

1

$$\begin{cases} u_{xx} + u_{yy} - u_x = 0 & 0 < x < 1 \\ & 0 < y < \pi \\ u(0, y) = \sin 3y \\ u(1, y) = y(\pi - y) \\ u(x, 0) = u(x, \pi) = 0. \end{cases}$$

1)  $u = X(x)Y(y)$   
 $X''Y + XY'' - X'Y = 0$   
 $\frac{Y''}{Y} = -\frac{X'' - X'}{X} = -\lambda$

$$\begin{cases} Y'' + \lambda Y = 0 \\ Y(0) = Y(\pi) = 0 \end{cases} \rightarrow \lambda = n^2, Y_n = \sin ny$$

$$\begin{aligned} X'' - X' - n^2 X &= 0 \\ r^2 - r - n^2 &= 0 \rightarrow \begin{cases} r_n = \frac{1}{2} + \sqrt{n^2 + \frac{1}{4}} \\ s_n = \frac{1}{2} - \sqrt{n^2 + \frac{1}{4}} \end{cases} \end{aligned}$$

$$X_n = A_n e^{r_n x} + B_n e^{s_n x}$$

2)  $u = \sum_{n=1}^{\infty} (A_n e^{r_n x} + B_n e^{s_n x}) \sin ny$

$$x=0) \quad A_n + B_n = \begin{cases} 1 & n=3 \\ 0 & n \neq 3 \end{cases}$$

$$x=1) \quad A_n e^{r_n} + B_n e^{s_n} = f_n = \frac{2}{\pi} \int_0^{\pi} y(\pi - y) \sin ny \, dy$$

$$\begin{aligned} f_n &= \frac{-2}{n^2 \pi} \left[ \underbrace{y(\pi - y) \cos ny}_{=0} \Big|_0^{\pi} - \int_0^{\pi} (\pi - 2y) \cos ny \, dy \right] = \frac{2}{n^2 \pi} \int_0^{\pi} (\pi - 2y) \cos ny \, dy = \\ &= \frac{2}{n^2 \pi} \left[ \underbrace{(\pi - 2y) \sin ny}_{=0} \Big|_0^{\pi} + 2 \int_0^{\pi} \sin ny \, dy \right] = \frac{-4}{n^3 \pi} \cos ny \Big|_0^{\pi} = \end{aligned}$$

$$= \frac{4}{n^3 \pi} (1 - (-1)^n)$$

$$B_n = \begin{cases} 1 & n=3 \\ 0 & n \neq 3 \end{cases} - A_n$$

$$A_n (e^{r_n} - e^{s_n}) + \begin{cases} e^{s_3} & n=3 \\ 0 & n \neq 3 \end{cases} = f_n$$

$$A_n = \frac{f_n - \begin{cases} e^{s_3} & n=3 \\ 0 & n \neq 3 \end{cases}}{e^{r_n} - e^{s_n}}$$

$$\begin{cases} u_t = a u_{xx} - k u_x \\ u(0, t) = f(x) \end{cases} \quad -\infty < x < \infty, t > 0$$

(k2)

$$u = e^{\alpha x + \beta t} v$$

$$e^{\alpha x + \beta t} (\beta v + v_t) = a e^{\alpha x + \beta t} (\alpha^2 v + 2\alpha v_x + v_{xx}) - k e^{\alpha x + \beta t} (\alpha v + v_x)$$

$$v_t = a v_{xx} + v_x [2\alpha a - k] + v [\alpha^2 - k\alpha - \beta]$$

$$\alpha = \frac{k}{2a}$$

$$\beta = a\alpha^2 - k\alpha = a \frac{k^2}{4a^2} - \frac{k^2}{2a} = -\frac{k^2}{4a}$$

$$u = e^{+\frac{k}{2a}x - \frac{k^2}{4a}t} v$$

$$\begin{cases} v_t = a v_{xx} \\ v(x, 0) = f(x) e^{-\frac{k}{2a}x} \end{cases} \quad -\infty < x < \infty, t > 0$$

$$v(x, t) = \frac{1}{2\sqrt{a\pi t}} \int_{-\infty}^{\infty} f(y) e^{-\frac{ky}{2a}} e^{-\frac{(x-y)^2}{4at}} dy$$

