1. Draw a polygon $P$ and place guards in it, such that the guards cover the boundary of $P$, but there exists a point in the interior of $P$ that is not seen by any of the guards.

2. The **stabbing number** of a triangulated simple polygon $P$ is the maximum number of diagonals intersected by any line segment contained in the interior of $P$. Describe an algorithm that computes a triangulation for a convex polygon that has stabbing number $O(\log n)$.

3. Let $S$ be a set of $n$ disjoint horizontal segments in the plane. Construct a data structure of size $O(n \log n)$ that supports **vertical ray shooting queries**. That is, construct a data structure that supports queries of the following form: Given a query point $p$ and a direction up/down, find the segment in $S$ which lies immediately above/below $p$ (if there exists such a segment), that is, find the segment that is hit first by a vertical ray emanating from $p$ and directed upwards/downwards. Analyze your solution in terms of preprocessing time, storage space, and query time. The desired bounds are $P(n) = O(n \log^2 n)$, $S(n) = O(n \log n)$, $Q(n) = O(\log^2 n)$.

4. Let $S = \{s_1, \ldots, s_n\}$ be a set of $n$ axis-parallel squares in the plane, and put $U = \bigcup_{i=1}^{n} s_i$. Prove that the combinatorial complexity of the boundary of $U$ is $O(n)$.

5. Let $S$ be a set of $n$ axis-parallel rectangles in the plane. We would like to be able to report all rectangles in $S$ that are fully contained in a query axis-parallel rectangle. Describe a data structure of size $O(n \log^4 n)$ that supports such queries in time $O(\log^4 n + k)$, where $k$ is the number of reported rectangles.

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