1. A rectilinear polygon is a simple polygon whose edges are horizontal and vertical. Show that \( \left\lfloor \frac{n}{4} \right\rfloor \) guards might be needed to guard a rectilinear polygon with \( n \) vertices.

2. The stabbing number of a triangulated simple polygon \( P \) is the maximum number of diagonals intersected by any line segment contained in the interior of \( P \). Describe an algorithm that computes a triangulation for a convex polygon that has stabbing number \( O(\log n) \).

3. Let \( S \) be a set of \( n \) disjoint horizontal segments in the plane. Construct a data structure of size \( O(n \log n) \) that supports vertical ray shooting queries. That is, construct a data structure that supports queries of the following form: Given a query point \( p \) and a direction up/down, find the segment in \( S \) which lies immediately above/below \( p \) (if there exists such a segment), that is, find the segment that is hit first by a vertical ray emanating from \( p \) and directed upwards/downwards. Analyze your solution in terms of preprocessing time, storage space, and query time. The desired bounds are \( P(n) = O(n \log^2 n), S(n) = O(n \log n), Q(n) = O(\log^2 n) \).

4. Let \( S = \{s_1, \ldots, s_n\} \) be a set of \( n \) axis-parallel squares in the plane, and put \( U = \bigcup_{i=1}^{n} s_i \). Prove that the combinatorial complexity of the boundary of \( U \) is \( O(n) \).

5. Let \( P \) be a set of \( n \) points in the plane, and assume that for each pair of points \( p, q \in P \), \( p_x \neq q_x \) and \( p_y \neq q_y \). Construct a data structure that supports queries of the following type: Given a north-east quadrant \( r \), determine the number of points that lie in \( r \), where a north-east quadrant is defined by two rays — a vertical ray directed upwards and a horizontal ray directed rightwards — with a common origin. Describe the algorithm for handling a query. What are the appropriate complexity bounds for the preprocessing, storage and query time.

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