$$u = v(x, t) e^{+\frac{k}{2a}x - \frac{k^2}{4a}t} \quad SK$$

1) 
$$\begin{cases} u_t = u_{xx} - 2u \\ u(x,0) = e^{-x^2} \end{cases} \quad u = v e^{-2t}$$

$$\begin{cases} v_t = v_{xx} \\ v(x,0) = e^{-x^2} \end{cases}$$

$$v(x,t) \sim G_1(x,t) (M_0 + o(t^{-1}))$$

$$M_0^v = \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} > 0$$

⊗ 
$$u(x,t) \sim \frac{1}{2\sqrt{\pi t}} e^{-\frac{x^2}{4t} - 2t} (M_0^v + o(t^{-1}))$$

2) W: 
$$M_0^w = \int_{-\infty}^{\infty} x e^{-x^2} dx = 0, \quad M_1^w = \int_{-\infty}^{\infty} x^2 e^{-x^2} dx > 0, \quad M_2^w = \int_{-\infty}^{\infty} x^3 e^{-x^2} dx = 0$$

$$W(x,t) \sim G_{100}(x,t) \left( M_0^w + \frac{2xM_1^w - (M_2^w(0))^2}{4at} + o(t^{-2}) \right)$$

$$W(x,t) \sim \frac{1}{2\sqrt{100\pi t}} e^{-\frac{x^2}{4 \cdot 100 t}} \left( \frac{2xM_1^w}{4at} + o(t^{-2}) \right)$$

(\*) 
$$u(1,t) e^{-2t} \text{ pilsa } \text{ pilsa } \quad : x=1$$

$$\lim_{t \rightarrow \infty} \frac{u(1,t)}{W(1,t)} = 0$$

$$u(0,t) = W(0,t) \quad x=0$$

SK

$$\lim_{t \rightarrow \infty} \frac{W(0,t)}{u(0,t)} = 0$$

$$\begin{cases} u_{tt} = g u_{xx} \\ u(x,0) = e^{-x}, u_t(x,0) = 0 \\ u(0,t) = e^{-t} \end{cases}$$

$$0 < x < \infty, t > 0 \quad (13)$$

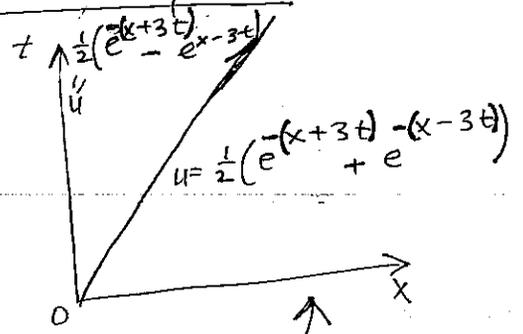
$$\begin{cases} u_{tt} = g u_{xx} \\ u(x,0) = e^{-x}, u_t(x,0) = 0 \\ u(0,t) = 0 \end{cases}$$

$$\begin{cases} u_{tt} = g u_{xx} \\ u(x,0) = u_t(x,0) = 0 \\ u(0,t) = e^{-t} \end{cases}$$

$$(I) \rightarrow u = u_I + u_{II} \leftarrow (II)$$

$$(I) \begin{cases} \tilde{u}_{tt} = g \tilde{u}_{xx} & -\infty < x < \infty, t > 0 \\ \tilde{u}(x,0) = \begin{cases} e^{-x} & x > 0 \\ -e^x & x < 0 \end{cases} = f(x) \\ \tilde{u}_t(x,0) = 0 \end{cases}$$

