Homework assignment no. 1

1. Let P be a set of n points in the plane. Construct a data structure of size $O(n)$ for answering in/out queries in $O(\log n)$ time. That is, given a query point q, determine in $O(\log n)$ time whether q lies in the convex hull of P.

2. Let C_1 and C_2 be two convex polygons with n_1 and n_2 vertices, respectively. (Each polygon is given by the sequence of its vertices in clockwise order.) Describe an $O(n)$-time algorithm for computing $C_1 \cap C_2$, where $n = n_1 + n_2$.

3. Prove that the problem of computing the convex hull of a set of n points in the plane has an $\Omega(n \log n)$ lower bound. Hint: Show that a set of n real numbers can be sorted in time $O(n)$, plus the time needed for a single convex hull computation.

4. A *strip* is a region of the plane that is defined by two parallel lines. Its *width* is the distance between the lines defining it. Let P be a set of n points in the plane. The *width* of P is the width of a minimum-width strip that contains P. Describe an $O(n \log n)$-time algorithm for computing the width of P. (Hint: Show that the width of P is determined by a pair of parallel lines supporting the convex hull of P, where at least one of them contains an edge of the convex hull.

5. Let S_1 be a set of n disjoint horizontal segments, and let S_2 be a set of n disjoint vertical segments. Describe an $O(n \log n)$-time algorithm for *counting* the number of intersections in $S_1 \cup S_2$.

6. Let C be a set of n circles. We wish to determine whether the circles in C are pair-wise disjoint. How fast can this be done?

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