

$$\left\{ \begin{array}{l} -\Delta u = xy \\ u(0, y) = u(\pi, y) = 0 \\ u(x, 0) = \sin x, \quad u(x, 1) = 1. \end{array} \right. \quad 0 < x < \pi, \quad 0 < y < 1 \quad (1)$$

$$u = \sum_{n=1}^{\infty} Y_n(y) \sin nx$$

$$xy = y \cdot \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x \sin nx dx = -\frac{2}{\pi n} \left(x \cos nx \Big|_0^{\pi} - \int_0^{\pi} \cos nx dx \right) = -\frac{2(-1)^n}{n}$$

$$\sum_{n=1}^{\infty} \left(n^2 Y_n - Y_n'' \right) \sin nx = \sum_{n=1}^{\infty} y b_n \sin nx$$

$$Y_n'' - n^2 Y_n = -y b_n$$

$$Y_n = C_n e^{ny} + D_n e^{-ny} + A_n y + B_n$$

$$-n^2(A_n y + B_n) = -y b_n \Rightarrow A_n = \frac{b_n}{n^2}, \quad B_n = 0$$

$$Y_n(y) = C_n e^{ny} + D_n e^{-ny} + \frac{b_n}{n^2} y$$

$$y=0) \quad Y_n(0) = \begin{cases} 1 & n=1 \\ 0 & n \neq 1 \end{cases} \Rightarrow C_n + D_n = \begin{cases} 1 & n=1 \\ 0 & n \neq 1 \end{cases}$$

$$y=1) \quad Y_n(1) = \frac{2}{\pi} \int_0^{\pi} 1 \cdot \sin nx dx = -\frac{2}{n\pi} \cos nx \Big|_0^{\pi} = \frac{2}{n\pi} (1 - (-1)^n) = d_n$$

$$\begin{cases} C_n e^n + D_n e^{-n} + \frac{b_n}{n^2} = d_n \\ C_n + D_n = \begin{cases} 1 & n=1 \\ 0 & n \neq 1 \end{cases} \end{cases}$$

$$\left\{ \begin{array}{l} u_{tt} - 9u_{xx} = e^{-4|x|} \\ u(x,0) = f(x) = \begin{cases} 1 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases} \\ u_t(x,0) = \cos \pi x \end{array} \right.$$

(2)

$$u = \frac{1}{2}(f(x+at) + f(x-at)) + \frac{1}{2a} \int_{x-at}^{x+at} \cos(\pi s) ds + \frac{1}{2a} \int_0^t \int_{x-a(t-\tau)}^{x+a(t-\tau)} e^{-is} ds d\tau \quad (1c)$$

$x = 9, t = 3, a = 3$

$$x-at=0, \quad f(0)=1 \quad \int_0^8 \cos \pi s ds = 0, \quad s \in (0, 18) \Rightarrow s > 0.$$

$$x+at=18, \quad f(18)=0$$

$$\int_{x-a(t-\tau)}^{x+a(t-\tau)} e^{-s} ds = -e^{-x-a(t-\tau)} + e^{-x+a(t-\tau)} = e^{-x} \left(e^{a(t-\tau)} - e^{-a(t-\tau)} \right)$$

$$I = \frac{e^{-x}}{2a} \int_0^t \left(e^{a(t-\tau)} - e^{-a(t-\tau)} \right) d\tau = \frac{e^{-x}}{2a^2} \left[-e^{a(t-\tau)} - e^{-a(t-\tau)} \right]_0^t =$$

$$= \frac{e^{-x}}{2a^2} \left[-1 + e^{at} - 1 + e^{-at} \right]$$

$$u = \frac{1}{2} + \frac{e^{-9}}{18} \left[2 + e^9 + e^{-9} \right] = \underline{\underline{\frac{1}{2} + \frac{1}{18} (1 - 2e^{-9} + e^{-18})}}$$

