Broadcasting in geometric radio networks

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Introduction

Non-Centralized Broadcasting

- A source node wishes to send a message to everyone
- The set of nodes isn’t known a-priori
- Algorithm advances in **synchronous rounds**
- How many rounds are needed to broadcast?
- How to tell when everyone got the broadcast?
- Minimal time of $\Omega(D)$ rounds
Problem Domain

- Omni-directional antennas
- Each node holds: location, label, range
- Node data isn’t known a-priori
- The set of available ranges is known
- The effect of knowledge radius is examined
Introduction

Previous Work

- Most algorithms assume full knowledge
- Deterministic neighborhood-knowledge algorithms require $\Omega(n)$ rounds
- Previous statement is false
- Best 0-knowledge algorithm achieved $O(n \log^2 D)$
Broadcast time

Symmetric

\(\varepsilon\)-knowledge radius with CD

\(O(D + \log n)\)

Proof of existence of networks requiring \(\Omega(D + \log n)\)

\(\Theta(D + \log n)\)

Large knowledge radius (no CD)

\(O(D + \log n)\)

Non-Symmetric

0-knowledge radius (no CD)

\(O(n)\)

\(\Omega(n\log n)\)

Large knowledge radius (no CD)

\(\Theta(D)\)

Nodes don’t know location
Non-symmetric large knowledge radius

Proposed Algorithm

- Minimal and maximal transmission ranges $r_{min}, r_{max}$ are known
- Fixed knowledge radius $s > r_{max}$
- An $O(D)$ algorithm is proposed:
  1. Plane is partitioned into squares of side $z = \frac{min(r_{min}, s - r_{max})}{\sqrt{2}}$ named “tiles”
  2. Tiles are grouped as blocks
  3. Tiles transmit to other tiles of the block in an organized fashion
Non-symmetric large knowledge radius: Tile Size

Any 2 nodes are within each other’s range

All nodes within the range of a tile’s node are known to all the tile’s nodes

Tile side $z = \min(r_{\text{min}}, s - r_{\text{max}})/\sqrt{2}$
Non-symmetric large knowledge radius

Block Grouping

- Tiles are grouped to non-overlapping blocks of $x \times x$ tiles where $x = 2\lceil \frac{r_{\text{max}}}{z} \rceil + 1$

- Nodes in the central tile of the block cannot reach outside of the block

\[ r_{\text{max}} \]

\[ \left\lceil \frac{r_{\text{max}}}{z} \right\rceil \]

\[ 1 \]

\[ \left\lceil \frac{r_{\text{max}}}{z} \right\rceil \]
Non-symmetric large knowledge radius

Simultaneous Transmission

- Corresponding tiles in 2 adjacent blocks will never collide
Non-symmetric large knowledge radius

Phase Rounds

For each tile pair \((t, j)\) (and for all the blocks simultaneously):

1. If \(t\) is in state \textit{uninformed} or has been \textit{informed} for an entire phase, do nothing. Else, transmit by label

2. If \(j\) is in state \textit{uninformed}, the \textit{smallest node} to get that message transmits. Tile \(j\) becomes \textit{informed}
Non-symmetric large knowledge radius

Observations

- Each phase consists of $y = x^2(x^2 - 1)$ steps of 2 rounds. $y$ is constant.
- By the choice of tile side nodes have enough knowledge to decide whom submits.

Proof

1. Given a node $v$ in tile $T$, since it first hears the message, after at most $y$ rounds, $T$ becomes informed (since $v$ is in the range of some node from the tile this message comes from).
2. A tile $T'$ with nodes in the range of $v$ becomes informed after at most $y$ rounds.
3. Therefore, $v$ gets the message and after at most $2y$ steps all nodes in $v$'s range get the message.

- Broadcast is completed in $O(D)$, which is also $\Theta(D)$.
What’s next?

Broadcast time

Symmetric

- $\varepsilon$-knowledge radius with CD
  - $O(D + \log n)$
  - Proof of existence of networks requiring $\Omega(D + \log n)$
  - $\Theta(D + \log n)$

Non-Symmetric

- Large knowledge radius (no CD)
  - $O(D + \log n)$

- 0-knowledge radius (no CD)
  - $O(n)$
  - Nodes don’t know location
  - $\Omega(n\log n)$

- Large knowledge radius (no CD)
  - $\Theta(D)$
Non-symmetric 0-knowledge radius

0-Knowledge drawbacks

- Fixed knowledge radius \( s = 0 \)
- Consequences of 0-knowledge radius:
  1. Nodes have no knowledge of other nodes in the same tile
  2. Nodes of a tile no longer know which nodes of another tile are accessible and by whom

Coping with the lack of knowledge

1. Solved by a preprocessing phase that attains knowledge radius \( r_{min} \)
2. Solved by changing the algorithm so that the entire area reachable from a tile is informed
Non-symmetric 0-knowledge radius

Proposed Algorithm

- Minimal and maximal transmission ranges $r_{\text{min}}, r_{\text{max}}$ are known
- Fixed knowledge radius $s = r_{\text{min}}$ attained by preprocessing
- An $O(n)$ algorithm is proposed:
  1. Plane is partitioned as before, with tiles of side $z = r_{\text{min}} / \sqrt{2}$
  2. Tiles transmit to the entire reachable area in a different organized fashion
### Preprocessing
- Every node transmits its data in its turn (determined by label)
- $O(n)$
- Requires knowing $n$
- A modification to the algorithm is shown such that knowledge of $n$ is not required (still $O(n)$)

### Initialization
- All tile’s nodes are numbered according to the labels
- Every tile is given a transmission counter initialized to 1
- Let $|T|$ be the number of nodes in tile $T$
Non-symmetric large knowledge radius

Phase Rounds

For each tile $i$ (and for all the blocks simultaneously):

1. If tile $i$ is *informed* and its *transmission counter* $< |i|$, a node transmits according to the *counter*

2. For each tile $j \neq i$:

3. If $j$ is *uninformed* and a node received the message in (1), the lowest labeled node to receive the message transmits. Tile $j$ becomes *informed* and *counter* is increased
Non-symmetric large knowledge radius

Observations
- A node that gets a message knows whom else in the node got it, and can decide who transmits

Proof
1. From the moment tile T becomes informed until all tiles with a node in the range of a node from T become informed, at most $|T|$ phases have passed
2. There is a path of length at most $D$ from the source to any node $v$
3. This path corresponds to a sequence of tiles $T_1, T_2, ..., T_d, d \leq D$
4. Hence, the time required for broadcasting is $O(|T_1| + |T_2| + ... + |T_d|) \subseteq O(n)$
What's next?

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  - $\Theta(D)$
Questions?