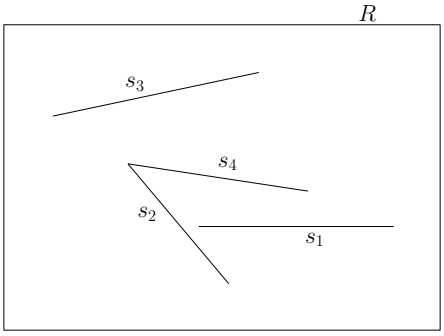
Homework assignment no. 3

1. Given n inequalities $a_i x + b_i y \geq 1$, for $i = 1, \ldots, n$, describe an expected linear-time algorithm that finds a point (x, y) (if exists) that (i) satisfies all these inequalities, and (ii) is closest to the origin under the L_1 distance (where $d_1(p, q) = |q_x - p_x| + |q_y - p_y|$).

2. Construct the search structure for the given scene, assuming s_i is inserted in the i'th iteration, $i = 1, 2, 3, 4$. Draw the structure after each insertion.



3. Let P be a set of n sites (i.e., points) in the plane. Describe an $O(n \log n)$-algorithm that computes for each $p \in P$ its closest site in P.

4. The Gabriel graph of a set $\mathcal{P}$ of points in the plane consists of all edges pq, $p, q \in \mathcal{P}$, such that the circle with diameter pq does not contain any point of $\mathcal{P}$ in its interior.

   (a) Prove that the Delaunay triangulation of $\mathcal{P}$ contains the Gabriel graph of $\mathcal{P}$.

   (b) Prove that pq is an edge of the Gabriel graph if and only if pq intersects the Voronoi edge between $\mathrm{Vor}(p)$ and $\mathrm{Vor}(q)$.

   (c) Show that the Gabriel graph can be computed in $O(n \log n)$ time.

5. Let P be a set of n points in the interior of an axis-parallel rectangle R. Assume that the points in P represent the houses in some living neighborhood R. One needs to determine the best location for a garbage dump in R. That is, one needs to find a point $g \in R$ that maximizes the expression $\min_{p \in P} \mathrm{dist}(p, g)$. Give an $O(n \log n)$-time algorithm for finding such a point g.

Submission: January 10, 2017.