11'31PJ2 11'30K5 11'3PJ10 K10 $\frac{1}{2}[f(x+at) + f(x-at)]$ 11'2'1c1 (2)
 f(x) 11'1P81 11'3PJ10 11'30 8e 812c3 f(x) 11'3D8 / 11' . x+at = ±1, 0
 11'3PJ10 11'30 8e 812c3 f(x) 11'112 8e 11'30 8K1
 11'3PJ10 11'30 8e 812c3 f(x) 11'112 8e 11'30 8K1
 11'3PJ10 11'30 8e 812c3 f(x) 11'112 8e 11'30 8K1
 11'3PJ10 11'30 8e 812c3 f(x) 11'112 8e 11'30 8K1
 11'3PJ10 11'30 8e 812c3 f(x) 11'112 8e 11'30 8K1

$$③ \quad u_t = 2u_{xx} + u + x(1-t - e^{-t})$$

$$u(x,0) = 1 - \cos 2\pi x$$

$$u_x(0,t) = u_x(1,t) = 0 \quad 0 < x < 1, \quad t > 0$$

$$\begin{aligned} 1) \quad u(x,t) &= V(x,t) + A(t)x + B(t)x^2 \\ V_x(0,t) &= V_x(1,t) = 0 \end{aligned} \quad \Rightarrow \quad A = 0, B = 0$$

$$u(x,t) = V(x,t) + xt \Rightarrow$$

$$\begin{cases} V_t = 2V_{xx} + V - xe^t \\ V_x(0,t) = V_x(1,t) = 0 \\ V_x(0,t) = 1 - \cos 2\pi x \end{cases}$$

$$2) \quad V(x,t) = W(x,t)e^{-t} \Rightarrow$$

$$\begin{cases} W_t = 2W_{xx} - x \\ W_x(0,t) = W_x(1,t) = 0 \\ W(x,0) = 1 - \cos 2\pi x \end{cases}$$

$$3) \quad W_t = 2W_{xx} + g(x,t) \Rightarrow g(x,t) = -x$$

$$W(x,t) = \frac{W_0(t)}{2} + \sum_{n=1}^{\infty} w_n(t) \cos n\pi x$$

$$-x = g(x,t) = \frac{g_0(t)}{2} + \sum_{n=1}^{\infty} g_n(t) \cos n\pi x$$

$$\frac{w_0'}{2} + \sum_{n=1}^{\infty} w_n' \cos n\pi x = -2 \sum_{n=1}^{\infty} n^2 \pi^2 w_n \cos n\pi x + \frac{g_0}{2} + \sum_{n=1}^{\infty} g_n \cos n\pi x$$

$$\begin{cases} w_0' = g_0 \\ w_n' + 2n^2 \pi^2 w_n = g_n \end{cases} \quad g_n(t) = 2 \int_0^1 -x \cos n\pi x dx = \frac{2(1-(-1)^n)}{n^2 \pi^2} = \delta_n$$

$$\Rightarrow g_0(t) + 2 \int_0^1 -x dx = -1, \quad g_n(t) = \frac{2(1-(-1)^n)}{n^2 \pi^2} = \delta_n$$

$$W(x,0) = 1 - \cos 2\pi x = \frac{w_0(0)}{2} + \sum_{n=1}^{\infty} w_n(0) \cos n\pi x \Rightarrow$$

$$w_0(0) = 2, \quad w_n(0) = \begin{cases} -1, & n=2 \\ 0, & n \neq 2 \end{cases}$$

$$\left. \begin{array}{l} w_0' = -1 \\ w_0(0) = 2 \end{array} \right\} \Rightarrow w_0(t) = 2 - t$$

$$\left. \begin{array}{l} w_2' + 8\pi^2 w_2 = 0 \\ w_2(0) = -1 \end{array} \right\} \Rightarrow w_2(t) = -e^{-8\pi^2 t}$$

$$\left. \begin{array}{l} \forall n \neq 2 \quad \left\{ \begin{array}{l} w_n' + 2n^2\pi^2 w_n = \gamma_n \\ w_n(0) = 0 \end{array} \right. \end{array} \right\} \Rightarrow w_n(t) = C_n e^{-2n^2\pi^2 t} + A_n$$

$$A_n = \frac{\gamma_n}{2n^2\pi^2}$$

$$w_n(0) = 0 \Rightarrow C_n = -A_n$$

$$w_n(t) = \frac{\gamma_n}{2n^2\pi^2} \left(1 - e^{-2n^2\pi^2 t} \right)$$

$$w(x, t) = \frac{2-t}{2} + e^{-8\pi^2 t} \cos 2\pi x + \sum_{\substack{n=1 \\ n \neq 2}}^{\infty} w_n(t) \cos n\pi x$$