: d/3 - 1/c (2EN)



$$\tilde{u}(x,t) = \frac{1}{2} (f(x-3t) + f(x+3t))$$

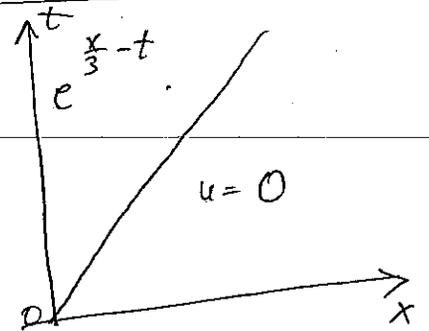
$$u_I(x,t) = \frac{1}{2} \left[ e^{-(x+3t)} + \begin{cases} e^{-x+3t} \\ -e^{x-3t} \end{cases} \right]$$

$$\left. \begin{matrix} x > 3t \\ x < 3t \end{matrix} \right\} \quad \begin{matrix} x > 0 \\ t > 0 \end{matrix}$$

$$(II) \quad u = \begin{cases} \varphi(x-3t) & x < 3t \\ 0 & x > 3t \end{cases}$$

$$\varphi(-3t) = e^{-t} \Rightarrow \varphi(u) = e^{u/3}$$

$$u_{II} = \begin{cases} e^{\frac{x-3t}{3}} & x < 3t \\ 0 & x > 3t \end{cases}$$



$$u = u_I + u_{II} = \begin{cases} \frac{1}{2} [e^{-(x+3t)} + e^{x-3t}] + e^{\frac{x}{3}-t} & x < 3t \\ \frac{1}{2} [e^{-(x+3t)} + e^{-(x-3t)}] & x > 3t \end{cases}$$

$$v = u_1 - u_2:$$

$$\begin{cases} v_{tt} = a^2 v_{xx} - k^2 v \\ v(x,0) = v_t(x,0) = 0 \\ v(t,0) = v(t,l) = 0 \end{cases} \rightarrow v_t(t,0) = v_t(t,l) = 0$$

$$E = \frac{1}{2} \int_0^l (v_t^2 + a^2 v_x^2 + k^2 v^2) dx.$$

$$\frac{dE}{dt} = \int_0^l (v_t v_{tt} + a^2 v_x v_{xt} + k^2 v v_t) dx =$$

$$= \int_0^l a^2 (v_t v_{xx} + v_x v_{xt}) dx = a^2 \int_0^l \frac{\partial}{\partial x} (v_t v_x) dx =$$

$$= a^2 \left. \underbrace{v_t v_x}_0^l \right|_0^l = 0 \quad \left. \begin{matrix} \\ E(0) = 0 \end{matrix} \right\} \Rightarrow E \equiv 0 \Rightarrow v \equiv 0.$$

(N5)

$$\begin{cases} u_x + u_y = u + x \\ u(x, 0) = x^2 \quad (x > 0) \end{cases}$$

$$\sigma_0: \begin{cases} x_0(s) = s, y_0(s) = 0, u_0(s) = s^2 \end{cases}$$

$$\frac{y}{x} \Big|_0^1 = \frac{0}{s} \Big|_0^1 = s \neq 0 \quad \text{für } s > 0$$

$$\begin{cases} x_\tau = 1 & x|_{\tau=0} = s \\ y_\tau = x & y|_{\tau=0} = 0 \\ u_\tau = u + x & u|_{\tau=0} = s^2 \end{cases}$$

$$x = \tau + s \quad y_\tau = \tau + s \Rightarrow y = \frac{\tau^2}{2} + s\tau$$

$$u_\tau = u + \tau + s$$

$$u = c(s)e^\tau + A(s)\tau + B(s)$$

$$A = \underline{A}\tau + \underline{B} + \underline{\tau} + \underline{s} \Rightarrow \begin{cases} A = -1 \\ A = B + s \\ B = -1 - s \end{cases}$$

$$u = c(s)e^\tau - \tau - 1 - s$$

$$\tau=0 \rightarrow c(s) - 1 - s = s^2 \\ c(s) = s^2 + s + 1$$

$$\begin{cases} x = \tau + s \\ y = \frac{\tau^2}{2} + s\tau \end{cases}$$

$$y = \frac{1}{2}(\tau + s)^2 - \frac{s^2}{2} = \frac{1}{2}x^2 - \frac{s^2}{2}$$

$$s^2 = x^2 - 2y \quad s = +\sqrt{x^2 - 2y}$$

$$\boxed{y < \frac{x^2}{2}}$$

$$\tau = x - s = x - \sqrt{x^2 - 2y}$$

$$u = (s^2 + s + 1)e^\tau - \tau - 1 - s = (x^2 - 2y + \sqrt{x^2 - 2y} + 1)e^{x - \sqrt{x^2 - 2y}} - x + 1$$