$$w(x, t) = \frac{2-t}{2} + e^{-8\pi^2 t} \cos 2\pi x + \sum_{\substack{n=1 \\ n \neq 2}}^{\infty} w_n(t) \cos n\pi x$$

$$u(x, t) = xt + w(x, t) e^t, \quad \gamma_n = \frac{2(1 - (-1)^n)}{n^2\pi^2}$$

(4)

$$\begin{cases} yu_x - xu_y = u + x^2 + y^2 \\ u(0, y) = 0 \quad (y > 0) \end{cases}$$

$$\begin{cases} x_\tau = y & x|_{\tau=0} = 0 \\ y_\tau = -x & y|_{\tau=0} = s \\ u_\tau = u + x^2 + y^2 & u|_{\tau=0} = 0 \end{cases} \quad (\underline{\underline{s > 0}})$$

$$x_{\tau\tau} = y_\tau = -x \quad x_{\tau\tau} + x = 0$$

$$x = A(s) \sin \tau + B(s) \cos \tau$$

$$y = x_\tau = A(s) \cos \tau - B(s) \sin \tau$$

$$\tau = 0) \quad B(s) = 0 \quad A(s) = s$$

$$x = s \cdot \sin \tau$$

$$y = s \cos \tau$$

$$s = \pm \sqrt{x^2 + y^2} \quad (s > 0 \text{ "+" } n \pi/2)$$

$$\tau = \operatorname{arctg} \frac{x}{y} + \pi n, \quad n = 0, \pm 1, \dots$$

$$: n = 0 \text{ SK } x = 0 \text{ ND } \tau = 0 \text{ IEC}$$

$$\tau = \operatorname{arctg} \frac{x}{y}$$

$$u_\tau = u + x^2 + y^2 = u + s^2$$

$$u = e^\tau c(s) - s^2$$

$$\tau = 0) \quad 0 = c(s) - s^2 \rightarrow c(s) = s^2$$

$$u = (e^\tau - 1)s^2 = \underline{(e^{\operatorname{arctg} \frac{x}{y}} - 1) \cdot (x^2 + y^2)}$$

$$\left\{ \begin{array}{l} u_t = 4u_{xx} - 3u \\ u(x,0) = f(x) = \begin{cases} 1 & -2 < x \leq 1 \\ 2 & 1 < x < 2 \\ 0 & 2 \leq |x| \end{cases} \end{array} \right. \quad -\infty < x < \infty, t > 0$$

(5)

$$1. \boxed{u = v e^{-3t}} : (v_t - 3v)e^{-3t} = 4e^{-3t}v_{xx} - 3e^{-3t}v$$

$$\left\{ \begin{array}{l} v_t = 4v_{xx} \\ v(x,0) = f(x) \end{array} \right.$$

$$2. f = \theta(x+2) + \theta(x-1) - 2\theta(x-2), \quad \theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$$

$$v(x,t) = \frac{1}{2} \left\{ 1 + \operatorname{erf} \frac{x+2}{4\sqrt{t}} \right\} + \frac{1}{2} \left\{ 1 + \operatorname{erf} \frac{x-1}{4\sqrt{t}} \right\} - 2 \cdot \frac{1}{2} \left\{ 1 + \operatorname{erf} \frac{x-2}{4\sqrt{t}} \right\}$$

$$\boxed{v = \frac{1}{2} \left\{ \operatorname{erf} \frac{x+2}{4\sqrt{t}} + \operatorname{erf} \frac{x-1}{4\sqrt{t}} - 2 \operatorname{erf} \frac{x-2}{4\sqrt{t}} \right\}}$$

$$3. M_2^v(t) = \int_{-\infty}^{\infty} x^2 v(x,t) dx. \quad \dot{M}_2^v = 4 \cdot 2 \cdot 1 \cdot M_0^v(t), \quad M_2^v = 8M_0^v t + M_2^v(0) \stackrel{= \text{const}}{=}$$

$$M_2^v(0) = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_{-2}^1 x^2 dx + 2 \int_1^2 x^2 dx = \frac{1}{3}(1+8) + \frac{2}{3}(8-1) =$$

$$M_0^v(0) = \int_{-\infty}^{\infty} f(x) dx = \int_{-2}^1 dx + 2 \int_1^2 dx = 5$$

$$M_2^v(t) = 40t + \frac{23}{3}$$

$$\boxed{M_2^v(t) = e^{-3t} \left(40t + \frac{23}{3} \right)}.$$